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Life Quality Index for Assessing Risk Acceptance in Geotechnical Engineering

D. Straub
Engineering Risk Analysis Group, TU München, Germany

A. Lentz
COWI A/S, Kongens Lyngby, Denmark

I. Papaioannou
Computation in Engineering, TU München, Germany

R. Rackwitz
TU München, Germany

ABSTRACT: The Life Quality Index (LQI) is a recently developed concept that establishes a relation between the resources invested in improving the safety of an engineering facility and potential fatalities that are avoided by the investment. In this way, the LQI provides a rationale for determining acceptability of decisions involving life safety risks in engineering, including the establishment of target reliabilities. In this contribution, the principle of the LQI is outlined and its relevance for making safety-relevant decisions in geotechnical engineering is highlighted. The methodology is illustrated by an application to the design of a slope, involving a FE-based reliability analysis.

Keywords: Risk acceptance, target reliability

1 INTRODUCTION

In geotechnical engineering, decisions or recommendations on actions must be made, which will affect life-safety risk. Whenever standards and codes do not apply (or when these are to be written), the engineer must answer the question "How safe is safe enough?". On the one hand, the engineer has the responsibility to ensure the safety of people involved in the construction and the use of the facility. On the other hand, he or she has the responsibility to use resources in an economical way. To find the right tradeoff between these two contradicting goals is the responsibility of the engineer. In geotechnical engineering (and general civil engineering), this tradeoff is selected mostly implicitly, i.e. safety-related decisions are made on the basis of past experience and calibration, thus implying an underlying (but unknown) weighting of safety vs. cost. In many instances, this approach leads to good engineering decisions, but in some circumstances it can give rise to inconsistent or even grossly misguided actions. This applies in particular for novel engineering applications or larger projects for which no or little experience is available. A procedure for explicitly defining the right tradeoff is therefore desirable, not least because it enables the documentation and justification of the decisions taken.

The Life Quality Index (LQI) is a recently developed concept for determining acceptability of decisions involving life safety risks in engineering, which provides a rationale for establishing target reliabilities for civil engineering systems (Nathwani et al. 1997, Rackwitz 2002, Lentz 2007). The LQI is a socio-economic utility function that depends on the wealth and life expectancy of a society. Any decision that increases the value of the LQI is deemed acceptable. This increase can be due to an increase in life expectancy (reduction of fatalities) or an increase in societal wealth (reduced use of resources). In this way, the LQI establishes a relation between the resources invested in improving the safety of an engineering facility and potential fatalities and injuries that are avoided by the investment, i.e. it provides a means to quantify the optimal tradeoff between safety and cost.

In this contribution, the principle of the LQI is outlined and its relevance for making safety-relevant decisions in geotechnical engineering is highlighted. The methodology is illustrated by an application to the design of a slope, involving a FE-based reliability analysis.

2 LIFE QUALITY INDEX

There are different ways of assessing whether a safety-related decision should be deemed acceptable or not. One of the most consistent approaches proposes to take a look at the personal utility an individual experiences due to different decisions. Utility, here, is seen as the result of several factors, such as long life in good health, wealth, intact family relations etc. This usage of the concept is common in socio-economics. Unfortunately, many contributors to utility -- or simply to life quality -- cannot be quantified properly. For this reason, income and life expectancy are generally used as representative indicators for life quality as a whole.

Since the 70ies, several economists such as Shepard & Zeckhauser (1984) have made proposals for the formulation of $L = L(e_0,g)$, where e_0 is life expectancy at birth and g denotes average income available for risk reduction measures. In the engineering domain, Nathwani et al. (1997) first formulated the so-called life quality index (LQI). The LQI is essentially a socio-economic utility function, which can be derived in different ways making use of different principles (e.g. Pandey et al. 2006). In its present form (Pandey & Nathwani 2004, Rackwitz 2004), it is written as

$$L = g^q l_d \quad \text{with} \quad q = \frac{1}{\beta}\frac{w^*}{1-w^*} \tag{1}$$

