

A probabilistic approach to risk assessment of slow slope movements

M. Ranalli & G. Gottardi

DISTART, University of Bologna, Italy

Z. Medina-Cetina

Zachry Department of Civil Engineering, Texas A&M University, USA

F. Nadim

International Centre for Geohazards, Oslo, Norway

ABSTRACT: Many regions are getting more and more vulnerable from a hydro-geological point of view. The causes are due to both the fragility of the territory and to the anthropic influence on its continuous modifications. To avoid or reduce the human life and property economic losses, the quantitative prediction of landslide occurrence and the estimation of the amount of consequent slope movements are necessary. In this paper a specific type of landslide, characterized by a viscous behaviour of soil, is discussed. To catch the displacements evolution of this type of landslide, an analytical dynamic-viscous model was set up. In order to develop an advanced analysis and prediction procedure of the behaviour of such creep landslides, a probabilistic approach, based on Bayesian theorem, is presented. The method is validated making use of a well established and highly reliable monitoring database (Alverà landslide), located in Cortina d'Ampezzo (Dolomites, Italy). The model calibration was then probabilistically performed solving the inverse problem. The solution is then characterized by probability density functions of model parameters, including their corresponding correlation structure. Furthermore, this analysis enables to describe statistically the model error, associating a degree of uncertainty to the predictions.

Keywords: Slow slope movements, dynamic-viscous model, probabilistic calibration, risk analysis

1 INTRODUCTION

Landslides represent one of the most common natural hazard in the world. In recent years, landslide risk assessment has gained significant and ever increasing importance. The identification of triggering factors, the investigation of soil properties, the understanding of kinematic mechanism, the prediction of mass movements are then fundamental issues to avoid or reduce the losses of property or people life due to their occurrence. The risk assessment and management process consists in studying all these elements, together with the consequences of landslides trigger and the possible mitigation measures. In particular, the prediction of landslide occurrence and the estimation of the amount of consequent slope movements are the main issues both for estimating the hazard and for designing the mitigation measures and warning systems. To this purpose, probabilistic analyses are powerful tools to combine the several elements of risk management process and take into account the uncertainties inside their definition.

The paper focuses on a particular type of landslide, characterized by shallow and translational movements, which involve fine, essentially clayey material. According to the Varnes classification, they can be identified as extremely or very slow slope movements, with velocities typically of few centimetres per year. The main triggering factor is hydrologic, since the movements show a periodical and seasonal activation, usually strictly connected to ground water level fluctuations. These characteristics can be related to the viscous response of soils.

To catch the displacement evolution of this type of landslides, a well-defined dynamic-viscous model, able to predict the displacements from groundwater level inputs and return a value of mobilised friction angle, was set up. It consists of introducing an additional viscous resisting force into the equation of motion of a block of soil, enabling to model the actual evolution of such slow and seasonal movements (Vulliet and Hutter, 1988; Van Asch and Van Genuchten, 1990; Corominas *et al.*, 2005; Van Asch *et al.*, 2007). Here, a slightly modified version (Ranalli *et al.*, 2010) of the original Gottardi and Butterfield's visco-plastic model (Gottardi and Butterfield, 2001; Butterfield, 2000) is considered.

The relevant and innovative feature of the work is the probabilistic approach used to calibrate the model. Referring on Bayesian theory, the joint probability distribution of model parameters (posterior) is obtained by means of the combination of their prior information (prior) and the performance measure of the predictive model (likelihood). It is thus possible to propagate quantitatively the uncertainty of the geotechnical parameters and the triggering factors into the model parameters (fully probabilistic solution). Starting from the prior and the likelihood, the use of Markov-Chain Monte Carlo method allows to sample the posterior, conditioned on a site specific data (Medina-Cetina, 2006). The so obtained probabilistic solution is then characterized by a full description of each model parameter, given in the form of their probability density functions, including their corresponding correlation structure. Furthermore, this analysis enables to describe statistically the model error, associating a degree of uncertainty to the predictions. Finally, it allows to update the knowledge on the model parameters every time a new information or observation becomes available.

In order to validate the approach, a well established and highly reliable monitoring database (Alverà landslide) located in Cortina d'Ampezzo (Dolomites, Italy), was used (Deganutti e Gasparetto, 1992; Angeli *et al.*, 1996; Gasparetto *et al.*, 1996; Panizza *et al.*, 1996). It is essentially made up of frequent and extensive ground displacement and piezometer records.

