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Optimization of Reinforced Concrete Retaining Walls Using Ant Colony Method

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ABSTRACT: Optimization of concrete retaining walls is an important task in geotechnical and structural engineering. Classical optimization search methods are rudimentarily based on direct search methods. Direct search methods belong to a class of optimization methods that do not compute derivatives. However, these algorithms suffer from both trapping in local minima and increasing run time. In order to reduce the possibility of suffering from this problem, the heuristic approaches are more favored among the scientists.

This paper applies a methodology to arrive at optimal design of concrete retaining wall using ant colony optimization (ACO) algorithm that is a general search technique for the solution of difficult combinatorial problems with its theoretical roots based on the foraging behavior of ants. The algorithm is used to find the minimum weight and cost for concrete retaining walls. Coulomb lateral earth pressure theory is used to derive the lateral total thrust on the wall. The results are compared with other available optimization scheme applied by other researchers. The results clearly indicate that ACO yields the solutions for all benchmarks due to its capability to explore and exploit the solution space effectively. As a result, it can be used for optimizing the reinforced concrete retaining walls.

Keywords: Ant Colony, Education, Optimization, Concrete Retaining Wall, Swarm Intelligent.

1 INTRODUCTION

Concrete retaining walls are most widely used structures in civil engineering practice. Such walls are commonly used to support earth, coal, ore piles, and water. Optimization of retaining walls is necessary due to economical consideration. Current optimization structural softwares for retaining wall design often lack the ability to find out optimal design because of their deterministic nature, while those employing stochastic methods are not tailored specifically for retaining walls and massive concrete structures. Classic optimization search methods are rudimentarily based on direct search methods. Direct search methods belong to a class of optimization methods that do not compute derivatives. Examples of direct search method are the Nelder Mead Simplex method, Hooke and Jeeves's pattern search, the box method, and Dennis and Torczon's parallel direct search algorithm employing a multi-sided simplex. However, these algorithms suffer from both trapping in local minima and increasing running time.

In this paper, a methodology is presented to arrive at optimal design of concrete retaining wall using Ant Colony Optimization (ACO) algorithm that is a general search technique for the solution of difficult combinatorial problems with its theoretical roots based on the foraging behavior of ants. ACO is based on the indirect communication of a colony of simple agents, called artificial ants, mediated by artificial pheromone trails. The pheromone trails in ACO serve as distributed numerical information, which the ants use to probabilistically construct solutions to the problem being solved.

Optimum design of retaining walls has been the subject of a number of studies. Saribas and Erbatur presented a detailed study on optimum design of reinforced concrete cantilever retaining walls using cost and weight of walls as objective functions. In their study, they controlled overturning failure, sliding failure, shear and moment capacities of toe slab, heel slab, and stem of wall as constraints [1]. Ceranic and Fryer proposed an optimization algorithm based on Simulated Annealing, which can compute the minimum cost design of reinforced concrete retaining walls [2]. Sivakumar and Munwar introduced a Target Reliability Approach for design optimization of retaining walls [3]. Ahmadi Nedushan and Varaee proposed an optimization algorithm based on Particle Swarm Optimization. They claim that this method require fewer number of function evaluations, while leading to better results in optimization of retaining wall [4].

2 ANT COLONY OPTIMIZATION

From years of study and observation, ethologists have found that ants, although almost completely blind, are able to successfully navigate between their nest and food sources and in the process, discover the shortest path between these points [6]. The ant colony is able to determine the shortest path to food sources using pheromone trails. As an ant moves, it deposits pheromones along its path. A single ant will move essentially at random, however, another ant following behind it will detect the pheromone trail left by the lead ant and will be inclined to follow it. Once an ant selects a path, it lays additional pheromones along the path, reinforcing the increasing pheromone level of the trail and increasing the probability that subsequent ants will follow this path. This type of collective feedback and emerging knowledge in the ant colony is a form of autocatalytic behavior [7].

