Evaluation of Failure Probability of Soil Cushions

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ABSTRACT: Article shows the typical examples of mathematical models realization of probabilistic description of random variables (RV) distribution of geotechnical characteristics and the heterogeneity parameters according to data of laboratory and field tests of compacted soils, mining industrial wastes and their mixtures. It also presents the variability of some technological parameters of soil cushions erection. The statistical parameters of RV distribution of designed and ultimate strengths of compacted soils, as well as settlement of foundations on cushions were determined by analytical methods. By means of numerical simulation of cushion tensely-deformed state (TDS) by method of ultimate elements (MUE) using the elastic plastic model, imitation simulation by Monte Carlo method and experimentally obtained distribution laws of RV of compacted soils characteristics, the statistic parameters and distribution laws of foundations settlements were obtained. Due to statistic analysis of settlements distribution of artificial foundations bases and their relative differential settlements the probability of failure was obtained.

Keywords: compacted soil, soil cushion, angle of internal friction, unit cohesion, modulus of deformation, distribution law, random variables, probabilistic design, method of ultimate elements, foundations settlements, soil design and ultimate strengths, probability of failure.

1 INTRODUCTION

1.1 Actuality of the problem

The engineering and geological conditions of building sites often are complicated. For example, for erection of modern building projects, the over flooded territories which are composed of poor-bearing soils are often used. For such circumstances, foundation engineering practice constantly replenishes the positive experience of filled earth massif erection with improved soil physical and mechanical properties. The improvement of soil properties is performed by compaction using modern vibrorollers, heavy tampers, compactors etc.

For utilization of industrial wastes and minimization of cost of soil cushions it is necessary to study the overburden rock and their mixtures as the material of artificial bases. In Poltava region (Ukraine) the large deposits of iron ore were explored. That’s why the problems of overburden rock utilization are actual.

Compacted soils are of inherent heterogeneity, the parameters of which are taken the RV of the soil characteristics, anisotropy of mechanical properties etc. These parameters depend on the type and nature of the material properties of artificial bases, technological parameters of its erection and so on. Modern methods of cushions design are deterministic and do not consider the real variation of values of compacted soils properties. These design methods put unreasonable reserves of strength and deformability in their erection. That’s why the geotechnical reliability is very actual problem, especially for artificial bases.

1.2 Analysis of previous investigations

The distribution curves of soil characteristics coincide with Gaussian normal distribution law. M.N. Goldshtein, 1971, believed that the soil mechanical properties were most characteristic for logarithmically normal distribution. From the study of D.C. Bugrov, 2003, the stochastic properties of the soil are best described by normal and improved Gram-Charlier distribution laws of RV. S. Macij believes that the RV of the angle of internal friction and unit cohesion of soils are better approximated by the normal or logarithmically normal distribution laws. The variation coefficients of natural soils properties are summarized in Table 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>sand</th>
<th>loamy sand</th>
<th>loam</th>
<th>clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture content w, %</td>
<td>30-50/4.4-49</td>
<td>10-30/6.2-27.7</td>
<td>8-28/3.8-15.0</td>
<td>4-25/12.65</td>
</tr>
<tr>
<td>Void ratio e</td>
<td>3-13/1.1-6.7</td>
<td>6-12/2.3-16.5</td>
<td>6-25/3.5-14.2</td>
<td>3-22/19.3</td>
</tr>
<tr>
<td>Density $\rho$, g/cm³</td>
<td>2-7.5/0.5-3.2</td>
<td>2-4.5/0.5-2.5</td>
<td>2.5-7.5/0.8-3.7</td>
<td>2-6/4.3</td>
</tr>
<tr>
<td>The density of particles $p_{v}$, g/cm³</td>
<td>/-0.3</td>
<td>/-0.2-0.65</td>
<td>/-0.2-0.6</td>
<td>/-0.8</td>
</tr>
<tr>
<td>Number of plasticity $I_p$</td>
<td>25-50/-</td>
<td>5-35/-</td>
<td>7-30/-</td>
<td></td>
</tr>
<tr>
<td>Limit of uncoiling $W_u$, %</td>
<td>6-17/-</td>
<td>5-25/-</td>
<td>7-27/-</td>
<td></td>
</tr>
<tr>
<td>Yield limit $W_y$, %</td>
<td>5-16/-</td>
<td>5-20/-</td>
<td>5-20/-</td>
<td></td>
</tr>
<tr>
<td>Shear strength $\tau$, kPa</td>
<td>/-</td>
<td>9-27/-</td>
<td>6-29/-</td>
<td></td>
</tr>
<tr>
<td>Deformation modulus $E$, MPa</td>
<td>/-</td>
<td>/-</td>
<td>15-35/18.6-65.4</td>
<td>/-</td>
</tr>
</tbody>
</table>

In the numerator are data by Ermolaev M.M. and Myheyev V.V., 1976 in the denominator – data by Bugrov D.C. and Shilin V.G., 2003

1.3 Study purposes

It is necessary to study and determine statistical parameters of strength and deformation characteristics of compacted soils, to investigate factors that influence the distribution of RV, to study patterns that occur in artificial masses during their service for implementation into engineering practice the stochastic models of artificial bases.