The final results aim at improving the now available tools for landslide risk analysis and management in a rational and effective way.

2 THE PROBABILISTIC APPROACH

The Bayes' paradigm is one of the most known probabilistic formulation used to solve the inverse problem (Congdon, 2007). In this process, the solution is inferred by a backward procedure, moving from the observations to the model parameters. So, it is important to use any available prior information on the model parameters and to have a representation of the data uncertainty in an explicit way. To this purpose, the Bayes' theorem is a suitable tool since it combines the prior knowledge about the model parameters with the information coming from a new observation, both defined in terms of probability. The result is an updating of the parameters knowledge, which represents the probabilistic solution to the inverse problem.

The analytical expression of Bayes' paradigm is:

$$\pi(\theta|d_{obs}) = \frac{f(d_{obs}|\theta, g(\theta))\pi(\theta)}{\int f(d_{obs}|\theta, g(\theta))\pi(\theta)d\theta} \quad (1)$$

where $g(\theta)$ is a generally non-linear function that relates the set of model parameters θ with the observations d_{obs} , $\pi(\theta)$ is the prior probability density function of parameters, $f(d_{obs}|\theta, g(\theta))$ is the likelihood, which represents a measure of model performance in terms of probability function, and $\pi(\theta|d_{obs})$ is the posterior probability density function of parameters and represents the solution of the inverse problem. The denominator is a normalization constant.

Usually, the Equation (1) can be solved only by means of numerical techniques. The Markov Chain Monte Carlo method allows to sample randomly a large vector of values according to the posterior probability distribution. This process consist in a sequence of random variables X_0, X_1, \dots, X_t such that the next state of the chain X_{t+1} depends only on the previous one. The stochastic walk was generated by implementing the Metropolis-Hastings algorithm, since it allows to sample probability distributions which are known except a constant.

3 THE DYNAMIC-VISCOUS MODEL

The viscous behaviour of soil is modelled by considering a linear damping force F_v , defined as:

$$F_v = C\dot{x} = (\eta/h_m)\dot{x} \quad (2)$$

where η is the dynamic viscosity (in kilopascal second) and h_m is the thickness of the shear zone (in meters). Assuming a infinitive slope scheme, which is able to model translational landslides, and referring to Figure 1, the equation of motion of a unit-block soil element, with mass M and thickness D , on an infinite slope inclined at an angle θ is:

$$M\ddot{x} = P - F_{M-C} - F_v \quad (3)$$

where $P = Mg\sin(\theta)$ is the driving force, $F_{M-C} = N'tg(\varphi')$ is the Mohr-Coulomb resisting force, depending, for continuously moving landslides, only on residual friction angle φ'_r .

Introducing a dimensionless variable β , defined as:

$$\beta = \frac{N'}{N} = 1 - \frac{\gamma_w}{\gamma_{sat}} \frac{d}{D} \quad (4)$$

where d is the groundwater level, the Equation (3) becomes:

$$M\ddot{x} + C\dot{x} = N(\tan \theta - \beta \tan \varphi') \quad (5)$$

Also, introducing the initial condition (v_0, β_0) , which identifies the step change from static to sliding slope, along with the assumption of acceleration equals to zero, we can estimate the corresponding value of mobilised shear strength angle φ'_0 :

$$\tan \varphi'_0 = \frac{1}{\beta_0} \left(\tan \theta - \frac{Cv_0}{N} \right) \quad (6)$$

Combining Equation (5) and (6) and introducing the following dimensionless variables:

$$\begin{aligned} T &= gt/v_0 \\ X &= xg/v_0^2 \\ \dot{X} &= (dX/dT) = \dot{x}/v_0 \\ \ddot{X} &= (d\dot{X}/dT) = \ddot{x}/g \end{aligned} \quad (7)$$

the solution in terms of predicted velocity and displacement is respectively:

$$\begin{aligned} \dot{X} &= \frac{H}{G} + (G - H)\exp(-GT) \\ X &= \frac{H}{G}T + \left(\frac{G - H}{G^2} \right) (1 - \exp(-GT)) \end{aligned} \quad (8)$$

which, translated into the physical space, yields:

$$\begin{aligned} \dot{x}_{pred} &= \dot{X}v_0 \\ x_{pred} &= X \frac{v_0^2}{g} \end{aligned} \quad (9)$$

where the capitol letters are dimensionless variables:

$$\begin{aligned} H &= B + (\beta/\beta_0)G \\ B &= \sin \theta (1 - \beta/\beta_0) \\ G &= Cv_0/Mg \end{aligned} \quad (10)$$

So, the model is characterized by three parameters: v_0 , β_0 and C , since γ_{sat} , D and θ are usually known. Once they are defined, it is able to return displacement predictions and an estimation of mobilised friction angle for assigned groundwater levels.

