PARAMETRIC STUDY OF A MODIFIED PANEL METHOD IN APPLICATION TO THE SHIP-TO-SHIP HYDRODYNAMIC INTERACTION

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SUMMARY

Properties of a new modification of the potential flow algorithm developed by the authors and called “dihedral panel method” are studied in application to the ship-to-ship interaction problem. The method uses quadrilateral dihedral panels with constant source density distributed. The non-penetration condition is satisfied in the integral sense over each panel using Gauss cubature formulae with various numbers of nodes. Numerical investigation was carried out for the case of a parallel overtaking manoeuvre modelled as a kinematic pseudo-simulation with output of the surge and sway forces and of the yaw moment. Responses for Gauss formulae with 1, 4 and 7 nodes per any triangular subpanel were compared with those obtained with the classic Hess and Smith algorithm demonstrating, at equal overall number of panels, better accuracy of the new method especially when an increased number of Gauss nodes is used.

NOMENCLATURE

\[ \begin{align*}
L & \quad \text{Ship length, m} \\
M & \quad \text{“Field” point} \\
n & \quad \text{Outer unity normal} \\
N & \quad \text{Yaw moment, kNm} \\
p & \quad \text{Pressure, Pa} \\
p_i & \quad \text{Source point} \\
r & \quad \text{Angular velocity of yaw, rad/s; or distance, m; or Gauss formula order} \\
S & \quad \text{Wetted surface, m}^2 \\
t & \quad \text{Time, s} \\
V & \quad \text{Local velocity of surface point, m/s} \\
V_{Oi} & \quad \text{Velocity of origin of } i \text{ th body, m/s} \\
V_i & \quad \text{Induction velocity, m/s} \\
x, y, z & \quad \text{Coordinates in body frame, m} \\
X & \quad \text{Surge force, kN} \\
Y & \quad \text{Sway force, kN} \\
\rho & \quad \text{Density of water, t/m}^3 \\
\sigma & \quad \text{Single layer density, m/s} \\
\xi, \eta, \zeta & \quad \text{Coordinates in fixed frame, m} \\
\phi & \quad \text{Velocity potential, m}^2/\text{s} \\
\end{align*} \]

1 INTRODUCTION

Ability to predict hydrodynamic interaction effects occurring during manoeuvring of surface displacement ships is rather important for adequate modelling of the ship’s reactions in the bridge simulators and importance of the latter for appropriate training of human operators necessary for safe navigation is evident. Importance of mathematical modelling of this kind of forces is especially high due to the simple fact that full-scale training of this kind is impossible because of safety and economic considerations.

There were many publications on hydrodynamic interaction and a rather comprehensive review can be found in [9]. Regarding some later developments, the perfect-fluid formulation which includes wave effects including those stemming from oncoming sea waves was proposed and handled by Yuan et al. [11]. Also, direct application of CFD methods for RANS equations is becoming more and more popular, see [3] as an example.

As dangerously close manoeuvres are mostly performed in slow speed, it is often acceptable to exploit the so-called Havelock hypothesis [1] stating that the hydrodynamic interaction is mainly caused by inertial hydrodynamic loads rather reliably estimated within the double-body potential flow model. Of course, the Havelock hypothesis does not hold when the velocities of the interacting ships are not sufficiently low and the wavemaking effects may become tangible especially in shallow water [5]. It is rather difficult to establish exact limits of the applicability of the waveless flow model as, for instance, the Froude number can be based on various linear distances even in the deep water case. In particular, if the Froude number based on the ship length is quite small (i.e. of the order 0.05–0.1) it may become quite large if one of the interacting ships is crabbing, which is possible when e.g. a tractor tug is interacting with a large assisted vessel, and its breadth must be considered as the characteristic length. Moreover, the characteristic length can be based on the distance between the ship hulls and indeed definite influence of the free-surface effects is always observed when the lateral clearance is of the order of 1m in full scale.

Viscosity is in general less important as viscous effects are much more localized but these also can be expected to be significant at large drift angles when developed separation of the flow happens but no definite conclusions on this matter can be drawn at present. However, in spite of the mentioned limitations, the double-body potential flow interaction model often gives reasonable predictions and is unique from the viewpoint of absence of kinematic limitations i.e. it can be applied online at any mutual position and motion of the interaction bodies. The “curse of dimension” associated with the interaction problem is not always well understood but becomes evident in view of the fact that for a system of two unconnected bodies in 2D motion the overall number of state variables completely defining their position and
motion is 12 of which only 3, i.e. position and heading of one of the bodies, will not affect the hydrodynamic interaction loads. The simple fact that the interaction loads depend on 9 independent kinematical parameters practically exclude any possibility of preliminary computations or experiments [10] which would result in a sufficiently complete database for further online estimation of interaction forces and moments. All existing methods of this kind are based on incomplete experimental designs and cannot supply credible predictions in all situations. At the same time, the double-body potential flow model can be applied in online simulations with sufficient speed and without necessity of any preliminary computations. During last several years such a model was developed by the authors and under their supervision at the Centre for Marine Technology and Ocean Engineering at the University of Lisbon [6–9], [12–16].

