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Challenges in Geotechnical Reliability Based Design

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ABSTRACT: The author has been proposing a reliability based design (RBD) scheme for practicing geotechnical engineers. The essence of the proposed scheme is the separation of the geotechnical design part from the uncertainty analysis part in geotechnical RBD. In this way, practical engineers are able to perform RBD in a more comfortable way compared to the traditional RBD procedure. Results of RBD on some structures are presented in this paper to highlight the characteristics of the geotechnical RBD. Based on the results, some discussions are made to identify the major issues geotechnical RBD is facing. It is concluded that spatial variability of soil properties is only one of the sources of uncertainty. In many design problems, statistical estimation error, design calculation model error and transformation error associated with estimating soil parameters (*e.g.* friction angle) from the measured quantities (*e.g.* SPT N-values) have higher uncertainty. It is important to recognize these aspects in developing the geotechnical RBD to the next and the higher stage.

Keywords: Reliability base design, Statistical analysis, Random field, Geotechnical design

1 INTRODUCTION

Needs for carrying out reliability analysis (RA) for complex geotechnical design problems are increasing due to the introduction of the limit state design worldwide. On the other hand, in the current practical design of geotechnical structures, many sophisticated calculation methods, *e.g.* commercially available user friendly FEM programs *etc.*, are employed. These methods become more and more user friendly, and can be used with very small efforts for preparing input data and summarizing calculation results.

It takes quite amount of effort for people to combine these programs with RBD. To connect these design tools to RBD tools is not an easy task. Furthermore, to understand and become proficient with these RBD tools need quite amount of time and efforts.

Considering these situations, the author has been proposing a new RBD scheme for geotechnical design. The essence of the issue that makes geotechnical engineers difficult to practice RBD, as I see, is the mixing of geotechnical design tools with RBD tools in the existing RBD procedure. Furthermore, if we mix them together, one tends to lose intuitive understanding to the design problem at hand, which is very important in geotechnical design to make engineering judgements in the course of design.

The RBD scheme we are proposing here attempts to take into account of characteristics of geotechnical design as much as possible. The scheme is for geotechnical engineers who are proficient in various aspects of geotechnical design but not very familiar with RBD tools.

In this presentation, only the overall outline of the scheme is described. The concept of the methodology is more focused, but details are not very well explained. For the details of the methodology, readers are requested to see papers listed in the reference list. I

It is also a purpose of this paper to identify the major sources of uncertainty that are important in geotechnical RBD through four examples. It may be generally recognized that the spatial variability of soil properties is the most important source of uncertainty in geotechnical RBD. However, from the results presented in this paper, it is only one of the sources of uncertainty. In many design problems, statistical estimation error, design calculation model error and transformation error associated with estimating soil parameters (*e.g.* friction angle) from the measured quantities (*e.g.* SPT N-values) exhibit higher uncertainty.

2 PROPOSED SCHEME FOR GEOTECHNICAL RBD

2.1 *Outline of the Scheme*

The basic concept of the scheme is illustrated in Figure 1. The scheme starts with the basic variables. The basic variables include all variables concerned in design: Various actions, environmental effects, geotechnical parameters, other material properties, configuration and size of structure and supporting ground, boundary conditions are all included in the basic variables.

The scheme proposed here is separated to three parts: (I) geotechnical design, (II) uncertainty analysis of basic variables and (III) reliability assessment.

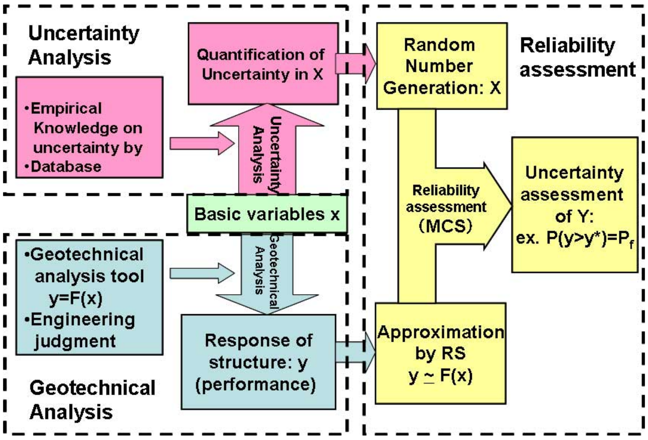


Figure 1. Proposed RBD scheme

Geotechnical design, (I), is almost the same as usual design procedure for geotechnical structures. The response of the structure (safety factor etc.), y, is obtained from the basic variables, x, by the design calculations. In some cases y can be related to x by a relatively simple performance function. In other cases, the response surface (RS) method can be used to relate x to y by a regression analysis (Box & Drepper, 1987).

The uncertainty analysis of basic variables, (II), is the main part of RA. Statistical analysis plays the major role in this analysis. Some basic knowledge on probability theory and statistical analysis are required in this step. Much accumulated knowledge in geotechnical reliability design is employed in carrying out the analyses. The author is recommending use of R language in this step which can make the analysis very easy and efficient. Actually, all the uncertainty analyses and reliability analyses presented in this paper are done by R.

The reliability assessment, (III), is carried out based on the results of the uncertainty analyses and the performance function by simple Monte Carlo simulation (MCS). MCS is recommended due to the following reasons:

(1) MCS is a very straight forward reliability analysis procedure that does not require detailed background knowledge of the probability theory in most cases.

(2) Since the performance function (or the response surface) introduced in the RBD calculation is simple, they do not require much calculation time. Therefore, it is not necessary to introduce any sophisticated reliability analysis methods that save the number of calculations of the performance function.

2.2 *Classification of Uncertainties and Their Treatment*

A classifications of the uncertainties encountered in geotechnical RBD is given in this section together with brief description how they are generally treated in this study. Not all the uncertainties classified here need to be considered in all geotechnical RBD. They need to be chosen according to the needs and the conditions of each design problem. It is assumed in this paper that the uncertainties on actions are separately given.

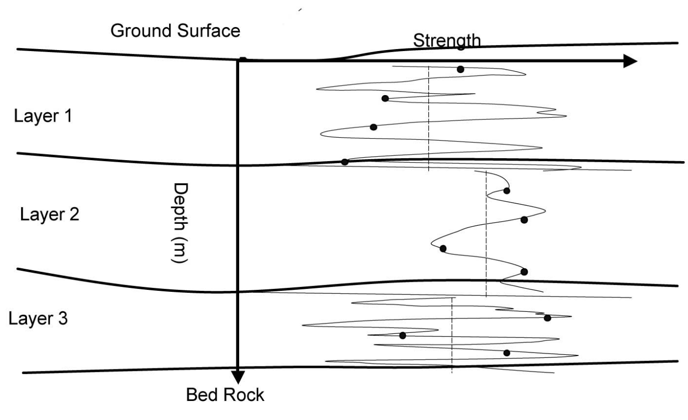


Figure 2. Modelling soil profile by random field

2.2.1 *Measurement error*

It is error involved in measurements in investigations and tests. In the traditional error theory, the measurement error is assumed to independently and identically follow a normal distribution. On the other hand, this error may include biases caused by the equipments and the operators. However, this error is usually ignored in geotechnical RBD because the influence of it may not be large compared to other uncertainty sources. Furthermore, it is very difficult to separate measurement error from observed spatial variability. Thus, the observed spatial variability may also include the measurement error.

2.2.2 *Spatial variability:*

The spatial variability of geologically identical geotechnical parameters are conveniently (or fictitiously) modelled by the random field (RF) theory in geotechnical RBD. The geotechnical parameters are determined by themselves and already exist at each location. However, because of our ignorance (*i.e.* lack of knowledge or Epistemic uncertainty (Baecher and Christian, 2003)), we model them using RF for our convenience. It is a simplification and an idealization of the problem.