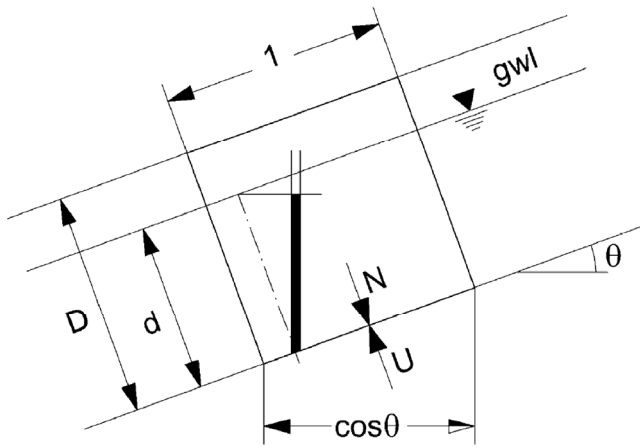


Figure 1. Scheme of infinite slope

4 THE PROBABILISTIC MODEL CALIBRATION

4.1 Alverà monitoring database

The Alverà landslide is a well-known and well-studied mudslide, located in the Italian Dolomites, near Cortina d'Ampezzo. It consists of a saturated clayey matrix (20-25 m thick) with irregular, poorly sorted blocks, which moves very slowly on a lower stable unit. A quicker shallow movement is identified within the upper unit, along a 5 m-deep surface. Laboratory tests on the clay fraction of samples recovered at this slip surface indicate the following geotechnical parameters: $\gamma_{\text{sat}} = 18.73 \text{ kN/m}^3$, $w_L = 0.95$, $I_p = 0.48$ and $\phi'_r = 15.9^\circ$ (residual shear strength angle).

An extensive and reliable monitoring system has been installed along the whole body of landslide (Gasparetto et al., 1996; Angeli et al., 1999; Corominas et al., 2000; Ranalli et al., 2010). To model calibration purpose, only the daily records relative to extensometer and piezometer installed in the more active and superficial movement are considered. The measurements are illustrated in the Figure 2.

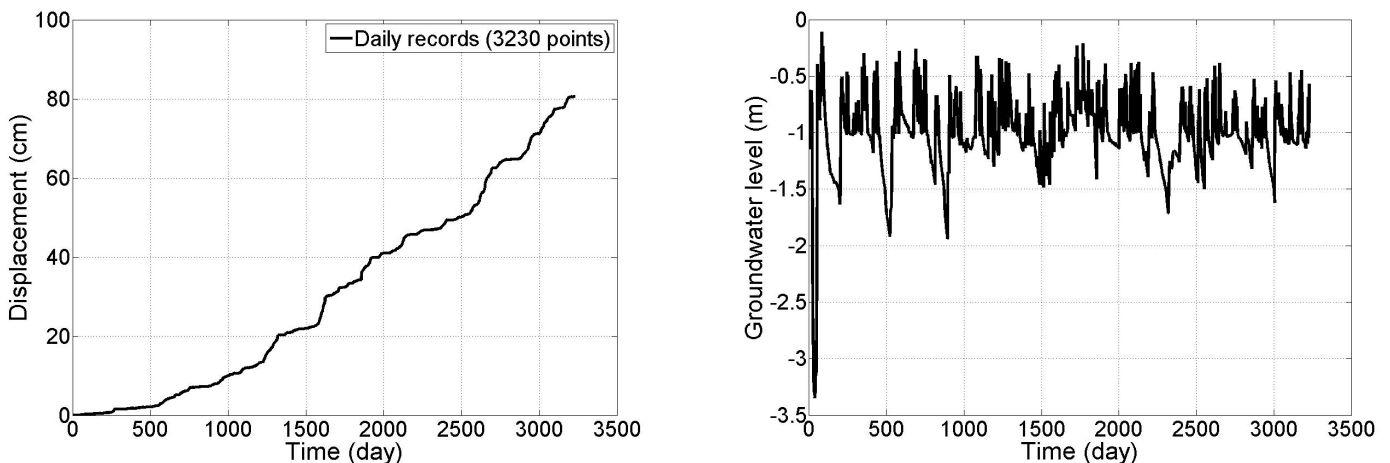


Figure 2. Displacements and groundwater levels records relative to Alverà landslide

4.2 Results of inverse problem

The model parameters v_0 , β_0 and C are calibrated by means of Equation (1), making use of Alverà landslide database. The almost 9-year monitoring period was divided in four sub-periods, so that four different initial conditions (that is different v_0 and β_0 values) were considered. Instead, the viscous parameter C was taken constant with the time.