This model was based from the beginning on the well-known Hess and Smith panel method [2] and the primary in-house code was developed in Fortran 90 for the case of deep water or shallow water with constant depth [6]. The number of arbitrarily moving interacting bodies was also arbitrary although most of the computations were carried out for two interacting ships. The Fortran version was later extended to embrace the case of uneven bottom with arbitrary bathymetry [12–16]. At the same time, to facilitate fusion with the offline manoeuvring simulation program [7], the version only applicable to a flat seabed was recoded in C++ and that code was later extended to include propellers modelled with disks of sinks [8].

While in general the codes based on the Hess and Smith method produced quite satisfactory results, they showed also some visible uncertainty in predicting the surge interaction force. Such an imperfection of the Hess and Smith panel method and results of its application to the prediction of interaction forces and moments obtained with various numbers of Gauss nodes on each subpanel compared also with results obtained with the classic Hess and Smith method.

2 PROBLEM FORMULATION AND MAIN RELATIONS

2.1 FORMULATION

The general formulation of the interaction problem is identical to that already presented in earlier publications by the authors and will only be briefly outlined here.

1. Unbounded perfect fluid is considered containing the plane \( O\hat{\xi}\hat{\eta} \) is considered to which the axis \( O\hat{\zeta} \) is perpendicular and the \( \hat{\xi}, \hat{\eta} \) -axes form a right-hand Cartesian frame fixed in space. As the gravity is not involved, the orientation of the frame can be arbitrary but in application to surface ships it is natural to assume that the \( \hat{\zeta} \)-axis is oriented vertically downwards and its positive half corresponds to the actual water volume.

2. Present are \( N \) arbitrary moving in the horizontal plane doubled bodies with wetted surfaces \( S_i, i = 0, \ldots, N-1 \) all symmetric with respect to the plane \( O\hat{\xi}\hat{\eta} \) intersecting them along the waterlines.

3. A body frame \( C_i x_i y_i z_i \) is associated with each \( S_i \) so that the axes \( C_i x_i, C_i y_i \) remain always parallel to \( O\hat{\zeta} \) and the planes \( C_i x_i, C_i y_i \) coincide with \( O\hat{\xi}\hat{\eta} \). In the case of a ship hull each axis \( C_i x_i \) lies in the centerplane of the hull and is directed from stern to bow while the axis \( C_i y_i \) is directed to the starboard.

4. The instantaneous position of each body is described by the position vector \( r_i \) connecting \( O \) with \( C_i \) and its motion—with the velocity \( V_{ci} \) and the angular velocity of yaw \( r_i \).

It is assumed that the flow is completely described by the absolute velocity potential \( \phi(\hat{\xi},\hat{\eta},\hat{\zeta},t) \) such as \( \Delta \phi = 0 \) over all the fluid volume; \( \frac{\partial \phi}{\partial \zeta} = 0 \) on the plane \( O\hat{\xi}\hat{\eta} \);
\[ \frac{\partial \phi}{\partial n} = \mathbf{V} \cdot \mathbf{n} \] on \( S_i \), where \( \mathbf{n} \) is the outer unity normal, and \( \mathbf{V} \) is the local velocity of a point on \( S_i \) depending on \( \mathbf{V}_o \) and \( r \). When the flow potential is known, the induced velocity is computed as \( \mathbf{V}_i = \nabla \phi \).

2.2 SOLVING EQUATIONS

The primary integral equation for the source (single layer) density \( \sigma \) is:

\[
2\pi \sigma(M) + \int_{S^+} \sigma(P) \frac{\partial G(M, P)}{\partial n} dS(P) = f(M), \quad (1)
\]

where \( M(\xi, \eta, \zeta) \) and \( P(\xi', \eta', \zeta') \) are the points on \( S^+ \), which is the part of \( S \) for \( \zeta > 0 \);

\[
f(M) = \mathbf{V}(M) \cdot \mathbf{n}(M) \] and the Green function is

\[
G(M, P) = \frac{1}{r} + \frac{1}{r'}, \quad (2)
\]

where

\[
r = \sqrt{(\xi - \xi')^2 + (\eta - \eta')^2 + (\zeta - \zeta')^2}. \quad (3)
\]

Collocation methods solving the equation (1) presume the following steps:

1. The actual wetted surface \( S^+ \) is approximated with some surface \( \hat{S} \) which can be easily partitioned into \( n \) non-intersecting panels \( S_i \): \( \hat{S} = \bigcup_{i=0}^{n-1} S_i \). In the case of the Hess and Smith method, first, the panels are formed and then they are united into \( \hat{S} \) representing a set of not necessarily connected flat quadrilaterals. In the dihedral method \( S \) is an inscribed polyhedron whose facets are organized in pairs forming quadrilateral dihedral panels \( S_i \).