It is a general procedure to model soil profile that belongs to a geologically identical layer by superposition of the trend and the random components (Lumb, 1974; Vanmarcke, 1977; Matsuo, 1984; Phoon and Kulhawy, 1999a *etc.*). The trend component gives a general overall behavior of the soil property, whereas the random component describes discrepancy of each observation from the trend (Figure 2):

$$z(x) = f(x|\beta) + \varepsilon(x|\sigma,\theta) \qquad \varepsilon \sim N(0,\sigma^2,\theta) \tag{1}$$

where

x : spatial coordinate vector (x_1, x_2, x_3), $f(x|\beta)$: a function showing the trend component

β : trend parameter vector, $\varepsilon(x|\sigma,\theta)$: the random component

σ^2 : variance of the random filed, θ : autocorrelation distance vector $\theta = (\theta v, \theta_h)$

θ_v : autocorrelation distance in vertical direction, θ_h : autocorrelation distance in horizontal direction

The random component $\varepsilon(\boldsymbol{x})$ is assumed to consist a stationary (=homogeneous) random filed (RF). The stationarity assumed in this study is that in a weak sense, which implies the RF can be described by the following three statistics:

$$\begin{aligned} \mu_z(x_1, x_2, x_3) &= 0 \\ \sigma_z^{\ 2}(x_1, x_2, x_3) &= \sigma^{\ 2} \\ \rho_z(x_1, x_2, x_3) &= \rho(\Delta x_1, \Delta x_2, \Delta x_3) \end{aligned} \tag{2}$$

The first equation states that the mean is a constant, *i.e.* independent of the coordinate $\boldsymbol{x}=(x_1, x_2, x_3)$. In the present context, this mean value is assumed to be 0. The second equation expresses that the variance is also constant. Finally, the third equation states that the autocorrelation function is given not by the absolute coordinate but by the relative distance between the two coordinate positions.

In addition to the above assumptions, the form of autocorrelation function is specified in this study. Due to the deposition process of soil layers, it is generally assumed that autocorrelation structure for the horizontal direction, *i.e.* x_1 and x_2, and for the vertical, *i.e.* x_3, are different. We assume that the autocorrelation function has separable property as suggested by Vanmarcke (1977):

$$\rho_\varepsilon(\sqrt{\Delta x_1^{\ 2} + \Delta x_2^{\ 2}}, \Delta x_3) = \rho_{\varepsilon h}(\sqrt{\Delta x_1^{\ 2} + \Delta x_2^{\ 2}}) \cdot \rho_{\varepsilon v}(\Delta x_3) \tag{3}$$

The exponential type autocorrelation function is assumed in this study

The typical values of these statistics for various types of soil are summarized, for example, in Phoon and Kulhawy (1999a and 1999b).

2.2.3 *Statistical estimation error*

Errors associated with the estimation of parameters of RF are termed the statistical estimation error. It further includes estimation error for parameter values estimated at a certain point in space by, say, Kriging. RF theory is used as a platform to evaluate statistical estimation errors.

In evaluating statistical estimation error, the author believes it very important to distinguish between the two cases below (Honjo and Setiawan,2007; Honjo, 2008).

General Estimation*:* The relative position of investigation location and of a structure to be built is not taken into account in soil parameter estimation. For example, if a large container yard to be designed, the bearing capacity of the ground at an arbitrary location may be evaluated considering general property of ground condition obtained in the whole area.

Local Estimation*:* The relative position of investigation location and of a structure to be built is taken into account in soil parameter estimation. Therefore, there would be considerable reduction in the estimation error if the two locations are very close. A straightforward example of this case is that if one wants to design a foundation for a house and made a detailed soil investigation at the spot, one need to consider very little uncertainty to ground condition.

The situation described here as General and Local estimation are rather common situations encountered by geotechnical engineers. The engineers surely have treated these conditions in an implicit way, and modified their design. These are a part of so called *engineering judgement* in the traditional geotechnical engineering. The difference here is that we explicitly take into account these situations and try to quantify the uncertainty.

Honjo and Setiawan (2007) has given formulation for these two cases for a particular situation. Honjo (2008) has discussed this problem in connection with actual design. A recent paper by Honjo et al. (2011) gives a general formulation for the general estimation, which is employed in the examples of this paper as well. For the local estimation in this paper, block Kriging is employed (*e.g.* Wachernagel, 1998).

The author believes that a general statistical theory need to be developed for these two situations based on RF theory. It is like the normal population theory gives a general theory for the mathematical statistics. Although any real situation do not exactly satisfy the simplified and idealized assumptions made in the theory, it can contribute quite a lot to give a basic platform for the evaluation of the statistical estimation error in geotechnical parameter estimation and geotechnical RBD.

2.2.4 *Transformation error*

Errors associated with the transformation of measured geotechnical parameters by a soil investigation to geotechnical parameters used in the design calculation are termed transformation error. There are usually both biases and scatters in the transformations.

Readers will see the examples of the transformation errors in the examples of this paper. The most comprehensive reference for this problem is a manual provided by Kulhawy and Mayne (1990), which gives considerable amount of quantitative information on this problem.

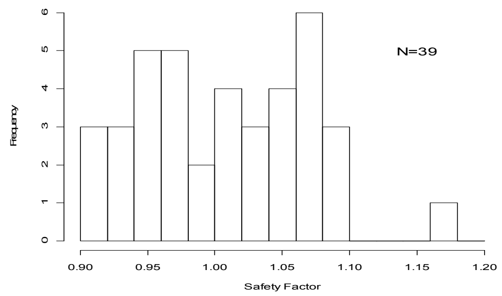


Figure 3. Error in Swedish circular slop analysis (Matsuo and Asaoka, 1976)

2.2.5 *Design calculation model error*

This is error associated with prediction capabilities of simplified and idealized design calculation models on the real phenomena. In geotechnical engineering, the tests and experiments closer to real structure scales (*e.g.* pile load tests, plate loading tests *etc.*) are more commonly performed, and many failure cases are available especially on earth structures such as embankments, cut slopes and excavations. These facts make it easier for us to evaluate the model errors in a quantitative manner in geotechnical design.

For example, the model error of the Swedish circular slip method in stability of embankment on soft cohesive soil is analyzed in detail by Wu and Kraft (1970) and Matsuo and Asaoka (1976). The latter has analyzed failed embankments on soft ground, and concluded that by the cancellations of many factors involved in the stability analysis, the final safety factors calculated follows an uniform distribution that lies between 0.9 and 1.1 (Figure 3). This conclusion is essen-

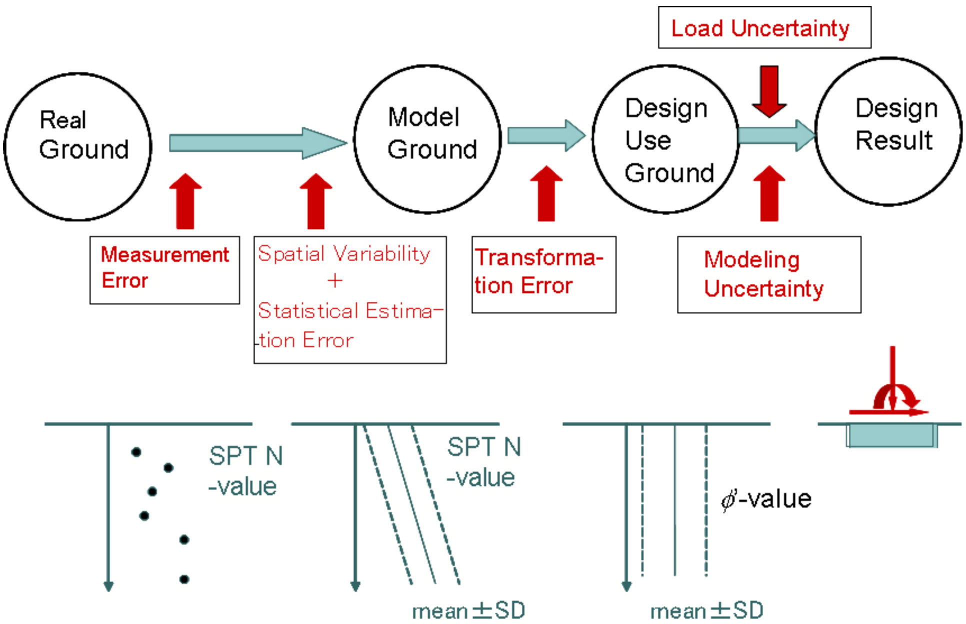


Figure 4. An example of a procedure for geotechnical RBD

tially in accordance with a comprehensive review on this problem by Wu (2009), where he stated that the combined uncertainty for limit equilibrium analysis with circular slip is estimated to be mean 1.0 (i.e. no bias) with COV 0.13-0.24.

By over viewing the uncertainties encountered in geotechnical design, most of uncertainty sources are Epistemic uncertainty (*i.e.* lack of knowledge) rather than Aleatory uncertainty (*i.e.* pure randomness) (Beacher and Christian, 2003). We are like playing cards with the ground where we peep through their cards by some investigations. (In this game, fortunately, the nature does not have any intention to circumvent us.)

An example of sequence of uncertainties entering into geotechnical RBD is illustrated in Figure 4.

