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Probabilistic Analysis of Bearing Capacity of Strip Footings

M. A. Shahin & E. M. Cheung
Department of Civil Engineering, Curtin University, Perth, Australia

ABSTRACT: Predicting bearing capacity of shallow foundations is a common practice in geotechnical engineering and an accurate estimation of its value is essential for a safe and reliable design. Traditional deterministic methods of estimating bearing capacity of shallow foundations do not explicitly consider the uncertainty associated with the factors affecting bearing capacity and rather employ a factor of safety that implicitly accounts for such uncertainty. This factor of safety is in reality "factor of ignorance" as it relies only on past experience and does not reflect the inherent uncertainty in relation to bearing capacity parameters, leading to unreliable bearing capacity predictions. In this paper, a more rational approach for estimating bearing capacity of strip footings subjected to vertical loads is proposed. The approach is based on probabilistic analyses using the Monte Carlo simulation and accounts for the uncertainty associated with two shear strength parameters, i.e. soil cohesion and soil friction angle. The probabilistic solutions negate the need for assuming a factor of safety and provide a more reliable indication of what the actual bearing capacity might be.

Keywords: Probabilistic analysis, Bearing Capacity, Strip footings, Shallow foundations.

1 INTRODUCTION

Bearing capacity and settlement are the two main components of design of shallow foundations; however, bearing capacity usually governs the design process. If bearing capacity is over-estimated, soil will fail, leading to serious consequences and fatalities. If, on the other hand, bearing capacity is under-estimated, undue costs are usually incurred. Consequently, an accurate prediction of bearing capacity is important for a safe and reliable design of shallow foundations. Traditional design methods of bearing capacity of shallow foundations are deterministic in the sense that they do not explicitly account for the inherent uncertainty associated with the factors affecting bearing capacity. Uncertainty associated with bearing capacity can be classified into the following three categories: (*i*) natural spatial variability; (*ii*) model uncertainty; and (*iii*) parameter uncertainty. Natural spatial variability is due the variation of soil properties from one point to another in space, which is caused by the variations in the mineral composition and characteristics of soil strata during soil formation. Model uncertainty is due to the inability of a selected mathematical model to mimic a real phenomenon (Frey 1998). Parameter uncertainty is due to inaccuracy in assessing the soil properties because of the limited number of soil sampling and testing data. It is also due to the inadequacy of interpreting the subsurface geology due to the measurements errors, data handling and transcription errors, inconsistency of data and inadequate representation of data sampling due to time and space limitations (Baecher and Christian 2003). Parameter uncertainty can also be due to the discrepancies between the in-situ implementation of structure and what appears in construction drawings.

In traditional deterministic methods, uncertainties associated with predicting bearing capacity of shallow foundations are implicitly dealt with by employing a fixed global safety factor that may lead to inappropriate bearing capacity predictions. In this paper, an alternative probabilistic approach that provides a more rational estimation of the bearing capacity of strip footings subjected to vertical loads is presented. The approach uses the Monte Carlo simulation to account for parameter uncertainty associated with the soil properties. Other types of uncertainties (i.e. natural soil variability and model uncertainty) are beyond the scope of this paper and are not considered. The current probabilistic approach provides the likely distribution of predicted bearing capacity, which enables the designer to make informed decisions regarding the level of risk associated with the design. To facilitate the use of the probabilistic approach, a computer algorithm using Excel software is developed and can be readily used by practicing engineers.

2 DETERMINISTIC BEARING CAPACITY OF STRIP FOOTINGS

In order to obtain probabilistic solutions for bearing capacity of shallow foundations, a deterministic model shall first be selected. In this work, the commonly used model proposed by Terzaghi (1943) is selected in which the deterministic ultimate bearing capacity of strip footings can be obtained as follows:

$$q_u = cN_c + qN_q + 0.5\gamma BN_\gamma \qquad (1)$$

where q_u is the ultimate bearing capacity, c is the soil cohesion, γ is the soil unit weight, B is the footing breadth, q is the overburden pressure (i.e. the soil unit weight × depth of foundation, D) and N_c, N_q and N_γ are the bearing capacity factors. The bearing capacity factors rely solely on the soil friction angle, ϕ, and are estimated as follows (Terzaghi 1943):

$$N_q = \frac{[e^{(0.75\pi-\phi/2)\tan\phi}]^2}{2\cos^2(45^\circ+\phi/2)} \qquad (2)$$

$$N_c = (N_q - 1)\cot\phi \qquad (3)$$

$$N_\gamma = \frac{1}{2}\left(\frac{k_{p\gamma}}{\cos^2\phi} - 1\right)\tan\phi \qquad (4)$$