2. On each panel the source density is approximated with some chosen shape functions depending on a number of parameters. In the both Hess and Smith and dihedral methods a 1-parameter constant density distribution is assumed.

3. Each panel serves as a platform for satisfying discretely the equation (1). In the Hess and Smith case it is satisfied locally at one control point (usually the centroid) per panel. In the dihedral method each panel has two different normals and it is not possible to keep the same approach. The non-penetration condition is then satisfied in the integral sense for the whole panel.

As result, in the dihedral method the equation (1) can be re-written in the following semi-discretized form:

\[
2\pi \sigma_i + \sum_{j=0}^{n-1} \sigma_j \int_{S_i} dS(M) \int_{S_j} \frac{\partial G(M, P)}{\partial n_M} dS(P)
\]

\[= \int_{S_j} f(M) dS(M), \quad M \in S_j, \quad i = 0, ..., n-1. \quad (4)
\]

The set above must be solved with respect to the densities \( \sigma_j \). After that, the induced velocities and the potential can be found as

\[
\mathbf{V}_i(M) = \sum_{j=0}^{n-1} \sigma_j \int_{S_j} \nabla \phi G(M, P) dS(P),
\]

\[\phi(M) = \sum_{j=0}^{n-1} \sigma_j \int_{S_j} G(M, P) dS(P). \quad (5)
\]

The pressure can then be calculated with the Bernoulli integral:

\[
p(M) = \rho \left[ -\frac{\Delta \phi(M)}{\Delta t} - \frac{1}{2} \left( \mathbf{V}^2(M) - \mathbf{V}_p^2(M) \right) \right], \quad (6)
\]

where \( \mathbf{V}_p = \mathbf{V}_i - \mathbf{V}_o \), and the force and moment acting on a body are:

\[
\mathbf{F} = -\sum_{k} \rho dS, \quad \mathbf{M} = -\sum_{k} \rho \mathbf{r} \times dS, \quad (7)
\]

where summations are only performed over the panels belonging to the body in concern.

All integrals over the panel \( S_j \) in the formulae above are calculated analytically using the formulae suggested by Hess and Smith with appropriate asymptotic simplifications at larger distances while the integrals over \( S_i \) or \( S_k \) are computed numerically, separately for each subpanel, with the Gauss integration formulae for triangles [17].

For any suitable function \( g(M) \) the integral over each subpanel \( S_k \) is represented as

\[
\int_{S_k} g(M) dS(M) \approx \sum_{i=1}^{m} w_i g(M_i), \quad (8)
\]

where \( m \) is the number of Gauss nodes \( M_i \) and \( w_i \) are the corresponding weights. Correspondence between the order of the Gauss formula \( r \) and the number of nodes is given by:

\[
r \quad 1 \quad 2 \quad 3 \quad 5 \quad 7 \quad m \quad 1 \quad 3 \quad 4 \quad 7
\]

The nodes of the second-order formula are located not inside the triangle but on its sides which makes this case unsuitable for the method applied. Also, it is clear that
the first-order Gauss formula with the node at the centroid of the triangle and with unity weight is nothing else then application of the average value theorem.

3 NUMERICAL INVESTIGATION

3.1 OBJECTIVES AND SCENARIO

The aim of the present study is testing the new potential flow dihedral panel method in application to the ship-to-ship interaction in overtaking manoeuvre using Gauss integration of various order. Also, performance of the new algorithm should be compared with the already well validated Hess and Smith method.

The scenario presumes kinematical simulation of the parallel motion of two identical vessels. The hull form corresponds to the “tanker” shape used in [9] was taken as basis but transformed to match the particulars of the S-175 container ship: \( L = 175m, B = 25.4m, T = 9.5m, V = 40842.6m^3 \). The lateral distance between the centerplanes remained constant and equal to 38m which corresponds to the distance between the sidewalls 12.6m. The overtaking ship (Ship 1) was advancing with 6kn while the target ship to be overtaken (Ship 2) had the speed of 4kn. The overtaking simulation started when Ship 1 was 300m behind Ship 2 and ended when it was 300m ahead. The output was represented by time histories of the forces of surge, sway and yaw represented, however as functions of the relative longitudinal shift

\[ \xi' = \frac{2(\xi_{c1} - \xi_{c2})}{L}. \]

3.2 GRID OF PANELS

All computations were performed with 2 grids: (1) coarse grid with 172 panels per hull and (2) a fine grid with 558 panels per hull. The panelled hull is shown in Figure 1.