Herein, $\beta \approx 0.7$ quantifies the share of labor in the creation of the GDP. w is the time fraction of life spent at work. The asterisk in w^* signifies that the trade-off between work time and leisure time is at its optimum from the point of view of the average citizen. l_d denotes the average remaining life expectancy of all currently living members of society of various ages a. In fact, age-averaged willingness-to-pay is the correct quantity to use as it must be assumed that a representative cross-section of the population is endangered by the event-type hazard:

$$l_d = E_A[l_d(a)] = \int_0^{a_u} l_d\,(a)h(a,n)\mathrm{d}a \tag{2}$$

The index d stands for discounting: Future income effects require discounting. For mathematical convenience, this effect is integrated in the life expectancy term l_d instead of the utility term g^q. The term $h(a,n)$ denotes the age distribution of a population growing at rate n, while $l_d(a)$ denotes the (discounted) remaining life expectancy of a person aged a:

$$\begin{aligned} l_d(a) &= \int_0^{a_u} S(t|a)\exp\left[-\int_a^t \gamma\,(\tau^*)\mathrm{d}\tau\right]\mathrm{d}t \\ &= \int_0^{a_u} \exp\left[-\int_a^t \mu\,(\tau)\mathrm{d}\tau\right]\exp\left[-\int_a^t \gamma\,(\tau^*)\mathrm{d}\tau\right]\mathrm{d}t \\ &= \int_0^{a_u} \exp\left[-\int_a^t \mu\,(\tau) + \gamma(\tau^*)\mathrm{d}\tau\right]\mathrm{d}t \end{aligned} \tag{3}$$

In the first line, $S(t|a)$ denotes the probability of surviving up to age t for a person aged a today. Survival probabilities are calculated from the age-dependent mortality rate $\mu(a)$. Discounting is performed at some rate $\gamma(\tau^*)$, where $\tau^* = \tau - a$.

If utility is made up of life expectancy l_d and disposable income g, it implies that life expectancy can be exchanged with income at a certain rate without changing overall utility. In fact, it can be observed that people are willing to give a certain amount of their income in order to increase their life expectancy by buying additional safety measures, e.g. when paying extra money for a car with additional safety features. This rate of exchange between income and life expectancy is referred to as willingness-to-pay (WTP). As outlined in Nathwani et al. (1997), this concept can be used for a criterion, by demanding that any safety-related decision shall not lower utility (life quality) L:

$$dL = \frac{\partial L}{\partial g}\mathrm{d}g + \frac{\partial L}{\partial l_d}\mathrm{d}l_d \geq 0 \tag{4}$$

Usually, engineering decisions have a simultaneous effect on safety levels and income. Safety measures lead to a rise in average life expectancy l_d, but their costs lead to a decrease in average available income g. According to the willingness-to-pay (WTP) approach, a decision is judged acceptable if the overall lifetime utility remains equal or rises. It is important to realize that this type of criterion is only suitable for risk prevention, i.e. saving the life of some member of society who cannot be identified in advance. The criterion is not applicable to identifiable persons already finding themselves in a state of immediate emergency.

Setting $dL = 0$ and inserting Eq. (1) yields

$$-\mathrm{d}g \leq \frac{\frac{\partial L}{\partial l_d}}{\frac{\partial L}{\partial g}} \mathrm{d}l_d = \frac{g}{q}\frac{\mathrm{d}l_d}{l_d} = \mathrm{WTP} \qquad (5)$$

Of principle reasons, it is more correct to replace $\mathrm{d}l_d/l_d = E_A[\mathrm{d}l_d(a)]/E_A[l_d(a)]$ by $E_A[\mathrm{d}l_d(a)/l_d(a)]$, see (Lentz 2007). The acceptable domain is then limited by

$$-\mathrm{d}g \leq \frac{g}{q} E_A\left[\frac{\mathrm{d}l_d(a)}{l_d(a)}\right] = \mathrm{WTP} \qquad (6)$$

or

$$\frac{\mathrm{d}g}{g} + \frac{1}{q} E_A\left[\frac{\mathrm{d}l_d(a)}{l_d(a)}\right] \geq 0 \qquad (7)$$

Note that safety investments lead to a negative change in income dg, so that $-dg$ adopts a positive value.