where $\pi = 3.14$ and $k_{p\gamma}$ is the passive earth pressure coefficient that relies on ϕ. From values of $k_{p\gamma}$ corresponding to ϕ given by Terzaghi (1943), the following matching empirical equations for $k_{p\gamma}$ can be proposed:

$$k_{p\gamma} = 10.49e^{0.0363\phi} \ (R^2 = 0.98, \text{ for } \phi = 0.0–15^\circ) \qquad (5)$$

$$k_{p\gamma} = 5.82e^{0.074\phi} \ (R^2 = 0.99, \text{ for } \phi = 15^\circ – 35^\circ) \qquad (6)$$

$$k_{p\gamma} = 0.364e^{0.1516\phi} \ (R^2 = 0.98, \text{ for } \phi = 35^\circ – 50^\circ) \qquad (7)$$

3 PROBABILISTIC BEARING CAPACITY OF STRIP FOOTINGS

In the present work, the probabilistic analysis for bearing capacity of strip footings is conducted by utilizing the Monte Carlo simulation and considering parameter uncertainty associated with the input variables in Equation (1). Detailed description of the Monte Carlo simulation can be found in many publications (e.g. Hammersley and Handscomb 1964; Rubinstein 1981). Among the five input variables of Equation (1), the soil cohesion, c, and soil friction angle, ϕ, are likely to include significant parameter uncertainty and thus are assumed to be random variables. The soil unit weight, γ, is assumed to be constant in the present work as it contributes to parameter uncertainty of a lesser degree, as demonstrated by Lee et al. (1983). In addition, the footing breadth, B, and depth of foundation, D, are likely to provide marginal parameter uncertainty and are thus assumed to be deterministic for practical purposes. It should be noted that model uncertainty is not considered in the present work and thus Equation (1) is assumed to be a perfect predictor (i.e. has no model uncertainty). For an individual case of bearing capacity prediction, the procedure used to obtain probabilistic solutions that account for the parameter uncertainty of c and ϕ is as follows:

1. For each of the bearing capacity input variables (i.e. c, ϕ, γ, B and D), a random value is generated in relation to parameter uncertainty of the input mean value, coefficient of variation (COV), known or assumed probability distribution function (PDF) and any correlation exists between that input variable and the other available input variables;
2. Using the generated input values from Step (1) and assuming that Equation (1) is a perfect predictor, a deterministic value of bearing capacity is obtained;
3. Steps 1 and 2 are repeated hundreds or thousands of times, as part of the Monte Carlo simulation, until certain acceptable convergence is met; and
4. Finally, all the bearing capacities obtained are collated and used to determine the cumulative distribution function (CDF) or to plot the cumulative probability distribution curve from which predictions associated with target reliability levels of 90% and 95% (the reliability levels that are usually needed for design) can be estimated.

In order to illustrate the probabilistic procedure set out above, the following case study is investigated. A strip footing of breadth B = 2.0 m is founded at a depth D = 1.5 m below the ground surface, and the soil is clayey sand with unit weight γ = 18 kN/m^3. The statistical values for c and ϕ are selected as follows: μ_c (mean of cohesion) = 5 kPa, μ_ϕ (mean of friction angle) = 30^o, COV$_c$ (coefficient of variation of soil cohesion) = 27%, COV$_\phi$ (coefficient of variation of soil friction angle) = 10% and $\rho_{c,\phi}$ (correlation coefficient between c and ϕ) = –0.6. The probability distribution functions for both c and ϕ are assumed to follow a lognormal distribution, as has been used in several geotechnical engineering applications. It should be noted that the above statistical values are within the practical ranges that are cited in the literature. For example, the mean of ϕ is typically between 20^o and 40^o (Abdel Massih et al. 2008), with COV ranging from 5% to 15% for sands and 12% to 56% for clays (Lee et al. 1983; Phoon and Kulhawy 1999). The COV for c varies between 10% to 70% (Cherubini 2000) with a recommended value of 30% (Lee et al. 1983). The COV between c and ϕ ranges between –0.24 and –0.7 (Lumb 1970; Wolff 1985; Yuceman et al. 1973) with a recommended value of –0.6 can be used in practice (Cherubini 2000).