2.3 *Local Average and Reliability Assessment*

There is a description on the characteristic value of a geotechnical parameter in Eurocode 7 (CEN,2004) as follows:

> *'The zone of ground governing the behaviour of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in situ test. Consequently the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value' (CEN EN1997-1, 2.4.5.2 (7)).*

The same fact has been pointed out much earlier by Vanmarcke (1977) that it is the local averages (LA) of soil properties that are important in controlling behaviour of geotechnical structures, such as piles, shallow foundations and slopes.

In geotechnical RBD, it is necessary to take the weighted average of geotechnical parameters to obtain the resistance. For example, the shaft resistance of a pile is integration of the soil strength along the pile shaft, resistance moment of a slip surface is integration of soil strength along the slip arc, and settlement of a pad foundation may be controlled by the average stiffness of a certain size of soil mass right under the foundation.

The local average (LA) of the geotechnical parameter for vertical direction over a length L is defined:

$$\overline{Z}_L = \frac{1}{L}\int_0^L Z(x)dx \tag{4}$$

It is apparent that the mean of the LA coincides with the original mean of the RF, μ. Furthermore, the variance reduction of the local average from the original variance of the RF has extensively studied by Vanmarcke (1977 and 1983), where he has derived so called the *variance function*, $\Gamma^2(L)$. If the autocorrelation function is of the exponential type, s_L^2, can be obtained by the variance function as,

$$s_L^2 = E\left[\left\{\frac{1}{\mathrm{L}}\int_0^L Z(x)dx - \mu\right\}^2\right] = \sigma^2\Gamma^2\left(\frac{L}{\theta}\right) = \sigma^2\left(\frac{\theta}{L}\right)^2\left[2\left(\frac{L}{\theta} - 1 + \exp\left(-\tfrac{L}{\theta}\right)\right)\right] \tag{5}$$

Vanmarke has further extended the theory to multidimensional space, and found that if the autocorrelation function is separable, the variance of local average over an area or a volume can be obtained by multiplying the variance functions for each dimension.

In this study, the resistance is calculated based on the local average of a certain soil mass that is controlling the behaviour of a geotechnical structure. Thus the uncertainty of resistance is a reflection of the variance of the local average of the geotechnical parameter.

3 GEOTECHNICAL RELIABILITY BASED DESIGN BY EXAMPLES

The proposed RBD scheme has been applied to several cases. 4 examples are chosen here to illustrate the procedure and highlight the characteristic of the method. Based on the results, some discussions are made to identify the major issues geotechnical RBD are challenged.

The first three examples are problems set by ETC10 for the purpose of a comparative study of the national annexes of Eurocode 7. The problems are relatively straight forward but not excessively simplified to lose the essence of real geotechnical design problems. Due to the limitation of the space, the details of RBD are not described. One should see Honjo et al. (2010, 2011) for the details.

The fourth problem is based on Otake et al. (2011) submitted to this conference. It is a reliability assessment of a 14 km long irrigation channel for liquefaction during expected Tokai-Tonankai earthquake. The difference between the general and the local estimation of the soil parameters on the results are emphasized.

3.1 *Pad foundation on sand (ETC10 EX2-1)*

3.1.1 *Problem description*

The problem is to determine the width of a square pad foundation on a uniform and very dense fine glacial outwash sand layer of 8 (m) thick on the underlying bedrock (Figure 5). It is requested that the settlement should be less than 25 (mm) (SLS) and stability should be secured (ULS). The design working life of the structure is 50 years.

It is specified that the pad foundation is to be built at embedded depth of 0.8 (m), and vertical permanent and variable loads of the characteristic values 1000 (kN) (excluding the weight of foundation) and 750 (kN) respectively are applied. The unit weight of the concrete is 25 (kN/m^3). No horizontal loading is applied.

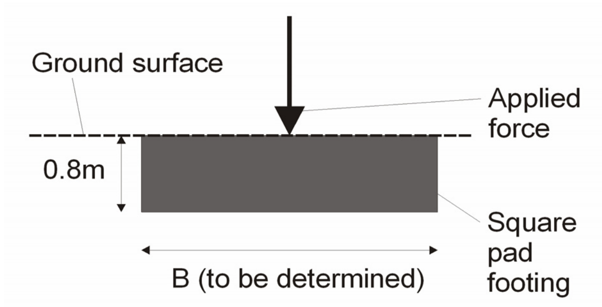


Figure 5. The pad foundation on sand

There are 4 CPT tests within 15 (m) radius from the point the pad foundation is to be constructed and digitized q_c and f_s values of 0.1 (m) interval are given to 8 (m) depth from the ground surface (Figure 6). The groundwater is 6 (m) below the ground surface. The unit weight of sand is 20 (kN/m^3).

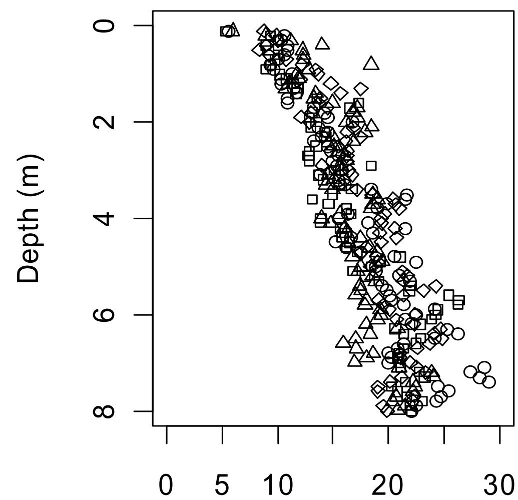


Figure 6. 4 CPT q_c results

3.1.2 *Uncertainty analysis*

There are two limits states to be examined: SLS where the settlement should be less than 25mm, and ULS where the stability should be secured.

For the SLS, the CPT q_c values are used to model the spatial variability of the ground. A linear model is used to describe the trend and the residuals follow a normal distribution. The vertical autocorrelation distance of 0.4 m is estimated. The horizontal autocorrelation distance of 4 m is assumed.

The general estimation is employed and estimation error is evaluated. Also reduction of the variance by taking the local average between the depth of 0.8 to 1.8 m is taken into account. The overall reduction of SD of CPT q_c value is estimated, where SD of 2.28 MPa reduced to 1.66 MPa.

The transformation of CPT q_c values to Yong's modulus is done considering the transformation error. The mean and SD of the error is estimated to be 1.14 and 0.94 respectively. This is considerably large error.

The uncertainty associated with the permanent and the variable loads are taken from Holicky et al. (2007). . These quantities are used in the code calibrations of the structural Eurocodes rather widely. The uncertainties evaluated are listed in Table 1 for SLS.

Table 1. List of basic variables for Ex.2-1 SLS settlement

Basic variables	Notation	mean	SD	Distribution type
Estimation error and local average variance of q_c	I_E is proportional to I_{qc}	qc=10.54+1.66x_3 (MPa)	7.2(MPa) COV=0.13(1) at z=1.5(m)	Normal
Transformation error on E' from q_c	δ_E	1.14	0.94	Lognormal
Permanent load	δ_{Gk}	1.0	0.1	Normal(2)
Variable load	δ_{Qk}	0.6	0.35x0.6=0.21	Gumbel distribution(2)

(Note 1) COV has been obtained by Eq.(3). (Note 2) Based on JCSS (2001) and Holicky et al. (2007).

Table 2. List of basic variables for Ex.2-1 ULS stability

Basic variables	Notation	Mean	SD	Distribution type
Spatial variability	ϕ'_{tc}	42.8 (degree)	0	Deterministic variable
Transformation error from q_c	ϕ'_{tc}	42.8 (degree)	2.8 (degree)	Normal
R_u model error	δ_{Ru}	0.894	0.257	Lognormal
Permanent action	δ_{Gk}	1.0	0.1	Normal
Variable action	δ_{Qk}	0.6	0.35x0.6=0.21	Gumbel distribution

For the ULS, the CPT q_c values are first converted to internal friction angle in a equation proposed by Kulhawy and Mayne (1990). The converted internal friction angle had very small variance, which made the spatial variability of this quantity null. The transformation error in this conversion is given in the same literature.

The model error in the bearing capacity calculation form the internal friction angle is obtained from a recent literature which compares the calculated values with the results of the plate loading test.

The evaluated uncertainties are listed in Table 2 for ULS.