Safety-relevant measures cause a change in mortality rate μ, which is defined as the number of deaths divided by the population size. Usually, this calculation is performed for each age group separately, leading to an age-dependent mortality rate $\mu(a)$. Absolute and proportional mortality changes constitute two of the most basic cases. In the first case, an age-independent increment $d\mu(a) = d\mu = \Delta$ is added to background mortality, so that $\mu_\Delta(a) = \mu(a) + \Delta$. In the second case, age-dependent background mortality is multiplied with a constant factor, so that $\mu_\delta(a) = \mu(a) \times (1 + \delta)$. The first case is more typical for accidents (e.g. structural failure), whereas the second case can be observed with the effects of toxic exposure. Other, more complex models exist as well.

For practical purposes, it is convenient to linearize the relationship between (small) changes in mortality $d\mu(a)$ and (small) changes in discounted life expectancy $\mathrm{d}l_d(a)$ (Rackwitz, 2004), so that

$$E_A\left[\frac{\mathrm{d}l_d(a,\Delta)}{l_d(a)}\right] = -J_\Delta \Delta \quad \text{or} \quad E_A\left[\frac{\mathrm{d}l_d(a,\delta)}{l_d(a)}\right] = -J_\delta \delta \qquad (8)$$

Linearization coefficients are in the vicinity of $J_\Delta \approx 13$–17 and $J_\delta \approx 14$–18 for industrialized countries (Lentz 2007). The latter result is multiplied with crude mortality $\mu = \int \mu(a)h(a,n)\, da$. For the absolute risk model, inserting in Eq. (7) leads to

$$-\mathrm{d}g \leq -\frac{g}{q} J_\Delta \Delta = -G_\Delta \Delta \qquad (9)$$

It can be shown that $G_\Delta = \frac{g}{q} J_\Delta$ is actually the *WTP for averting one fatality*. In the literature it is known as the 'value of a statistical life' (VSL). However, this terminology appears to be unluckily chosen with respect to ethical considerations. Typical values come close to 2 million PPP US$ for industrialized countries.

Empirical investigations basically confirm this number, e.g. Mrozek & Taylor (2002). However, some cases indicate significantly elevated values. Presumably, this deviation from the analytically derived VSL is due to the psychological phenomenon that people dread events disproportionally, if their perceived control over the situation is small or if a large number of victims are not killed in several small accidents but by one single big accident. Both criteria apply to aircraft passengers – and in fact, civil aviation is known for costly measures against very small residual risks.

3 HUMAN CONSEQUENCE MODELING

The previous section assesses engineering decisions by comparing changes in human mortality with changes in income (caused by project costs). However, the directly controllable result of a safety-related decision is not a change in mortality $\mathrm{d}\mu$, but a change in failure rate $\mathrm{d}r$. Obviously, $\mathrm{d}\mu$ is a function of $\mathrm{d}r$

if the failure is related to some potentially fatal hazard. The present section reviews some basic concepts of how to establish this link.

Most potentially fatal events in civil engineering share some basic properties: They occur at an unpredictable moment and practically all fatalities occur at once. A basic methodology for this type of event-type hazards was introduced in Lentz (2007). According to its basic idea, the expected number of fatalities in case of a failure event F can be written as

$$N_{D|F} = N_{PE}(1 - P_Q)P_{D|F} = N_{PE}k \quad (10)$$

Here, N_{PE} is the number of people endangered. It corresponds to the number of people actually expected to be present at the onset of the event. This is a subset of all people *potentially* present N_{pop}. P_Q is the probability of successful escape and $P_{D|F}$ is the probability of death given no successful escape. The latter probabilities are united in a single factor $k = (1 - P_Q)P_{D|F}$ in order to keep the notation short in long expressions. The strength of the approach lies in the fact that the determination of N_{PE} and P_Q follows the same principles regardless of the specific event-type, such as building collapse after an earthquake, dam failure or tunnel fire. The same statistical information on human behavior and physiology can be used in all cases. Only the last component of Eq. (10), $P_{D|F}$, requires case-specific modeling. All three components of $N_{D|F}$ are made up of several sub-quantities that have been numerically described in Lentz (2007) and elsewhere.