As a consequence of this assumption, the number of parameters to be calibrated adds up to nine. In order to apply the Bayes' theorem, a prior for model parameter should be assumed or evaluated. For this purpose, it was defined by an uncertainty identification and propagation analysis, as widely illustrated in Ranalli et al. (2010).

As mentioned above, the integration of Equation (1) was performed through the Markov Chain Monte Carlo method (MCMC), which allows to sample randomly a large vector of values according to the posterior. By way of example, the samplings relative to v_{01} and β_{01} parameters are shown in Figure 3. The solution of probabilistic model calibration consists of marginal probability distribution for each model parameter and their correlation structure. Figure 4 shows the cumulative density functions of v_{01} and β_{01} . Information about their correlation can be obtained by the correlation matrix of parameters or their joint cumulative density function.

Pursuant to probabilistic definition of parameters, it is possible to associate a degree of uncertainty to model predictions. Figure 5 shows the variability of model predictions compared with the observations, highlighting a global good fit. Figure 6 presents instead the trend of residuals with the time, together with the residuals mean and standard deviation. The fluctuation of residuals around the zero value indicates that the model generates unbiased predictions. So, all this statistical information allows to assess the reliability of predictive model.

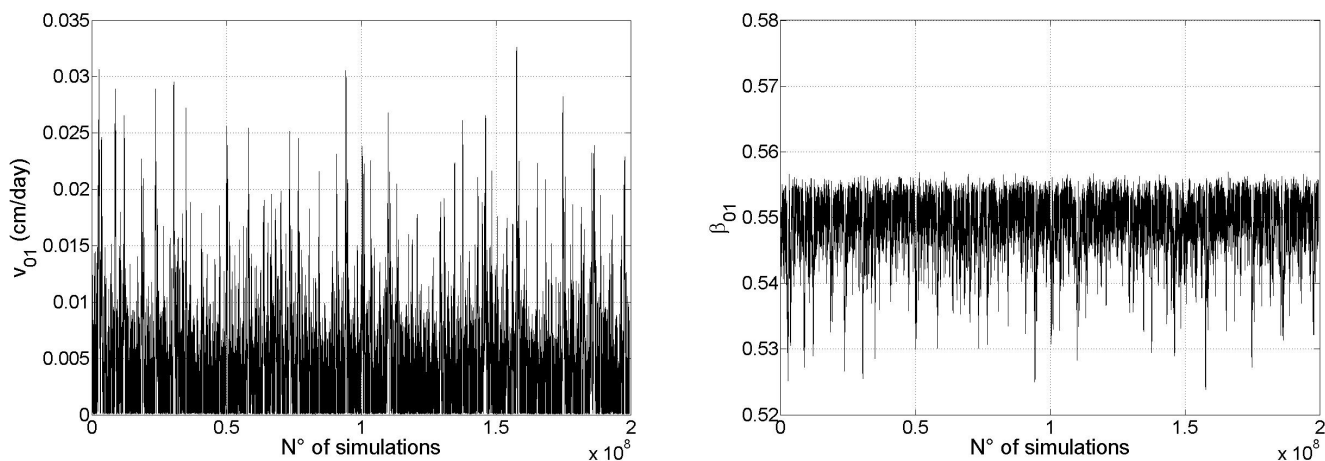


Figure 3. Sampling of v_{01} and β_{01} parameters by MCMC method

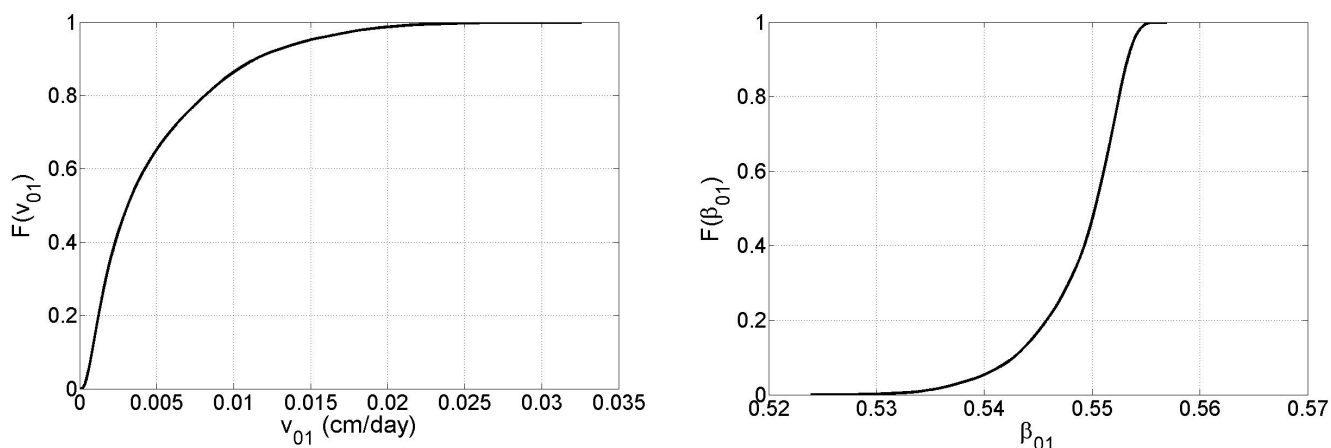