In addition, the computations with the Hess and Smith method were carried out for even finer grid with 1258 panels per hull.

It can be seen that the grids are not perfect in the sense that they do not represent a polyhedron without gaps. This is caused by the fact that the initial set of the hull offsets was subdivided into 5 sub-bodies with different number of contour points on each of them. This inequality was kept in the transformed hull representation. Although it can be noticed that some panels are definitely non-plane, their dihedral nature is not clearly demonstrated because of absence of the dividing diagonals on the sketch.

3.3 RESULTS

Numerical results in form of responses for the surge and sway interaction forces and for the yaw moment are shown in Figures 2 and 3 for the coarse grid and in Figures 4–5 for the finer grid.

Besides the responses obtained with the dihedral code with various order of the Gauss integration formulae, every plot contains also the response obtained with the classic Hess and Smith algorithm which had been validated by the authors earlier [6], [9]. In general, it must be

![Figure 1. Panel grids used in computations](image-url)
Figure 2. Interaction forces and moment responses for Ship 1 and coarse grid

Figure 3. Interaction forces and moment responses for Ship 2 and coarse grid
Figure 4. Interaction forces and moment responses for Ship 1 and fine grid

Figure 5. Interaction forces and moment responses for Ship 2 and fine grid
understood that no one of the shown responses represents “true” or “exact” values although some judgement can be made. The most pronounced influence of the collocation method is observed for the surge interaction force, especially on the overtaking vessel. Here it is evident that the results provided by the Hess and Smith method certainly are not dependable as the surge interaction force must change its sign in course of the overtaking manoeuvre and with the Hess and Smith method it does not happen at all. The situation is already substantially improved with the dihedral first-order variant but seemingly most consistent results are obtained with 3rd and 5th order formulae.

The difference between the results obtained with various methods is much smaller for the sway force and yaw moment especially for Ship 2 where it can be practically neglected. For Ship 1, however, it is possible to note that the peak values (both global and local) depend on the method non-negligibly: the relative difference between the peak values of the suction sway force reaches more than 25 percent and even more than 100% for the initial repulsion peak. This deserves some attention as the estimates obtained with simpler methods are non-conservative.

Differences in the integrated loads obviously are caused by variations in the pressure distribution as can be illustrated by Figure 6 where snapshots of this distribution are shown for the Hess–Smith and dihedral methods. Although the pressure differences may seem insignificant, they are quite sufficient to produce significant difference in the estimated surge forces.

As could be expected, the influence of the method and of the Gauss order becomes much weaker when a finer grid is used although this influence is still significant for the surge force. Considering the trends in the behaviour of the data it can be concluded that even with the finest grid the accuracy of the Hess and Smith method is comparable with that of the dihedral method with coarser grids and 1st-order Gauss integration.

At the same time, it was noticed that the dihedral method is substantially slower at a given number of panels as the necessity of at least two computations of normal components of the induced velocities for each panel instead of only one required by the Hess and Smith method increases accordingly the time required for formation of the induction matrix. In the case of the 3rd and 5th-order Gauss scheme the corresponding time augmentation factor becomes 8 and 14 respectively.

4 CONCLUSIONS

The following conclusions can be drawn on the basis of the study performed:

1. The new variant of the potential flow algorithm based on dihedral panels has confirmed its applicability for studying ship-to-ship interaction problems.
2. Comparison of the numerical results obtained with the new method with those produced by the classic Hess and Smith algorithm has demonstrated potential superiority of the former in terms of accuracy, especially at small number of panels.
3. The surge interaction force turned out the most sensitive to the method, integration parameters and number of panels in the grid, so that application of the Hess and Smith algorithm can even result in qualitatively wrong estimates.
4. At the same time, the dihedral method may at present seem too slow for online real-time simulations and can only be immediately recommended for benchmark and validation computations.

Figure 6. Pressure distribution: top – Hess and Smith method, bottom – 5-order dihedral method; the overtaking vessel is on the left
Regarding the last conclusion, the method has some reserves for increasing its speed. In particular, integrated induction on each subpanel from distant panels represented asymptotically by point sources can be computed not with the Gauss scheme but using analytic formulae proposed by Söding [4] which not only can promise faster computation but also somewhat better accuracy.

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6 REFERENCES


7 AUTHORS’ BIOGRAPHIES

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