The change in mortality caused by a failure is $N_{D|F}/N_{pop}$, where N_{pop} is the number of people in the entire population (country). By multiplying with the failure rate, the change in mortality is obtained as

$$\Delta = \frac{N_{D|F}}{N_{pop}} \mathrm{d}r = \frac{N_{PE}k}{N_{pop}} \mathrm{d}r \quad (11)$$

For many engineering facilities, the failure rate r is not constant with time, but approximate results can be obtained with a constant (asymptotic) value of r when failed facilities are systematically rebuilt (Rackwitz 2005). In the application presented in this paper, it is assumed that failure events occur as a homogenous Poisson process, and the failure rate therefore is constant.

4 APPLICATION TO TECHNICAL FACILITIES

In design and operation of technical facilities, system parameters $\mathbf{p}$ are selected, which determine the performance of the facility. (In the application example presented later, the parameter is the slope angle of a embankment.) These parameters determine both the life-cycle cost of the facility, which causes a change in societal income dg, as well as the failure rate r, which causes a change in the mortality risk associated with the fatality.

To apply the LQI criterion, both the costs as well as the change in mortality are expressed as annual values. Let $C_a(\mathbf{p})$ be the annualized net present life-cycle cost of the facility and $r(\mathbf{p})$ be the failure rate of the facility, which is here assumed to be constant. Following Rackwitz (2002), we can set the negative change in income of the total population equal to the change in the annualized life-cycle cost of the facility, i.e. $-dg = dC_a(\mathbf{p})/N_{pop}$. (The division with N_{pop} is introduced because g is the per-capita GDP.) Furthermore, the expected change in mortality is given by Eq. (11). Inserting in Eq. (9), it is

$$\frac{\mathrm{d}C_a(\mathbf{p})}{N_{pop}} \leq -\frac{g}{q} J_\Delta \frac{N_{PE}k}{N_{pop}} \mathrm{d}r(\mathbf{p}) \quad (12)$$

N_{pop} cancels out, and rearranging the terms leads to the acceptance criterion:

$$-\frac{\mathrm{d}C_a(\mathbf{p})}{\mathrm{d}r(\mathbf{p})} \geq \frac{g}{q} J_\Delta N_{PE}k = WTP \cdot N_{PE}k \quad (13)$$

Here, $\mathrm{d}C_a(\mathbf{p})/\mathrm{d}r(\mathbf{p})$ is the change in annualized cost with respect to the change in failure rate and will take negative values for reasonable engineering situations (the cost increases with decreasing failure rate).

Eq. (13) is the criterion that engineering decisions $\mathbf{p}$ must fulfill to comply with societal values as expressed through the LQI. The right hand side depends on the willingness to pay WTP as determined from the LQI principle, as well as the number of people exposed N_{PE} and the probability k that a person exposed is killed during a failure. The left-hand side depends on the effectiveness of measures for reducing the failure rate. When more effective measures (i.e. less costly measures) are available, implicitly a higher level of safety will be required. The application of the principle in Eq. (13) is illustrated in the following for a simple but representative design decision in geotechnical engineering.

5 APPLICATION OF THE LQI PRINCIPLE TO THE ACCEPTABILITY OF SLOPE DESIGN

5.1 *Problem statement*

Consider the embankment shown in Figure 1 to be constructed for a railway line. The height of the embankment h as well as the width at the top is prescribed, but the slope angle α can be selected by the designer. Clearly, an increase in the slope angle will lead to a reduction of cost but also to an increase in the failure rate r. It will be demonstrated how the LQI principle can be used to find the acceptable value of α.