3.1.3 *Geotechnical analysis and performance function*

As for SLS, 3D PLAXIS is used to obtain the relationship between the settlement and the foundation size, B at the mean values of Young's modulus and the loads. It is found that the settlement has a linear relationship with $log(B)$. Since the ground is assumed to be a elastic body, the settlement is doubled if Young's modulus is half or the load is doubled. These relationships are taken into account, and a performance function is obtained:

$$s = \frac{(17.0 - 9.73\log(B))}{I_E \cdot \delta_E}\left(\frac{\gamma \cdot D_f \cdot B^2 + G_k\delta_{Gk} + Q_k\delta_{Qk}}{\gamma \cdot D_f \cdot B^2 + G_k + Q_k}\right) = \frac{(17.0 - 9.73\log(B))}{I_E \cdot \delta_E}\left(\frac{20 \cdot B^2 + 1000\delta_{Gvk} + 750\delta_{Qvk}}{20 \cdot B^2 + 1750}\right) \tag{6}$$

The performance function for ULS is given as follows:

$$M = Ru(B,\ \phi'_{\mathrm{tc}}) \cdot \delta_{Ru} - G_k \cdot \delta_{Gk} - Q_k \cdot \delta_{Qk} \tag{7}$$

Where R_u is a classic bearing capacity formula, and M is the safety margin. The definitions of other notations are given in Table 2.

3.1.4 *Reliability assessment and results*

Simple Monte Carlo simulation is employed to carry out the reliability analysis. The uncertainty listed in Table 1 and Eq.(6) are used to evaluate the probability that the settlement exceeds 25 mm for SLS. The same procedure is taken to evaluate the failure probability of the pad foundation based on Table 2 and Eq.(7).

Figure 7 shows the results of MCS on ULS of the pad foundation. The MCS is repeated several times by removing each uncertainty sources to see the impact, which the results are also presented in the figure. The necessary width of the foundation based on the result for both SLS and ULS are presented in Table 4.

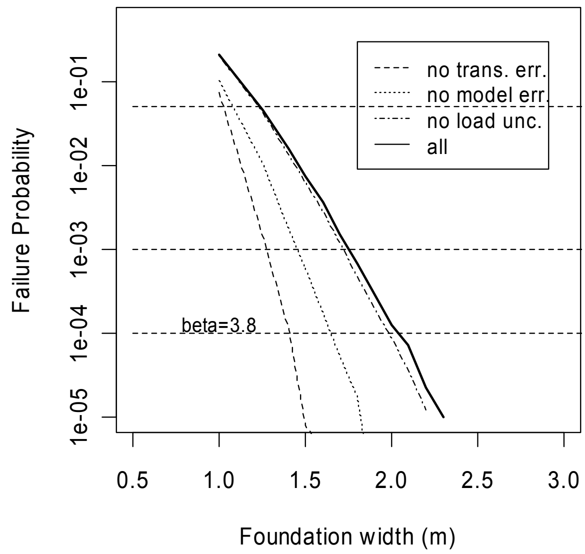


Figure 7. The results of MCS on the stability of the pad foundation.

Table 4. summary of the results for the pad foundation

Limit state	Target β for 50 years design working life. (P_f)	Required width (m)
S.L.S.(s < 25 mm)	1.5 (0.067)	B > 2.4 (m)
U.L.S.(stability)	3.8 (10^{-4})	B > 2.2 (m)

Table 5(a) rate of contribution of each uncertainty source for settlement analysis (B=1.0 m)

Uncertainty sources	All uncertainties Considered	transformation error	spatial variability	load uncertainty
β and β_{-i}	0.595	2.804	0.623	0.590
contribution	100 %	92 %	8 %	0 %

Table 5(b) rate of contribution of each uncertainty source for stability analysis (B=1.0 m)

Uncertainty sources	All uncertainties considered	transformation error	model error	load uncertainty
β and β_{-i}	0.811	1.443	1.261	0.840
contribution	100 %	51 %	44 %	5 %

The influence of each uncertainty source is listed in Table 5(a) and (b). An approximation method to estimate the contribution of each factor is explained in Appendix A. A discussion will be made on these results in the latter section of this paper.

3.2 *Pile foundation in sand (ETC10 EX2-6)*

3.2.1 *Problem description*

The problem is to determine pile length L (m) of a pile foundation of a building. The pile is a bored pile (D = 0.45 m) embedded entirely in a medium dense to dense sand spaced at 2.0 (m) interval (Figure 8). Each pile carries a characteristic vertical permanent load of 300 (kN) and a characteristic vertical variable load of 150 (kN). The soil profile includes Pleistocene fine and medium sand covered by Holocene layers of loose sand, soft clay, and peat (see Table 6).

There is one CPT (q_c measurement only) close to the spot to determine the strength profile of the ground. The water table is about 1.4 (m) below the ground level.

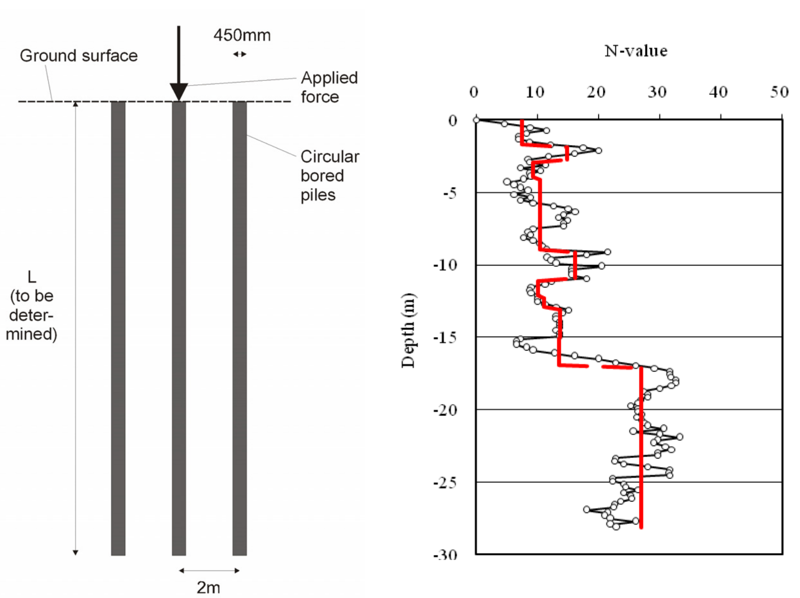


Figure 8. The configuration of the bored pile and soil profile by SPT N-value transformed from CPT q_c value.

3.2.2 *Uncertainty analysis*

The bearing capacity estimation equation for pile the author used is based on SPT N-value. Thus CPT q_c value is converted to SPT N-value by a equation given in Kulhawy and Mayne (1990). This transformation equation has the transformation error of mean 1, COV 1.03 and follows a log normal distribution.

Since there is only one CPT test result, and the layer have quite complex structure, the soil profile is modeled by 10 layers and the mean and the SD of each layer is estimated from the CPT test result.

The model error in the empirical bearing capacity estimation equation used widely in Japan is obtained from a literature which is based on the results of a number of pile loading test results. The model error for estimating shaft resistance and pile tip resistance are given separately as shown in Table 6.

The uncertainties on permanent and variable loads are taken from the same literature used in the previous example, and given in Table 6.

Table 6. Statistical properties of the basic variables

Basic variables		Notations	Mean	SD	Distribution	Note
uncertainty on characteristic value of permanent load		δ_{Gk}	1.0	0.1	Normal	G_k = 300 (kN) (1)
uncertainty of characteristic value of variable load		δ_{Qk}	0.6	0.21	Gumbel	Q_k = 150 (kN) (1)
uncertainty of estimating pile shaft resistance		δ_f	1.07	0.492	Log Normal	Okahara *et.al* (1991)
uncertainty of estimating pile tip resistance		δ_{qd}	1.12	0.706	Log Normal	Okahara *et.al* (1991)
uncertainty of transformation from CPT q_c to N		δ_t	1	1.03	Log Normal	Kulhawy & Mayne (1990)
Layer 1	Clay with sand seams	$N1^{(2)}$	7.51	3.66	Normal	Depth 0.0 - 1.9 (m)
Layer 2	Fine sand	$N2^{(2)}$	14.80	4.58	Normal	Depth 1.9 - 2.9 (m)
Layer 3	Clay with sand seams	$N3^{(2)}$	9.24	1.44	Normal	Depth 2.9 - 4.0 (m)
Layer 4	Fine silty sand	$N4^{(2)}$	10.33	3.22	Normal	Depth 4.0 - 9.0 (m)
Layer 5	Fine silty sand with clay & peat seams	$N5^{(2)}$	16.17	3.31	Normal	Depth 9.0 - 11.0 (m)
Layer 6	Clay with sand seams	$N5^{(2)}$	10.08	1.45	Normal	Depth 11.0 - 12.3 (m)
Layer 7	Clay with peat seams	$N7^{(2)}$	11.14	1.51	Normal	Depth 12.3 - 13.0 (m)
Layer 8	Clay with peat seams	$N8^{(2)}$	13.68	0.54	Normal	Depth 13.0 - 15.0 (m)
Layer 9	Fine sand	$N9^{(2)}$	13.56	7.24	Normal	Depth 15.0 - 17.0 (m)
Layer 10	Fine sand	$N10^{(2)}$	26.98	3.71	Normal	Depth 17.0 (m) below