The abovementioned statistical data are used to generate sample values of c and ϕ (Step 1) and the corresponding deterministic bearing capacity is calculated using Equation (1) of Terzaghi's model (Step 2). As mentioned previously, Terzaghi's model is assumed to be a perfect predictor with no model uncertainty and uncertainty associated with the natural variability of soil is not considered. Consequently, parameter uncertainty associated with the shear strength properties c and ϕ is the only source of uncertainty considered in this work. Steps 1 and 2 are repeated many times until a convergence criterion is achieved (Step 3). To determine whether convergence has been achieved, the statistics describing the distribution of the predicted bearing capacities are calculated at fixed numbers of simulations and compared with the same statistics at previous simulations. Convergence is deemed to have occurred if the change in the statistics describing the distribution of predicted bearing capacity is 1.5% or less. The predicted bearing capacities obtained from the many simulations conducted are used to plot the cumulative probability distribution curve from which bearing capacity predictions that assure target reliability levels are obtained (Step 4). It should be noted that the probabilistic simulation described in Steps 1 to 4 are conducted with the aid of the PC-based software @Risk (Palisade 2000) and the results are shown in Figure 1, which also includes the predicted deterministic value of bearing capacity. For the case study above, the predicted deterministic bearing capacity is obtained using Equation (1) and is found to be equal to 1067kPa. For target reliability levels of 90% and 95%, the corresponding bearing capacities are estimated from the cumulative probability function (or from Figure 1) to be equal to 730 kPa and 658 kPa, respectively. These values give equivalent factors of safety of 1067/730 = 1.5 and 1067/658 = 1.6, respectively. These results indicate that, for the case study above, the factor of safety of 3 that is usually used in the deterministic analysis is conservative. The results also demonstrate that the uncertainty associated with c and ϕ can considerably affect the bearing capacity of strip footings and thus should not be neglected.

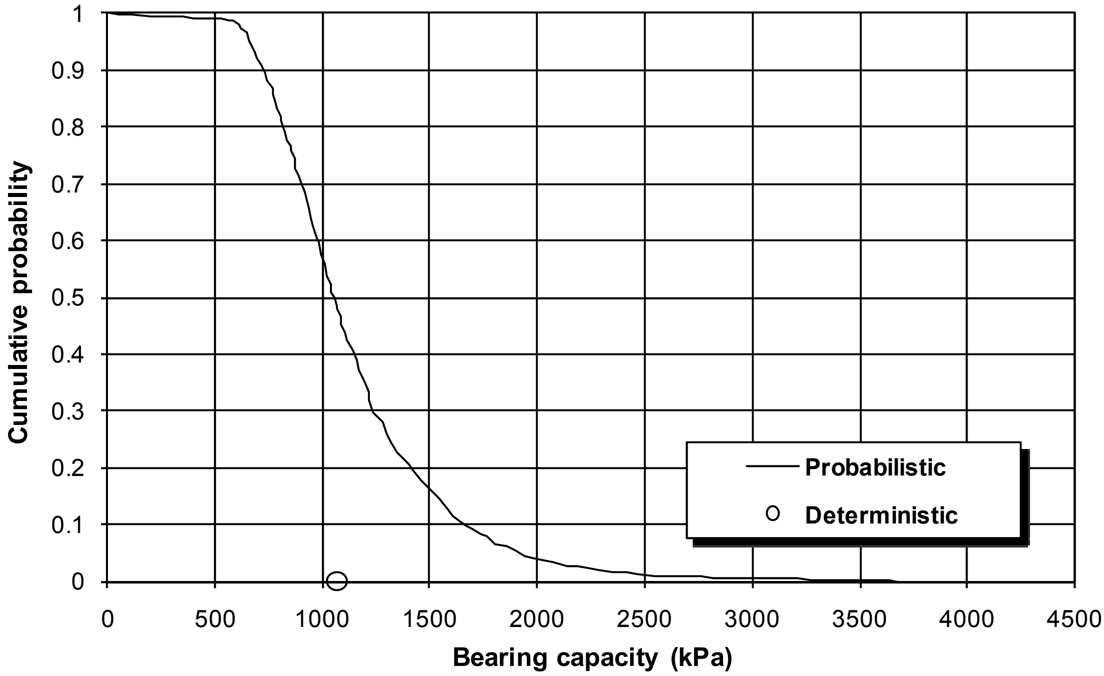

Figure 1. Cumulative probability distribution incorporating parameter uncertainty of c and ϕ for the case study considered

4 DEVELOPMENT OF PROBABILISTIC BEARING CAPACITY GENERIC SOLUTIONS

The probabilistic simulation applied to the case study described in Section 3 is used to develop a generic set of probabilistic solutions for routine use in practice, from which predicted bearing capacity corresponding to 90% and 95% reliability levels can be readily obtained. The solutions are based on the practical recommended parameter uncertainty of $COV_c = 30\%$ and $COV_\phi = 20\%$, and a coefficient of correlation between c and ϕ of -0.6 with lognormal distribution for both c and ϕ. The procedure that is used to develop the solutions is as follows:

1. A combination of input values for c, ϕ, γ, B and D are selected so as to be within the ranges that can be expected in practical applications, as given in Table 1;
2. The probabilistic approach, outlined previously, which incorporates parameter uncertainty for c and ϕ is applied and the corresponding CDF is obtained;
3. From the CFD, bearing capacities corresponding to 90% and 95% reliability levels are determined; and
4. Another combination of values of c, ϕ, γ, B and D are selected from Table 1 and Steps 2 to 3 are repeated until all possible combinations of values of c, ϕ, γ, B and D given in Table 1 are chosen and their probabilistic simulations are conducted. The results are used to develop probabilistic design solutions corresponding to 90% and 95% reliability levels.

Table 1 Values of the input variables used for development of the probabilistic design solutions

Input variable	Values	Number of values
Cohesion, c (kPa)	0, 20, 40, 60, 80, 100	6
Friction angle, ϕ (degrees)	0, 10, 20, 30, 40	5
Soil unit weight, γ (kN/m^3)	16, 18, 20	3
Footing breadth, B (m)	0.5, 1, 2, 3	4
Depth of foundation, D (m)	0, 1.5, 3	3

To facilitate the use of the obtained probabilistic solutions by practicing engineers, a computer code using Excel software is developed and can be readily used. Figure 2 shows the main menu of the developed Excel software with an illustrative example that will be explained below. By considering the number of values given in Table 1 for c, ϕ, γ, B and D, it can be derived that the number of probabilistic simulations conducted in order to develop the probabilistic design solutions are: $6 \times 5 \times 3 \times 4 \times 3 = 1080$. In order to illustrate the use of the design solutions using the developed Excel computer codes, the following numerical example is examined. A copy of Excel software program is available by the authors upon request.

PROBABILISTIC BEARING CAPACITY CALCULATOR COV_c = 30%, COV_ϕ = 20%		
INPUTS (CTRL+Q to Reset)		
Soil Properties		
Cohesion, *c* (kPa)	20	
Soil friction angle, ϕ (degrees)	35	
Unit Weight of Soil, γ (kN/m^3)	20	
Footing Properties		
Width of Footing, *B* (m)	1.5	
Depth of Foundation, *D* (m)	3	
Factor of Safety for Deterministic Design	3	
OUTPUTS		
Probabilistic Bearing Capacity, q_u (kPa)		
95% Confidence (kPa)	**1515**	
90% Confidence (kPa)	**1787**	
Deterministic Bearing Capacity, q_u (kPa)		
Ultimate Bearing Capacity (kPa)	**4211**	
Allowable Bearing Capacity (kPa)	**1404**	
Equivalent Safety Factor		
95% Confidence	**2.8**	
90% Confidence	**2.4**	

Figure 2. Main menu of Excel software: Example

Example: A strip footing of breadth B = 1.5 m is to be constructed at a depth D = 3 m below the ground surface in a soil that has the following properties: c = 20 kPa, $\phi = 35^o$ and γ = 20 kN/m^3. It is required to find the bearing capacity corresponding to reliability level of 90%, and also estimate the equivalent FOS.

Solution: For a reliability level of 90%, the Excel spreadsheet program shown in Figure 2 is used to obtain the bearing capacity corresponding to γ = 20 kN/m^3, leading to a bearing capacity of 1787 kPa. The deterministic bearing capacity is obtained to be equal to 4211 kPa, and for this case, the equivalent FOS = 4211/1787 = 2.4.

5 SUMMARY AND CONCLUSIONS

Probabilistic approach that utilizes the Monte Carlo technique was used to obtain probabilistic bearing capacity of strip footings from the commonly used deterministic Terzaghi's model. The proposed probabilistic approach accounts for parameter uncertainty of soil cohesion and friction angle, and enables bearing capacity to be quantified in the form of a cumulative probability distribution function that provides bearing capacity predictions corresponding to certain reliability levels. The approach was applied to a case study for illustration. A series of probabilistic solutions that incorporate parameter uncertainty of coefficient of variation of 30% and 20% for soil cohesion and friction angle, respectively, were carried out and computer code using Excel was developed to facilitate the use of the proposed approach for routine use by practitioners. A numerical example was given to illustrate the use of charts. The results indicate that the suggested factor of safety of 3 that usually used by available deterministic models is conservative. This indicates the importance of adopting probabilistic analyses in favor of the factor of safety. It was also shown that the developed probabilistic method can be used to predict bearing capacity of strip footings for reliability levels of 90% and 95%. The charts are believed to be a useful tool that can be readily used by practitioners for design of strip footings.

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