5.2 *Mechanical and probabilistic modeling*

The embankment is modeled in 2D with plain-strain finite elements. The material model used is an elasto-plastic model with a prismatic yield surface according to the Mohr-Coulomb criterion and a non-associated flow rule with zero dilatancy. The elasto-plastic deformations are computed as the converged pseudo time-dependent elasto-viscoplastic solution, applying the viscoplastic strain method (e.g. see Smith and Griffiths 2004).

The considered random variables are those relevant to shear failure, i.e. the strength parameters and specific weights of the soil and fill material, as well as the train loading (Table 1). The correlation coefficient between the strength parameters of the same materials is taken as –0.3. For the stiffness parameters, deterministic values are chosen ($E = 10^5$ kPa, $\nu = 0.3$ for both materials). Random spatial variability of the soil properties is not included in the analysis for simplicity.

Table 1. Random variables

Parameter	Distribution	Mean	COV
Friction angle (Fill) φ_F [º]	Lognormal	21	0.1
Cohesion (Fill) c_F [kPa]	Lognormal	12	0.2
Specific weight (Fill) γ_F [kN/m^3]	Normal	20	0.05
Friction angle (Clay) φ_C [º]	Lognormal	20	0.1
Cohesion (Clay) c_C [kPa]	Lognormal	15	0.2
Specific weight (Clay) γ_C [kN/m^3]	Normal	19	0.05
Train load q [kN/m^2]	Gumbel	50	0.2

Figure 2 shows the deformed mesh at failure for a slope angle $\alpha = 26.6$º, with displacements magnified by a factor of 200.

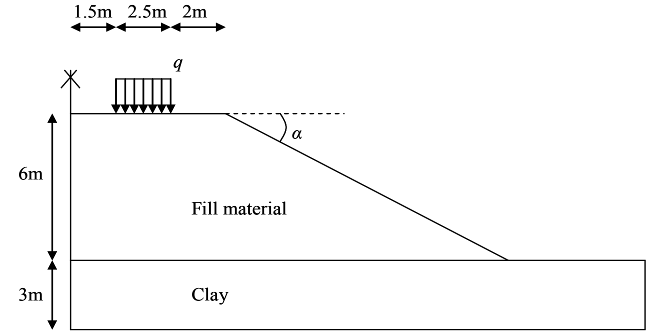

Figure 1. Embankment with train load

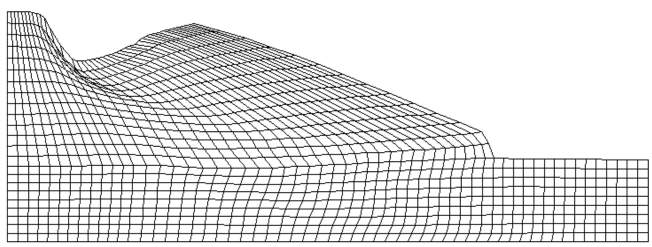

Figure 2. Deformations at failure. Slope angle $\alpha = 26.6$º (2:1), factor of safety FS = 1.66.

The factor of safety (FS) of the slope is computed applying the shear strength reduction technique (Matsui and San 1992). It is defined as the number by which the original strength parameters must be divided to reach the failure state. According to this approach, the strength parameters are gradually reduced by an increasing factor and an elasto-plastic finite element computation is performed at each step.

5.3 *Reliability analysis*

The limit-state function, with negative values defining the failure event, is expressed as:

$$g(\mathbf{X}) = FS(\mathbf{X}) - 1 \quad (14)$$

where **X** is the vector of random variables given in Table 1. A series of reliability analyses are carried out for selected values of the slope angle α by means of the first-order reliability method (FORM), resulting in corresponding values of the reliability index β. For convenience, a 2^{nd} order polynomial function is fitted to the computed values of β:

$$\beta(\alpha) \approx 11.61 - 0.415\alpha + 0.0049\alpha^2 \quad (15)$$

Figure 3 shows the reliability index β as a function of the slope angle, together with the corresponding failure rate r [yr^{-1}], which is related to the reliability index β by $r \approx \Phi(-\beta)$, with $\Phi(.)$ being the standard Normal cumulative distribution function.