（Note 1）Based on Holicky, M, J. Markova and H. Gulvanessian (2007). (Note 2) Unit of soil layers are SPT N-values

3.2.3 *Geotechnical analysis and performance function*

The performance function employed in this example is given as follows:

$$M = U\delta_f \sum_{i=1}^{n} \delta_{ti} f_i(\delta_t N_i) L_i + \delta_{qd} q_a(\delta_t N_n) A_p - \delta_{Gk} G_k - \delta_{Qk} Q_k \tag{8}$$

where, U: perimeter of the pile (m), f_i: maximum shaft resistance of each soil layer (kN/m2), L_i: thickness of each soil layer (m), N: standard penetration test (SPT) blow count, q_d: ultimate pile tip resistance intensity per unit area (kN/m2), and other notations are listed in Table 6. The details of f_l and q_d is given in SHB (2002).

3.2.4 *Reliability assessment and results*

Monte Carlo simulation using R language is carried out for different pile length L (m) to obtain the reliability index (or probability of failure). In this analysis, the number of random numbers generated for each case is 500,000 sets. The obtained reliability index for different pile length is shown in Figure 9.

Since the case considered is the ultimate limit sate, the reliability index, β, of more than 3.8 may be required. The pile length of more than 18 (m) is necessary.

In order to investigate the contribution of each uncertainty sources, reliability analyses are carried out by removing each uncertainty source at a time. These results are shown in Figure 9 as well. The rate of contribution of each source is further presented in Table 7. The contributions are estimated based on the approximation method explained in Appendix A. The result of this table will be discussed later.

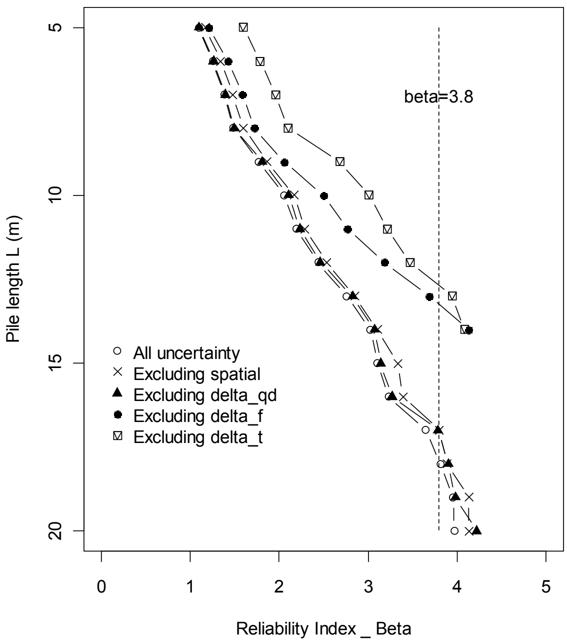
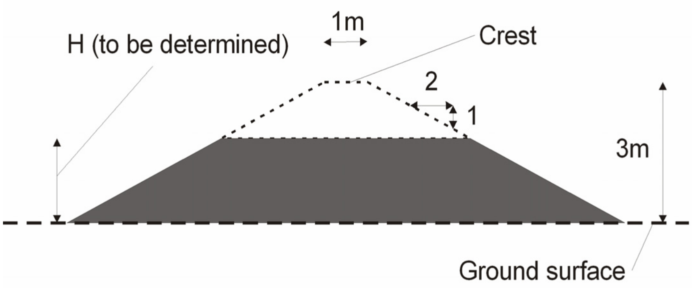


Figure 9. The results of MCS on the stability of the pile foundation.

Table 7. rate of contribution of each uncertainty source for a pile bearing capacity (at L=13 m)

Uncertainty sources	All uncertainty	Spatial variability	Pile tip resistance	Pile shaft resistance	Transformation error
β and β_{-i}	2.75	2.85	2.82	3.69	3.94
contribu-tion	100 %	6 %	5 %	41 %	48 %



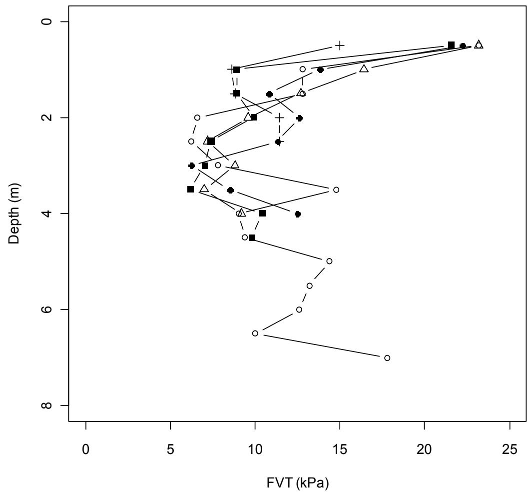


Figure 10. The configuration of an embankment on peat and the results of 5 FVT

3.3 *Embankment on peat ground*

3.3.1 *Problem description*

An embankment is to be designed on a soft peat ground whose final height should be 3 (m) above the ground surface (Figure 10). The problem here is to determine the first stage embankment height. The inclination of the embankment slope is 1:2, whereas the crest width 1 (m). The unit weight, γ, of the embankment soil is 19 (kN/m^3) and the friction angle ϕ'_k=32.5 (degree).

The ground surface is horizontal. The ground consists of a few dm of topsoil and normally consolidated clay (γ=18 (kN/m^3) and γ' = 9 (kN/m^3)) on a 3 to 7 (m) thick peat layer with γ' =2 (kN/m^3) overlaying Pleistocene sand of γ' =11 (kN/m^3) and ϕ'_k =35 (degree). 5 filed vane test (FVT) results are given whose testing interval is 0.5 (m) in the vertical direction and the length varies between 2.5 and 7.0 (m).

Only ultimate limit state needs to considered and no variable loads have to be taken into account.

3.3.2 *Uncertainty analysis*

The five FVT results are plotted in Figure 10. It is observed that s_u at surface layer of about 0.5 (m) is considerably larger than the bottom peat layer indicating different soil layer. It is determined to separate these data, and group them as topsoil. The trend component of the underneath peat layer is obtained as a quadratic curve, and the residual random component fits to a normal distribution with a constant variance of 2.40^2 (kPa^2).

The statistical estimation error for estimating the local average of peat layer is obtained, whose SD is estimated to be 0.528 (kPa), whereas the variance reduction by local averaging for 4 m depth makes SD of spatial variability to be 1.12 (kPa). The resulting SD for the local average of the peat strength is $\sqrt{0.528^2 + 1.12^2} = 1.24$ (kPa).

The uncertainty concerning the thickness of the top soil is introduced, so as the undrained shear strength, s_u. They are all listed in Table 8.

The design calculation model error is obtained based on Matsuo and Asaoka (1976), where an uniform distribution of [-0.1, 0.1] is introduced.

Table 8. Basic variables of embankment on peat

Basic variables	Notations	mean	SD	Distribution
Topsoil s_u	$s_{utopsoil}$ ($I_{topsoil}$)	21.04 (kPa) (1.0)	3.44 (0.163)	Normal
Peat s_u	s_{upeat} (I_{peat})	14.73-3.51z +0.536z^2 (kPa) (1.0)	1.20 (0.13)$^{(1)}$	Normal
Topsoil thickness	D_t	[0.5, 1.0] (m)		Uniform$^{(2)}$
Uncertainty of ϕ'=0 method	δ_{Fs}	[-0.1, 0.1]		Uniform$^{(3)}$
Unit weight of embankment	γ_f	19.0(kN/m^3)	—	Deterministic
Friction of embankment	ϕ_f	32.5 degree	—	Deterministic
Unit weight of topsoil	γ_c'	9.0(kN/m^3)	—	Deterministic
Unit weight of peat	γ_P'	2.0(kN/m^3)	—	Deterministic
Friction of sand	ϕ_s	35 degree	—	Deterministic
Unit weight of sand	γ_s'	11.0(kN/m^3)	—	Deterministic

(Note 1) s_{upeat} (at z=4.0(m)) = 14.73 - 3.5x4.0 + 0.53x4.0^2 = 9.27, COV=1.24/9.27=0.13

(Note 2) It is assumed that the boundary of the topsoil and the peat layer lies somewhere between z = 0.5 to 1.0 (m).