Therefore, the purpose of work is taken to carry out the investigations of physical and mechanical properties of compacted soils and to get the statistic data of these characteristics variation; to study experimentally the influence of properties variation of cushion compacted soils on its deformation, to estimate the heterogeneity of compacted soils and to decide on the correct application of distribution laws for RV of soil characteristics of artificial bases; to analyze the cushion TDS by MUE during the use of elastic-plastic model involving the imitation simulation; to estimate the probability of soil cushion failure. Probability of failure criteria adopted safety characteristic $\beta = F_{ser}/\sigma F > 3$ – number of deviations in the range from $F = 0$ to $F = F_{ser}$ (where $F = \bar{p}_u - \bar{p}$, $\sigma_F$ – deviation, $p_u$ – ultimate strength of compacted soil, $p$ – pressure on the base), maximum foundations settlements $S_u = 10$ cm and their relative differential settlements ($\Delta S / L_u = 0.002$ and ($\Delta S / L_i)_u = 0.004$.

2 LABORATORY AND FIELD TESTS OF COMPACTED SOILS

2.1 Objects of investigation

Authors carried out scientific and technical support of erection of some artificial bases (Vynnykov Y.L., 2010). On three sites a comprehensive field and laboratory studies of the properties of compacted soils were carried out.

Object № 1 – cushion of thickness $h = 4.0 – 4.4$ m and composed of loess loam. It was erected by surface compaction using heavy tamper and layer-by-layer rolling (50 cm) and loaded lorries weighing 20 tons and doing 8 – 12 passages on one track to the project value of dry density $\rho_d = 1.65$ g/cm³ ($k_s = 0.90$ – compaction coefficient).

Object № 2 – cushion of thickness $h = 3$ m and diameter $d = 22$ m under oil tanks with capacity of 3000 m³. Material was loess loamy sand and loam. Cushion was erected by layer-by-layer rolling doing 10 – 12 passages on one track by loaded lorries and carrying out 12 – 14 impacts by a tamper weighing 2 tons, which was thrown down from a height of 5 – 6 m.

Object № 3 – the fill with area of more than 1.9 million m³ and thickness 4 – 6 m for constructions of electrometallurgical plant. Material was overburden rocks, (fine, silty and medium grained sands, loamy sand and loam). Then the fill was compacted by vibrorollers (weight 14 – 16 tons, frequency 30/1.95 – 40/0.9 hertz/mm) and by pneumatic rollers (weight 22 tons) doing 4 – 8 passages on one track.

250
2.2 Methods of testing

The first stage consisted of sampling soil in foundation pits or determination of its grain-size composition in the laboratory, indicative characteristics and optimum moisture content for different shock impulses, the maximum dry density of soil and the values of the mechanical characteristics after achieving the project degree of soil compaction. For identification of optimum soils indexes the standard and modification Proctors’ tests were used.

The second stage consisted of the control of soil type brought to the site, fixation of the type of compacting mechanism, mode and number of passages on one track and measuring the thickness of layers on hooks before and after compaction. Third stage – sampling soil from each layer of cushion and their laboratory tests (compression, direct shear, penetration).

3 STATISTICAL ANALYSIS OF EXPERIMENTAL DATA

3.1 Methods of statistical analysis

The passive single-way analysis of variance plan of experiment for obtaining statistical data of physical and mechanical properties of compacted soil and variability of technological factors making the research program (Augusti G., 1988) was used. Using multi-way analysis of variance the mutual influence of variability of layers thickness, soil grain-size distribution, number of passages on one track and mode of compacting mechanism on variability of compacted soil characteristics was determined.

Mean value (expectation) for discrete and continuous RV is defined by (1):

\[
\bar{x} = \sum_{i=1}^{n} P_{x_i} \quad \bar{x} = \int x \cdot f(x)dx ,
\]

(1)

where \(P_{x_i}\) = probability value \(x_i\); \(x_i\) = possible values \(x\); \(f(x)\) = density of probability of continuous RV.