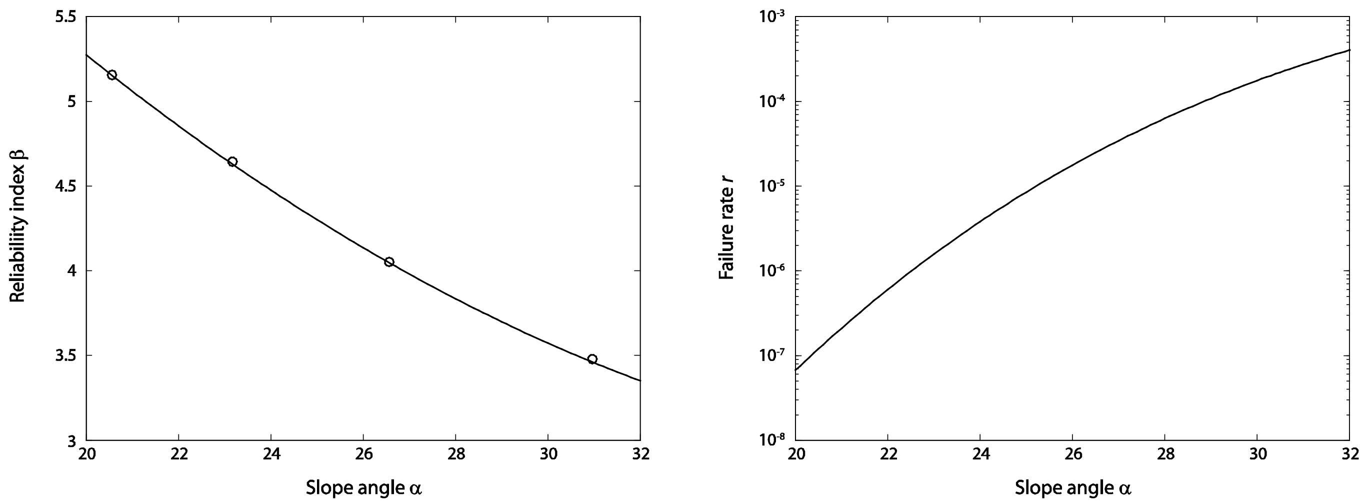

Figure 3. Reliability index β, failure rate r, as a function of slope angle α.

5.4 *Life-cycle cost*

The net present value of the annualized life cycle cost is a function of the slope angle, $C(\alpha)$. Since we are interested only in changes of the cost, $dC(\alpha)$, it is sufficient to consider incremental costs. Simplifying, we can write the construction costs as

$$\begin{aligned} C_c(\alpha) &= c_0 + c_1 \cdot Land\ use(\alpha) + c_2 \cdot Material(\alpha) \\ &= c_0 + c_1 h (\tan\alpha)^{-1} + c_2 \frac{h^2}{2} (\tan\alpha)^{-1} \\ &= c_0 + c \cdot (\tan\alpha)^{-1} \end{aligned} \quad (16)$$

Where the constant is $c = c_1 h + c_2 h^2/2$. For $h = 6$m, a value of $c = 10^5$€ is taken in the following (Note: this value of c is based on assuming that the value of the constant is 10^4€ per meter of embankment and that the embankment can be modeled as a series system whose components have length 10m. The latter asumption depends on the spatial correlation of material properties.)