(Note 3) Based on Matsuo & Asaoka (1976).

3.3.3 *Geotechnical analysis and performance function*

A response surface (RS) that relates embankment height, h, s_u of the topsoil layer, s_u of the peat layer, the thickness of the topsoil, D_t, and the safety factor, F_s, is obtained by a regression analysis based on the results of the stability analysis of 75 combinations of these parameters. Swedish circular method is employed for the stability analysis. In order to make the response surface equation simple, s_u of the peat layer and the topsoil layer are normalized at their mean values

$$I_{peat} = s_u / (\text{mean of } s_u \text{ of the peat layer})$$
$$I_{topsoil} = s_u / (\text{mean of } s_u \text{ of the topsoil}) = s_u / 21.04 \quad (9)$$

Based on the obtained response surface, a performance function is obtained as follows:

$$Fs = 1.783 - 1.351\, h + 0.213\, h^2 + 1.156\, I_{peat} + 0.272\, I_{topsoil} + 0.091\, D_t + \delta_{Fs} \quad (10)$$

where the notations are given in Table 8.

3.3.4 *Reliability assessment and results*

The performance function obtained in Eq.(10) is employed to evaluate the failure probability of embankment, Prob[$Fs \leq 1.0$], by MCS. The uncertainties considered in the analysis are listed in Table 8.

The MCS results are plotted in Figure 11. It is difficult to determine what level of reliability is required in this structure. If the failure probability of 1 %, which is $\beta = 2.32$ is chosen as a target, the height of the embankment for the fist stage may be 2.1 (m). The safety factor by the Swedish method is about 1.4 if the mean values of soil parameters are used in the stability calculation

The failure probability is evaluated by removing each uncertain source to find out the impact of each source. These results are also presented in Figure 11. The contribution of each source is approximately estimated by the method explained in Appendix A, where the results are listed in Table 9. In this case, the peat soil strength is the dominant source of uncertainty which is followed by the model error.

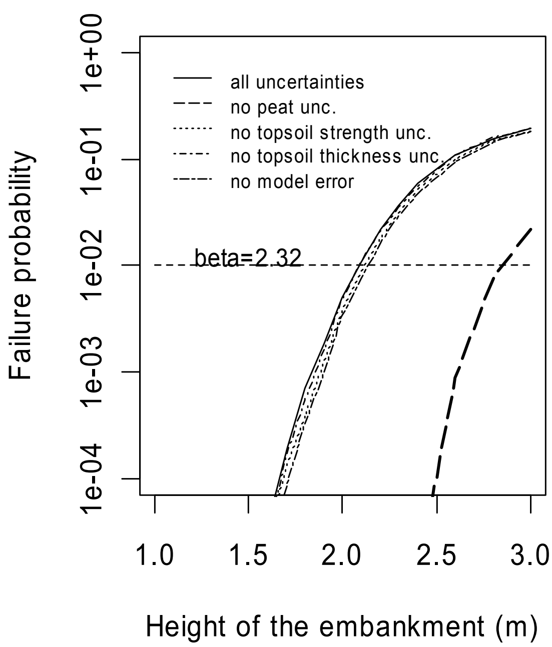


Figure 11. An embankment on peat MCS results

Table 9. The rate of contribution of each uncertainty source for embankment stability (H=2.1 m)

Uncertainty sources	All uncertainty	Peat strength	Top soil strength	Top soil thickness	Model error
β and β_{-i}	2.27	4.58	2.38	2.29	2.44
Contribution	100 %	75 %	9 %	2 %	13 %
Notes		Statistical: 14 % Spatial: 61 %			

3.4 *Liquefaction risk along 12 km long irrigation channel*

3.4.1 *Problem description*

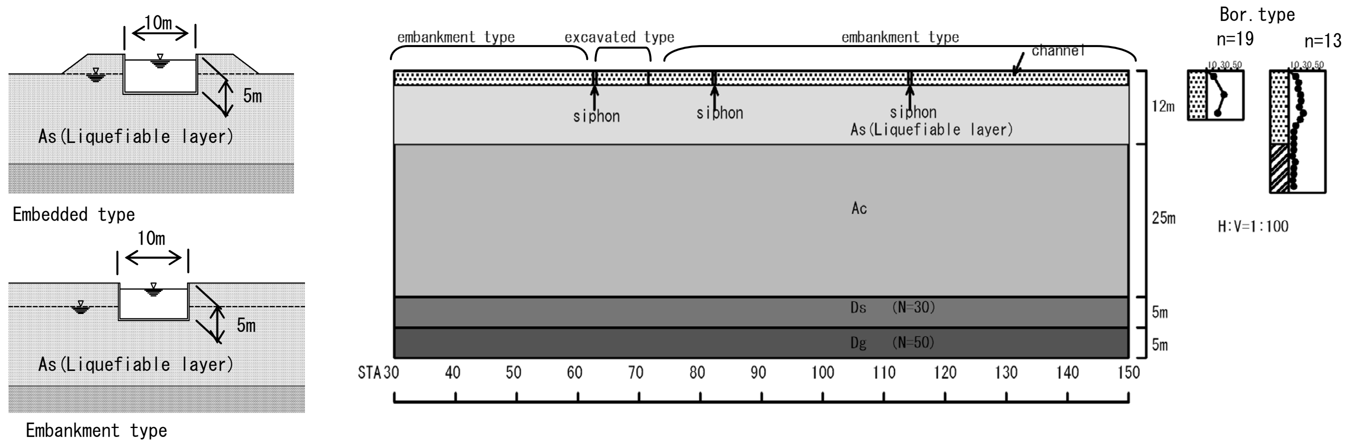
The case described here is based on a paper by Otake et al. (2011) which is one of the papers submitted to this conference. Therefore, only outline of the analysis and the results are given. The parts related to the purposes of this presentation are referred to.

The irrigation channel under study is 25 km long and completed in 1970 (Figure 12). The geology under the channel can be divided into three parts, where 12 km long central part (STA30 – 150) is described in the paper. It is an open channel RC frame structure and 90 % is build in the embankment (Figure 12(a), embankment type), whereas 10% is excavated channel (embedded type) including siphons. The RC frame channel has width of about 10m, height 5m and 10m long.

The channel is located on one of major Alluvial panes in Japan and geology is relatively homogeneous. There is a potentially liquefiable sand layer (As layer) of about 12m thick whose SPT N-value is about 15 and the fine contents (Fc) less than 10%.

The area is in the region where near future occurrence of Tokai-Tonankai earthquake is suspected. Model earthquake motion provided by the central disaster mitigation conference for the earthquake is employed in this study. The downstream part is more susceptible to stronger earthquake motion because it is closer to the epicentre. By the peak ground surface acceleration (PGA), it is 135gal at the most upstream point, 175gal at the middle point and 241gal at the most downstream point. The distinguished characteristics of this earthquake motion are its very long continuous time (about 120 sec) and dominance of the long period components (2 – 4 sec).

The performance requirements of this irrigation channel are *to keep the water level that is sufficient for the natural distribution of water to the surrounding area and to provide sufficient quantity of water to the destinations*. Thus, a limit was set to the absolute settlement of the RC frame for maintaining the water level, and to the relative settlement of the adjacent frames to preserve necessary quantity of water flow. To be more specific, the limit state was set to 60 cm for the absolute settlement based on the free board of the channel, and to 60 cm for the relative settlement due to the frame base thickness.



(a) Typical cross section (b) longitudinal section of soil condition

Figure 12. Characteristics of structure and soil condition

3.4.2 *Geotechnical analysis*

It is necessary to select a geotechnical parameter that is appropriate to represent ground characteristic in evaluating potential of liquefaction. S_n value proposed by Goto et al. (1982) is selected in this study to represent the strength of ground for liquefaction. This is weighted integration of adjusted SPT N-value, N1, over 20m depth. N_1 is defined as $N_1 = 170 \cdot N / (\sigma_v' + 70)$, where σ_v' is the effective overburden stress.

$$S_n = 0.264 \cdot \int_0^{20} e^{-0.04 N1(x) - 0.24x} dx - 0.885 \qquad (11)$$

The characteristic of the sand layer is solely evaluated by N1 value in this index. This is justified in this case because As layer is very homogeneous and the grain size distribution is similar throughout the area, thus S_n is an effective index to evaluate the liquefaction strength of ground at least relatively.