Dispersion for discrete and continuous RV is defined by (2):

\[
\overline{x^2} = \sum_{i=1}^{n} P_{x_i} (x_i - \bar{x})^2 ; \quad \overline{x^2} = \int (x - \bar{x})^2 f(x)dx ,
\]

(2)

Deviation (mean root square deviation) \(\sigma\) and variation coefficient \(\nu\) are defined by (3):

\[
\sigma = \sqrt{\overline{x^2}} ; \quad \nu = \frac{\sigma}{x}
\]

(3)

Central moments of \(k\)-th degree for discrete and continuous RV are defined by (4):

\[
\mu_k(x) = \sum_{i=1}^{n} P_{x_i} (x_i - \bar{x})^k ; \quad \mu_k(x) = \int (x - \bar{x})^k f(x)dx ,
\]

(4)

Asymmetry (skewness) \(A\) and excess (kurtosis) \(E\) are defined by (5):

\[
A = \frac{\mu_3}{\sigma^3} , \quad E = \frac{\mu_4}{\sigma^4} - 3 ,
\]

(5)

where \(\mu_3, \mu_4 = \) central moment of third and fourth order respectively.

Estimation of approximations of RV of characteristics of compacted soils, overburden rock and their mixtures in cushion is based on these types of distribution laws:

1) normal distribution (Gaussian distribution):

\[
p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \bar{x})^2}{2\sigma^2} \right) ,
\]

(6)

where \(x = RV\); \(n = \) number of values.

2) logarithmically normal (log-normal) distribution:

\[
p(x) = \frac{1}{\sigma \sqrt{2\pi}} \frac{1}{z} \exp \left( -\frac{\ln(x - \bar{z})^2}{2\sigma^2} \right) ,
\]

(7)
where $z = \ln \bar{x}$; $\sigma_z = \sigma_{\ln \bar{x}}$. Parameters of logarithmically normal law: mean value $\bar{x} = \exp(z + \sigma_z^2/2)$; deviation $\sigma_z^2 = (\exp(z + \sigma_z^2/2)(\exp(\sigma_z^2) - 1)$; asymmetry $A = (\exp(\sigma_z^2) + 2)^{\sqrt{(\exp(\sigma_z^2) - 1)}}$; median $Me = \exp(\bar{z})$; mode $M_0 = \exp(z - \sigma_z^2)$.

3) distribution that can describe a Gram-Charlier’s range:

$$p(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(- \frac{(x - \bar{x})^2}{2\sigma_x^2}\right) \cdot \left(Ex^4 + Ax^3 - 6Ex^2 - 3Ax + 3E + 1\right),$$  \hspace{1cm} (8)

where $A = \mu_3/6\sigma_3^3$; $E = (\mu_4 - 3\sigma_4^4)/(24\sigma_4^4)$.

4) exponential distribution

$$p(x) = I/\bar{x} \cdot \exp\left(- x/\bar{x}\right),$$  \hspace{1cm} (9)

5) polynomial-exponential distribution

$$p(x) = \exp(C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4),$$  \hspace{1cm} (10)

where $C_0…C_4 =$ polynomial coefficients, which are determined by estimation of mean value, deviation, asymmetry, excess and central moments of 1-4-th degree by solving the nonlinear equations.

Strength characteristics of soil ($c$ & $\varphi$) are totality of two RV (system of two RV). Geometrically it is interpreted as a random point with coordinates ($c; \varphi$) or random vector that is directed from the beginning to the point ($c;\varphi$). Distribution function of random vector is the probability of simultaneously realization of two inequalities: $X < x$ and $Y < y$, scilicet, $p(x,y) = P((X < x)(Y < y))$. For such distributions the central moment of k+s-th-degree for discrete and continuous RV is defined by (11):

$$\mu_{k+s}(x,y) = \sum_{i=1}^{x} \sum_{j=1}^{y} P_i(x_i - \bar{x})^i(y_j - \bar{y})^j; \mu_{k+s}(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^i(y - \bar{y})^j f(x,y)dx dy,$$  \hspace{1cm} (11)

Approximation of random distribution function $F(c, \varphi)$ was done on the basis of normal or logarithmically normal law (depending on that, which distribution law will have each of the RV).

$$p(c, \varphi) = \frac{1}{\sigma_c \sigma_\varphi \sqrt{2\pi}} \exp\left(- 0.5 \left(\frac{(c - \bar{c})^2}{\sigma_c^2} + \frac{(\varphi - \bar{\varphi})^2}{\sigma_\varphi^2}\right)\right);$$

$$p(x) = \frac{1}{\sigma_{\ln c} \sigma_{\ln \varphi} \sqrt{2\pi}} \exp\left(- 0.5 \left(\frac{(\ln c - \ln \bar{c})^2}{\sigma_{\ln c}^2} + \frac{(\ln \varphi - \ln \bar{\varphi})^2}{\sigma_{\ln \varphi}^2}\right)\right),$$  \hspace{1cm} (12)

To check the adequacy of the adopted theoretical distribution law, the Pearson’s test $\chi^2$ was used.