It is assumed here that the construction costs are the only relevant costs, i.e. that maintenance costs and other costs occurring after construction can be neglected. To compute the annualized life cycle cost, we consider an interest rate of γ of 2%, reflecting a long-term sustainable interest rate (corresponding to economical growth). If the embankment is utilized over a period of t_s years, the costs can be split into constant annuities C_a of

$$C_a(\alpha) = \frac{\gamma C_c(\alpha)}{1 - \exp(-\gamma t_s)} \tag{17}$$

which follows from $C_c(\alpha) = \int_0^{t_s} C_a(\alpha)\exp(-\gamma t)\,\mathrm{d}t$. To avoid selecting an arbirtrary service life time t_s, we consider an infinite time horizon $t_s = \infty$, Rackwitz (2010). In this case, the denominator in Eq. (17) becomes one (it would be 0.86 in the case of $t_s = 100\text{yr}$) and we get

$$\begin{aligned} C_a(\alpha) &= \gamma C_c(\alpha) \\ &= \gamma[c_0 + c \cdot (\tan\alpha)^{-1}] \end{aligned} \tag{18}$$

More sophisticated computations of life cycle costs based on renewal models are presented in Streicher and Rackwitz (2004) and Joanni and Rackwitz (2007). However, the presented calculation is accurate enough for the envisaged application.

5.5 *Consequence model*

Following Section 3, the expected value of the number of people killed given the event of a failure is determined as a function of (a) the number of people present N_{PE} at the onset of the event, which here can be considered as the number of people in a train, (b) the probability of successful escape P_O, which here is equal to the probability that the failure is detected timely and any train can be stopped before approaching the location, (c) the probability of death given no successful escape $P_{D|F}$, which here is the probability of a person being killed given that the train derails due to a slope failure at this location.

With numerical values $N_{PE} = 200$, $P_O = 0.3$, $P_{D|F} = 0.3$, the probability of an exposed person being killed in the case of a failure becomes $k = (1 - 0.3) \cdot 0.3 = 0.21$. The expected value of the number of people killed given the event of a failure is $N_{PE} \cdot k = 42$.

5.6 *Willingness to pay (WTP)*

The WTP is defined in Eq. (13) as $WTP = J_\Delta\, g/q$. All input values depend on socio-economic indicators and must therefore be defined country-specific. Here, values valid for Germany in 2010 are taken and are derived from the data provided in (OECD 2011) following (Lentz 2007). The disposable per-capita income is obtained as $g = 25'300€$, the constant q of Eq. (1) is $q = 0.13$ and the linearization coefficient J_Δ of Eq. (8) is $J_\Delta = 14.4$. It follows that

$$WTP = \frac{25'300€}{0.13} 14.4 = 2.8 \cdot 10^6€$$

5.7 *Acceptable slope angle*

The acceptable slope angle is now found by application of Eq. (13). The left-hand side of Eq. (13), which represents the efficiency of mitigating risk by decreasing α, is obtained from Eqs. (15) and (18) as

$$\begin{aligned} \frac{\mathrm{d}C_a(\alpha)}{\mathrm{d}r(\alpha)} = \frac{\frac{\mathrm{d}C_a(\alpha)}{\mathrm{d}\alpha}}{\frac{\mathrm{d}r(\alpha)}{\mathrm{d}\alpha}} &= \frac{\frac{\gamma[c_0 + c \cdot (\tan\alpha)^{-1}]}{\mathrm{d}\alpha}}{\frac{\mathrm{d}\Phi[-(11.61 - 0.415\alpha + 0.0049\alpha^2)]}{\mathrm{d}\alpha}} \\ &= \frac{\gamma c(\sin\alpha)^{-2}}{\varphi[-(11.61 - 0.415\alpha + 0.0049\alpha^2)](-0.415 + 2 \cdot 0.0049\alpha)} \end{aligned}$$

φ is the standard normal probability density function. $\mathrm{d}C_a(\alpha)/\mathrm{d}r(\alpha)$ is plotted in Figure 4. The right hand side of Eq. (13) is readily obtained as $WTP \cdot N_{PE}k = 2.8 \cdot 10^6€ \cdot 42 = 118 \cdot 10^6€$. Figure 4 illustrates how the acceptable slope angle is determined as $\alpha_{acc} = 23.8°$.