Then, the problem is to evaluate the residual settlement of the irrigation channel for the earthquake with considerably long duration and of long dominant period. The dynamic FEM based on the effective stress analysis, LIQCA2D07, is employed in order to take into account of the mobilization and dissipation of the excess pore pressure. The effectiveness and the limitations of the program was checked by analyzing shaking table test which had modeled the channel.

The settlement of the RC frame is predicted by LIQCA2D07 for various possible conditions. Based on this parametric study, a response surface (RS) is built which is to be used in the reliability assessment.

The settlement induced by the liquefaction is a complex phenomenon which is influenced by many factors. In stead of building a very complex RS, relatively simple RS was introduced in this study. The uncertainty associated to the RS, which is the residual of the regression analysis of the settlement by various factors are also introduced in the reliability assessment.

The vertical displacement is related to S_n and τ by a linear regression line:

$$D = a \cdot S_n + b \cdot \tau + c + \varepsilon \qquad (12)$$

where D: vertical displacement(cm) obtained by LIQCA2D07, τ: shear stress(kN/m^2) acting at the centre part of liquefiable sand layer, a,b and c: regression coefficients, and ε: residual error.

Table 10. Input to reliability analysis

Uncertain sources	Notation	mean	SD	Distribution type
S_n-value	Sn	-0.34※1)	0.85※1)	Normal
Earthquake shear stress	τ	[12-17.5]	0	Deterministic
Model error of RS	δ_{RS}	1.0	0.09※2) (0.06) ※2)	Normal
Model error of LIQCA2D07	δ_{FEM}	1.0	0.23	Normal

※1 : values by the General estimation. ※2 : COV=10.24/110=0.09(embankment type) 2.83/48=0.06(embedded type)

3.4.3 *Uncertainty analysis and performance function*

Uncertainties considered in this study are model uncertainty of LIQCA2D07, spatial variability of soil parameter represented by the spatial variation of S_n, statistical estimation error and error associated to the approximation by RS. These uncertainties are quantitatively analysed by the statistical means. The results of the statistical analysis, which is quantified uncertainty of each uncertainty source, is presented in Table 10.

The performance functions for the embankment type and the embedded type are respectively given as follows:

$$D_{embk} = (-212 \cdot S_n - 18.8 \cdot \tau + 120) \cdot \delta_{RS} \cdot \delta_{FEM} \qquad (13)$$

$$D_{embd} = (100 \cdot S_n + 1.97 \cdot \tau + 51) \cdot \delta_{RS} \cdot \delta_{FEM} \qquad (14)$$

where D_{emgk}: vertical movement of the embankment type RC frame, and D_{embd} : that of the embedded type.

3.4.4 *Reliability assessment and results*

Figure 11 and Figure 12 shows the mean elevation after shaking of each RC frame (10 m long) for the general estimation and the local estimation of S_n-value respectively. It can be seen, in both cases, the displacement is lager in the downstream because of the stronger earthquake motion. In the downstream part, the mean settlement exceeds the threshold value of 60 (cm). The lager relative displacement occurs at location where the embankment type switches to the embedded type, which implies danger of leakage of water from the channel.

Although the general feature of the vertical displacement is similar for the general and local estimation of S_n, one can see more detailed behavior of each RC frame in the local estimation. For example, there is location where the mean settlement exceed 60 (cm) near STA90 in the local estimation.

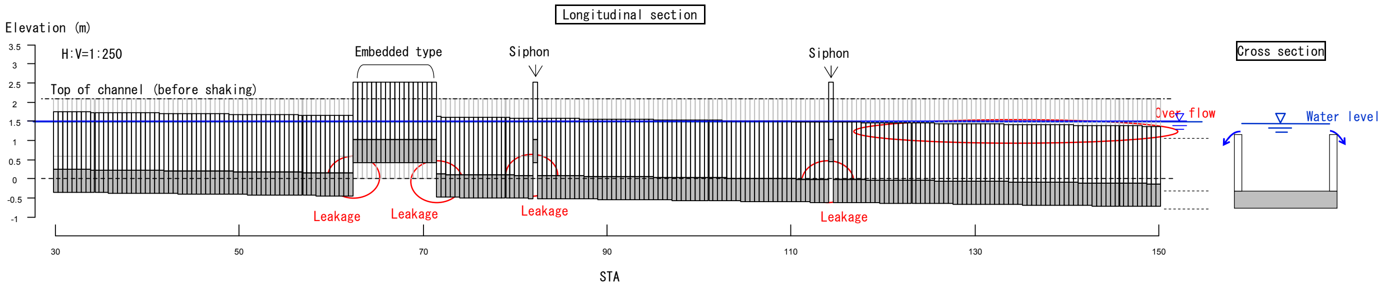


Figure 13. Mean elevation after shaking (general estimation of Sn-value)

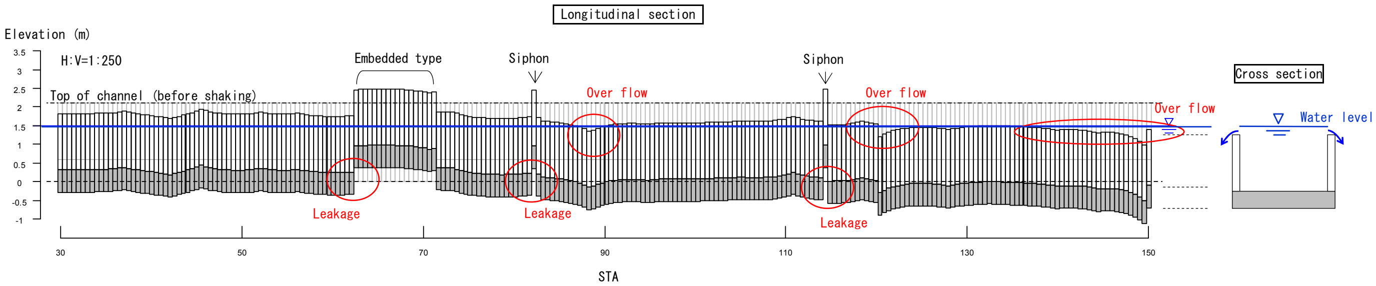


Figure 14. Mean elevation after shaking (local estimation of Sn-value)

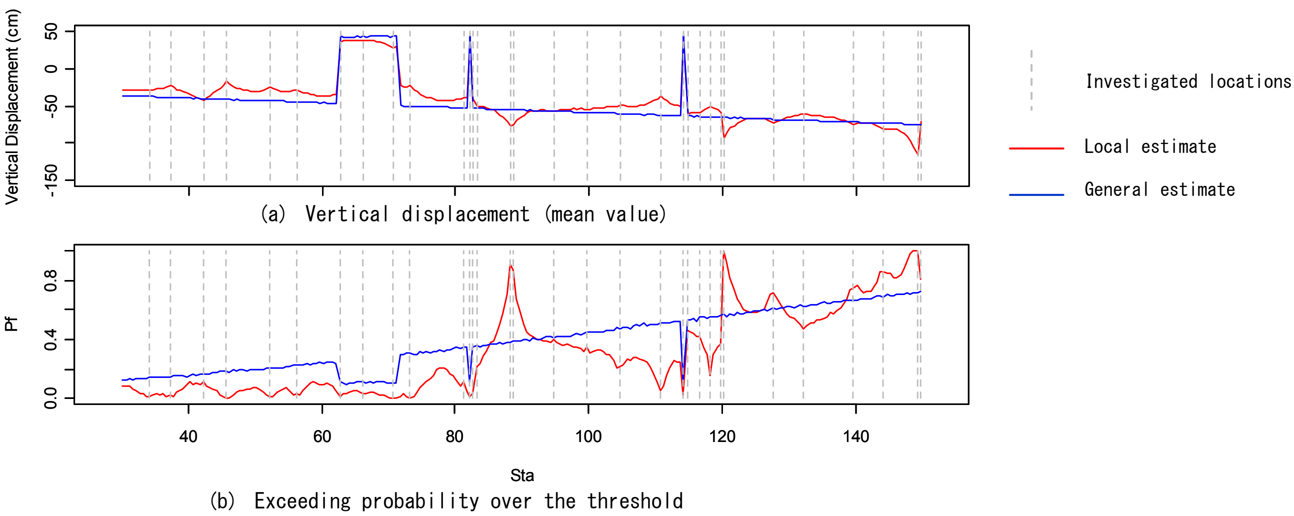


Figure 15. Result of Reliability analysis

Figure 15 presents the mean vertical displacement and the exceeding probability of it over the threshold values (i.e. 60 cm) are presented for the general and local estimation of S_n. The two cases are superposed in these figures for the comparison. The prediction based on the local estimation generally gives smaller exceeding probability, however there are several locations where this relationship is reversed. These probability can be used to determine the optimum enforcement plan of this irrigation channel.