3.2 The volume of experimental sampling

As a result of field studies, the RV sampling of test characteristics and technological parameters were obtained. Their number was for: humidity $w n = 100$, dry density of soil $\rho_d n = 55$, angle of internal friction $\varphi$ and unit cohesion $c n = 78$ in horizontal rings and $n = 28$ – in vertical (object № 1); $w n = 155$, $\rho_d n = 140$, unit soil penetration resistance $R n = 104$ (object № 2); humidity $w n = 3000$, dry density of soil $\rho_d n = 3000$, angle of internal friction $\varphi$ and unit cohesion $c n = 50$, modulus of deformation $E n = 1500$, measurements of passages on one track by compacting mechanism $n = 20$ and layer thickness $n = 100$ (object № 3).

3.3 Results of statistical analysis

To describe the experimental distribution of physical characteristics RV of compacted soils it is reasonable to use the normal distribution law and for dry density of soil mixture – polinomo-exponential distribution. Graphical interpretation of these results is shown in Figure 1.

Deformation modulus $E$ of compacted soils and their mixtures it is better to describe by log-normal law. Statistical parameters of these RV depend on the pressure in the oedometer. Research results are
presented in Figure 2. Angle of internal friction $\phi$ and unit cohesion $c$ of compacted soils and their mixtures are random vectors and are best described by normal and log-normal distribution laws. Graphical interpretation of research results is shown in Figure 3.

4 PROBABILISTIC DESIGN OF FOUNDATIONS ON SOIL CUSHIONS

4.1 Grounds of application of the probability design theory for estimation the TDS of foundations artificial bases

To apply the design scheme as linearly-deforming half-space basis for determining its settlement it is necessary that the average pressure under the foundation should not exceed the soil design strength $R$. The value of $R$ is a RV due to variability of internal friction angle $\phi$, unit cohesion $c$ and soil unit weight $\gamma$, which are included as arguments to the function $R = f(\phi, c, \gamma)$. The pressure under the foundation $p$ is also RV because loads and actions are random.

Application limit of base model as a linearly-deforming half-space to calculate the settlement of the foundation using function of random arguments $P = R - \rho_0$ is presented in Figure 4.

Values of foundation settlement $S_L$ at linear stage of soil deformations are also function of random arguments due to variability main modulus of deformation $E$, which varies within a layer by the applied law.

The modulus of deformation $E$ depends on the type and condition of the soil, the additional stress in the layer. The additional vertical normal stress in soil $\sigma_{zp}$, which depends on distribution of foundation external load (Pichugin S.F., 2009, Rethaty, 1988), and the soil unit weight $\sigma_{zg0}$ that lies above foundation are influenced by the settlement variability. These parameters are included as arguments to the function $S_L = f(E, \sigma_{zp})$. 
Figure 2. Density distribution of random variables of soil deformability characteristics of cushions: $n$ – number of random variables.

Around plastic deformations zones the nonlinear settlement $S_{NL}$ of bases foundations take place. These values are also RV due to variability of foundations settlements by pressure $p$ equal to the soil design strengths $R$, the soil ultimate strength $p_u$ and vertical stresses of its own weight of soil at the bottom of foundation $\sigma_{zg0}$. This is due to the heterogeneity of physical and mechanical soils properties. These parameters are included as arguments to the function $S_{NL} = f(p, R, p_u, \sigma_{zg0})$.

Figure 3. Density distribution of random variables of soil strengths characteristics of cushions.
4.2 Probability design of artificial bases

The statistical distribution parameters of RV of soil design and ultimate strengths (Table 2) using methods of linearization, Monte-Carlo (Raizer V.D., 1995), A.S. Lykov’s, 2008, V.P. Chirkov’s, 2006, and experimental data of RV of compacted soils characteristics were obtained. The statistical distribution parameters of RV of foundations settlements on single- and multi-layered soil cushions in linear stage and accounting the limits of variability of bases linear deformation were obtained (Figure 5).