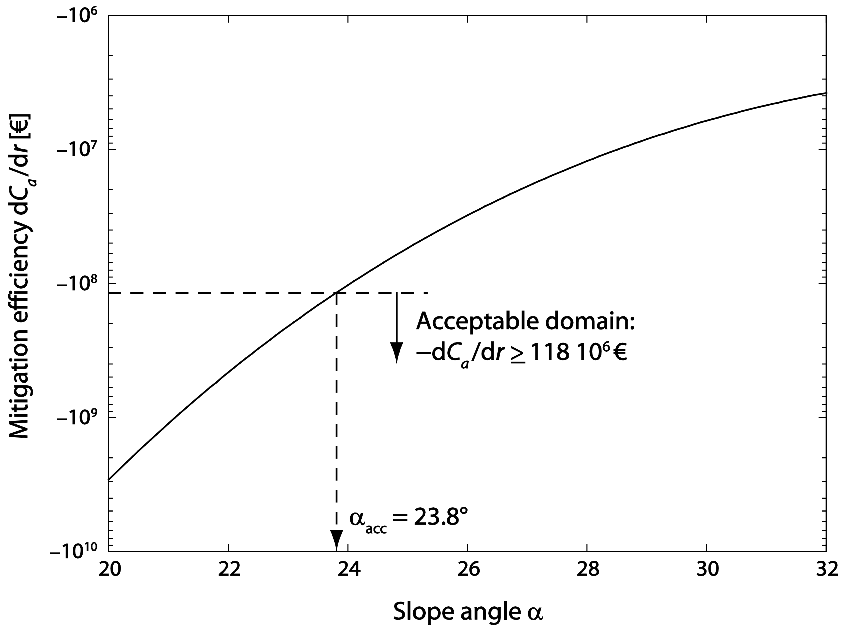

Figure 4. The relative cost of reducing the failure rate, $\mathrm{d}C_a(\alpha)/\mathrm{d}r(\alpha)$, and the acceptable slope angle derived according to the LQI criterion.

The minimum acceptable slope angle $\alpha_{acc} = 23.8°$ corresponds to a reliability index $\beta = 4.5$, as seen from Figure 3. The corresponding global safety factor is $FS = 1.83$.

6 CONCLUDING REMARKS

The paper summarizes and illustrates the use of the LQI principle for determining the acceptability of geotechnical engineering designs. The central idea is the formulation of an index (LQI) that serves as a proxy for societal utility and is formulated as a function of life expectancy and income (which in turn is a proxy for available resources). By requiring that any engineering decision must not decrease the value of the LQI, a minimum requirement on the resources to be spent for risk-reduction can be deduced.

The presented example serves for illustrational purposes only. No general conclusions must be drawn from this example, since the results are case-specific and are obtained from a simplified probabilistic model. The purpose of the example is purely to demonstrate the steps involved in the application of the LQI principle.

It is pointed out that the LQI is not a tool to be used directly for standard geotechnical projects, where decisions are – and should be – made based on global or partial safety factors concepts. However, the LQI principle can be used to determine the values of the safety factors prescribed by codes and standards. This can be achieved by computing a larger set of examples similar to the one presented in this paper and then calibrate safety factors (e.g., the acceptable slope angle shown in the example above corresponds to a global safety factor of 1.83). Optimally, safety factors are defined as a function of the consequences of a failure; the safety factors should increase with increasing consequences. The LQI principle enables to quantify this dependence. As an example, if the consequences of failure in the above example are reduced by installing a warning system that would increase the probability that trains can stop timely from 0.3 to $P_O = 0.9$, the acceptable slope angle increases to $\alpha_{acc} = 27.3°$ (with corresponding $\beta = 3.9$).

There has been some discussion in the scientific community on the exact formulation of the LQI, in particular on the definition of the factor q in Eq. (1) (see e.g. Ditlevsen 2004). It is noted, however, that the different formulations give results in the same order of magnitude and the dispute is thus of little practical relevance. More relevant is the fact that the LQI in its present form is restricted to considering fatalities. Failures of engineering systems can lead to other types of relevant societal consequences, including environmental damages. The LQI concept has yet to be extended to account for such consequences. A first step in this direction is suggested in Lentz (2007), namely to additionally account for injuries caused by a failure event, by considering only the life spent in good health in the formulation of the LQI concept.

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