Table 11. Contribution of Uncertainty sources

Uncertainty sources		All uncertainty	Sn-value	Model error	
				FEM	RS
β and β_{-i} (contribution)	Site-r1 (STA63)	1.87 (100%)	1.88 (0%)	8.49 (95%)	1.92 (5%)
	Site-r3 (STA56)	1.58 (100%)	2.32 (54%)	2.05 (41%)	1.63 (6%)
	Site-nr (STA60)	1.02 (100%)	1.42 (48%)	1.33 (41%)	1.08 (11%)

Note) Site-r1:N-values at every 1m, Site-r3: N-values at every 3 m, Site-nr：no investigation at the site

3.5 *Discussions*

It is also one of the purposes of this paper to identify some of the major issues geotechnical RBD is challenged based on the results of the examples. The important sources of uncertainty in geotechnical RBD can be found by carefully discussing the results presented in Tables 5(a), 5(b), 7, 9 and 11. The following observations are possible for RBD of SLS and ULS of the pad foundation, the pile foundation and the embankment on peat:

- It is found from SLS design of the pad foundation that uncertainty is quite large which makes necessary size of the foundation massive (Table 4). This is due to the large uncertainty in transforming CPT q_c to Young's modulus, which can be seen from the results in Table 5(a) that 92% of the uncertainty comes from this transformation error. It is well recognized among geotechnical engineers that estimating stiffness characteristics of ground from the penetration type investigations such as SPT and CPT is not reliable, and the result is ascertaining this fact. Traditionally, therefore, SLS is not checked in the shallow foundation design, and fairly large safety factor, *e.g.* 3, is introduced in ULS design to secure the performance for SLS.
- In stability problem of the foundation, *i.e.* ULS of the pad foundation and the pile foundation, the transformation error and the design calculation model error dominate the uncertainty. In both examples these two uncertainty sources contribute about 40 to 50 % of all uncertainty in the RBD respectively that they are actually controlling the results of the design (Tables 5(b) and 7). The transformation error in the pad foundation design is estimating ϕ' from q_c, whereas in the pile foundation design from q_c to SPT *N*-value. The model errors of the design calculation equations for the both examples are obtained by comparing the calculated results to the observations (i.e. the results of plate loading tests and pile loading tests). If the author was familiar with the pile capacity calculation formula based on q_c, the transformation error in the pile design may have been considerably reduced. The spatial variability of the soil property in the two examples are small because (1) the variance reduction by the local averaging, and (2) very small fluctuation of ϕ' in the pad foundation example.
- Only in the embankment example, the soil spatial variability is the major source of the uncertainty (Table 9). The spatial variability of the peat and top soil undrained shear strength occupies 70% of the total uncertainty. The statistical estimation error and the design calculation model error contribute 14 and 13 % respectively. This consequence comes partly from the accuracy of the design calculation formula, i.e. Sweetish circular slip method, as presented in Figure 3. The model error in this example is much smaller compared to the former examples.

The soil properties in the first three examples are essentially obtained by the general estimation concept, where we did not take into account the relative location of the soil investigation and the structures. The comparison of the general and the local estimation is specifically made in the irrigation channel example, where the followings are observed:

- By comparing the reliability indices, β, of three locations in Table 11, Site-r1 has the highest β, followed by Site-r3 and then Site-nr. It is actually the reflection of the amount of reliable soil property information at each site. Site-r1 has SPT N-value at each 1 m interval through the sand layer, whereas Site-r3 only in 3 m interval. Site-nr does not have any soil property information at the location and it has to be extrapolated from the nearby investigation results. Note that more information does not necessary means more safety of the structure. There are some locations that the exceeding probability is very high and yet the soil investigation was made (Otake *et al.*, 2011). The more information just implies more precise prediction, and if the soil property is near the average, the location with more information gives higher reliability due to the elimination of statistical estimation error.
- As far as the contribution of each uncertainty source is concerned, the error in estimating S_n and the model error contribute evenly at both Site-nr and Site-r3 (Table 11) to the total uncertainty. The error in estimating S_n includes effects of the spatial variability, the variance reduction by local averaging and the estimation error. (Actually, Kriging and the conditional simulation technique are used in estimating S_n) The model error consists of the FEM model error and the RS model error, where the former is far dominant. At Site-r1, there is no error for S_n estimation, thus the model error overrules the total uncertainty.

The readers may have found by now that the selection of uncertainty sources and their assigned extents may be different from one geotechnical engineer to another based on his knowledge and experiences. If one is more familiar with the local soil property, he/she can narrow down the uncertainty compare to a stranger. Actually, this is one of the essences of geotechnical design and the fact should be reflected in geotechnical RBD as well.

4 CONCLUSIONS

All the examples exhibited in this paper, the description is orders in "*problem description*", "*uncertainty analysis*", "*geotechnical analysis and performance function*" and then "*reliability assessment*". It is expected that readers would comprehend the philosophy of the proposed RBD scheme through these descriptions that the geotechnical analysis part is separated from the uncertainty analysis part. The uncertainty analysis part does require some knowledge in statistical analysis. However, other parts need only small knowledge on probability and statistics. It is anticipated that the readers are able to perceive some engineering judgments introduced in geotechnical analysis part, such as some geotechnical interpretation of the transformation equation from q_c to ϕ' in the pad foundation ULS example, the introduction of top soil layer thickness into embankment stability example, and the introduction of S_n in characterizing the potentially liquefiable layer in the irrigation channel example.

Through these examples, it may be understood that it is not necessarily soil properties spatial variability that controls the major part of uncertainty in many geotechnical design problems. The error in design calculation formulas, transformation of soil investigation results (*e.g.* SPT N-values, FVT, CPT q_c) to actual design parameters (*e.g.* s_u, ϕ', resistance values), and statistical estimation error are more important sources in some cases.

All the statistical and reliability calculations carried out in this paper are done by R language. Due to the restriction of space, it was not possible to explain the superiority of this language in this paper. By using R language, these operations become much user friendly and less time consuming.

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APPENDIX A

The estimation method given here is used to estimate the contribution of each uncertain source to the reliability analyses presented in this paper. It is really an approximate method to know these contributions so as to give materials for discussions on geotechnical RA.

The contributions are basically measured by contribution of each variance to the total variance. Suppose the performance function is given by a linear combination of all uncertain sources of resistances and forces. Let $\overline{R}$ be the average of total resistance, $\overline{S}$ that of force. The reliability index, β, can be given as follows:

$$\beta = \frac{\overline{R}-\overline{S}}{\sqrt{\sigma_1^2+\sigma_2^2+\cdots+\sigma_n^2}} = \frac{\overline{R}-\overline{S}}{\sqrt{\sigma^2}} \qquad \text{(A-1)}$$

where σ_i^2 : variance of uncertainty source i.

Also, let us define β_{-i} as

$$\beta_{-i} = \frac{\overline{R}-\overline{S}}{\sqrt{\sigma_1^2+\sigma_2^2+\cdots+\sigma_{i-1}^2+\sigma_{i+1}^2+\cdots+\sigma_n^2}} \qquad \text{(A-2)}$$

Based on Equations (B-1) and (B-2), contribution of σ_i^2 to all uncertainty, σ^2, can be calculated as

$$\frac{\sigma_i^2}{\sigma^2} = \frac{(\overline{R}-\overline{S})^2}{\sigma^2}\left(\frac{1}{\beta^2}-\frac{1}{\beta_{-i}^2}\right) = 1-\frac{\beta^2}{\beta_{-i}^2} \qquad \text{(A-3)}$$

As stated above, this method is only very rough approximation. The actual performance function is not a linear combination of uncertain sources. Furthermore, some basic variables have biases which changes total mean values of resistance and force. Thus, the interpretation of the results should be done with some care. However, in spite of all these restrictions, the author believes that the information provided by this calculation may give interesting and useful information in the geotechnical reliability analyses.

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