Figure 4. Posterior CDF's of v_{01} and β_{01} parameters; representing the probabilistic solution of model calibration

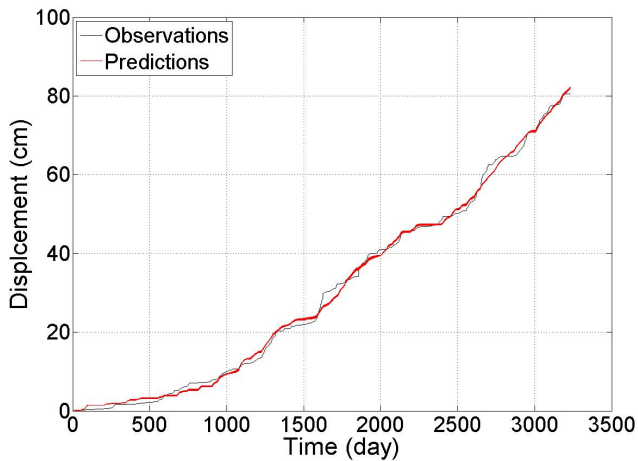


Figure 5. Comparison between model predictions and observations

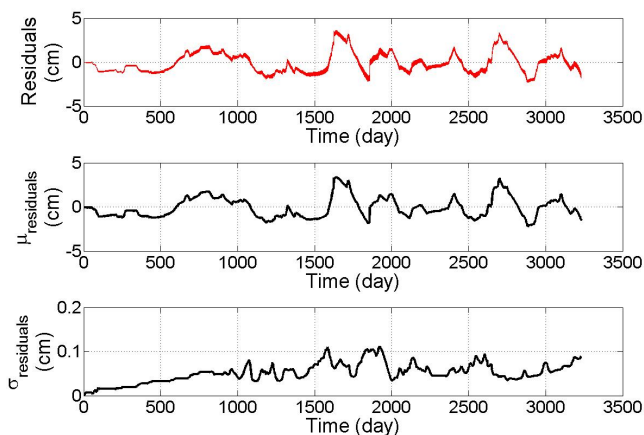


Figure 6. Statistics of model error

5 CONCLUSIONS

The paper concerns with the development of a new methodology for the analysis of slow slope movements, to be included within the landslide risk assessment and management process. In particular, an analytical and dynamic model, able to simulate landslides characterized by soil viscous behaviour, for which periodical movements depend on groundwater fluctuations, was set up.

The proposed methodology consisted in applying the Bayesian probabilistic approach to model calibration. The applicability and the effectiveness of such method was illustrated by using a case study, for which a reliable and extensive monitoring database is available.

The probabilistic solution of model calibration allowed to fully describe the parameters of the model, in terms of their marginal probability distributions and correlation structure. In addition, the error of the model was statistically described, since an uncertainty degree of predictions could be identified. In other words, it was possible to define and quantify the reliability of the model. This aspect is particularly crucial when the model is used as predictive tool into the risk analysis process.

A further advantage of such approach deals with the possibility of updating statistically the information on the parameters. In fact, the probabilistic solution obtained from initial observations can be improved every time new information become available.

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