4.3 Probability design of TDS of soil cushions by numerical simulation method

The simulation results of TDS of artificial bases by MUE using the elastic-plastic model and Monte-Carlo method with $10^4$ iterations (Won J.Y., 2009, Zeigler M., 2006, Staveren M.T., 2009) are shown in Figure 6. Comparative analysis of statistical characteristics of foundations settlements for single- and multi-layered soil cushions by different methods of probability design are presented in Table 3.

Table 2. Statistical parameters of random variables distribution of design and ultimate strengths of compacted soil

<table>
<thead>
<tr>
<th>Characteristic name</th>
<th>Simulation by approximating polynomial</th>
<th>Monte-Carlo method</th>
<th>O. Lykov’s and V. Chyrkov’s methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value, kPa</td>
<td>268.3/1208.5</td>
<td>273.5/1606.3</td>
<td>276.2/1660</td>
</tr>
<tr>
<td>Deviation, kPa</td>
<td>59.6/442.3</td>
<td>59.6/550.3</td>
<td>102.8/621.1</td>
</tr>
<tr>
<td>Variation coefficient, %</td>
<td>22.2/36.6</td>
<td>21.8/34.4</td>
<td>36.3/37.5</td>
</tr>
</tbody>
</table>

Compacted soil design strength $R$ / Compacted soil ultimate strength $p_u$
For single-layered soil cushion in linear stage of base deformation

For multi-layered soil cushion in linear stage of base deformation

For single-layered soil cushion accounting the limits of variability of bases linear deformation

For multi-layered soil cushion accounting the limits of variability of bases linear deformation

Figure 5. Density distribution of random variables of foundations settlements on cushion according the results of statistical simulation: n – number of random variables

Figure 6. Results of numerical simulation by method of ultimate elements in the probabilistic formulation

Table 3. Comparative analysis of statistical characteristics of foundations settlements for single- and multi-layered soil cushions

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Foundation settlements in linear stage of base deformation</th>
<th>Foundation settlements taking into account the limits of variability of bases linear deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linearization method</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Single-layered cushion</td>
<td>Multi-layered cushion</td>
</tr>
<tr>
<td>Mean value, cm</td>
<td>0.67</td>
<td>2.07</td>
</tr>
<tr>
<td>Deviation, cm</td>
<td>0.22</td>
<td>0.46</td>
</tr>
<tr>
<td>Variation coefficient, %</td>
<td>33</td>
<td>22</td>
</tr>
<tr>
<td>Probability of failure (excess S &gt; S_u = 10 cm)</td>
<td>(S_u - SMV &gt;&gt; 5σ)</td>
<td></td>
</tr>
</tbody>
</table>
CONCLUSIONS

For physical soils properties of cushions the correct application of the normal distribution law, for soil mixtures – polynomial-exponential distribution, for modulus of deformation of soils and soil mixtures – log-normal distribution, for angle of internal friction of soil – normal distribution, for unit cohesion of soil – log-normal distribution has been grounded. By comparison of variation coefficients of characteristics of natural and compacted soils it was proved that in cushion the soil is more homogeneous than in natural state. Variation coefficient of soil dry density is 2 – 4.4 %, moisture – 23 – 36 %, soil unit weight – 4 – 4.6 %, soil modulus of deformation – 33 – 57 %, angle of internal friction of soil – 11 %, unit cohesion of soil – 25 %.

The variation coefficient of design strength of compacted soil is 21.8 – 36.3 %, the ultimate strength – 34.4 – 37.5 %. Therefore the probabilistic approach shows that even without excess pressure under the foundation the design strength of soil is probability as linear so non-linear stages of base deformations. It is due to variability of characteristics of compacted soils and random loads on foundations. The simulation of TDS of artificial bases by MUE using the elastic-plastic model and Monte-Carlo method correctly describes the deformation of cushions. For multi-layered cushion the variation coefficient of settlement $v_s$ is less than single-layered, while mean value of settlement is 2.4 times more. Ratio $v_s$ increases with increasing heterogeneity of layers, particularly by larger compressibility of the upper layers than subsoil and by increasing the ratio of modulus of deformation in them. The way of cushions erection with different degrees of layers compaction reduces the variability of foundations settlements. The failure probability of cushion according to safety characteristics is acceptable $\beta = 4.82 > 3$. According to the criterion of relative differential settlements of foundations on single-layered cushion the failure probability reaches 10 % by ultimate value $(\Delta S/L)_u=0.002$ and 3 % by $(\Delta S/L)_u=0.004$. For multi-layered cushion, these values are only 0.02 % and 0.0006 %.

REFERENCES