In the past few years, ant colony optimization (ACO) algorithms have undergone many changes throughout their development, but each different system retains the fundamental ant behavioral mechanisms. The fundamental theory in an ACO algorithm is the simulation of the autocatalytic, positive feedback process exhibited by a colony of ants. This process is modeled by utilizing a virtual substance called ‘‘trail’’ that is analogous to pheromones used by real ants. Each ACO algorithm follows a basic computational structure outlined by the pseudocode in Fig. 1. An ant begins at a randomly selected point and must decide which of the available paths to travel. This decision is based upon the intensity of trail present upon each path leading to the adjacent points. The path with the most trail has a higher probability of being selected. If no trail is present upon a path, there is zero probability that the ant will choose that path. If all paths have an equal amount of trail, then the ant has an equal probability of choosing each path, and its decision is random.

An ant chooses a path using a decision mechanism and travels along it to another point. Some ACO algorithms now apply a local update to the trail (Fig. 1). This process reduces the intensity of trail on the path chosen by the ant. The idea is that when subsequent ants arrive at this point, they will have a slightly smaller probability of choosing the same path as other ants before them. This mechanism is intended to promote exploration among the ants, and helps prevent early stagnation of the search and premature convergence of the solution. The amount of this trail reduction is not great enough to prevent overall solution convergence. The ant continues to choose paths to travel between points, visiting each point, until all points have been visited and it arrives back at its point of origin. When it returns to its starting point, the ant has completed a tour (Fig. 1).

```
Initialize Trail
Do While (Stopping Criteria Not Satisfied)- Cycle Loop
        Do Until ( Each Ant Completes a Tour)- Tour Loop
                Ant Decision Mechanism
                Local Trail Update
        End Do

        Global Trail Update
End Do
```

Figure 1. Ant Colony Optimization algorithm in pseudocode.

The combination of paths an ant chooses to complete a tour is a solution to the problem, and is analyzed to determine how well it solves the problem. The intensity of trail upon each path in the tour is then adjusted through a global update process. The magnitude of the trail adjustment reflects how well the solution produced by an ant’s tour solves the problem. The paths that make up the tours that best solve the problem receive more trail than those paths that make up poor solutions. In this way, when the ant begins the next tour, there is a greater probability that an ant will choose a path that was part of a tour that performed well in the past. When all the ants have completed a tour and all of the tours have been analyzed and the trail levels on the paths have been updated, an ACO cycle is complete [10]. A new cycle now begins and the entire process is repeated. Eventually almost all of the ants will make the same tour on every cycle and converge to a solution. Stopping criteria are typically based on comparing the best solution from the last cycle to the best global solution. If the comparison shows that the algorithm is no longer improving the solution, then the criteria are reached [9].

The first ant algorithm was developed by Dorigo, referred to as ant system (AS) [8]. AS improves on SACO by changing the transition probability, P_{ij}^{k}, to include heuristic information, and by adding a memory capability by the inclusion of a tabu list. In AS, the probability of moving from node i to node j is given as

$$P_{ij}^{k}(t)=\begin{cases}\dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum_{j}\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)} & if \quad j\in N_i^k \\ 0 & if \quad j\notin N_i^k\end{cases} \tag{1}$$

where τ_{ij}^{α} represents the *a posteriori* effectiveness of the move from node i to node j, as expressed in the pheromone intensity of the corresponding link, *(i, j)*; η_{ij} represents the *a priori* effectiveness of the move from *i* to *j* (i.e. the attractiveness, or desirability, of the move), computed using some heuristic. The pheromone concentrations, τ_{ij}, indicate how profitable it has been in the past to make a move from *i* to *j*, serving as a memory of previous best moves [8].

3 CONCRETE RETAINING WALL DESIGN

Consider a concrete retaining wall shown in Fig. 2 with a height of H. Expressions for factors of safety against overturning failure, sliding failure, eccentricity failure and bearing failure are given in the following section.

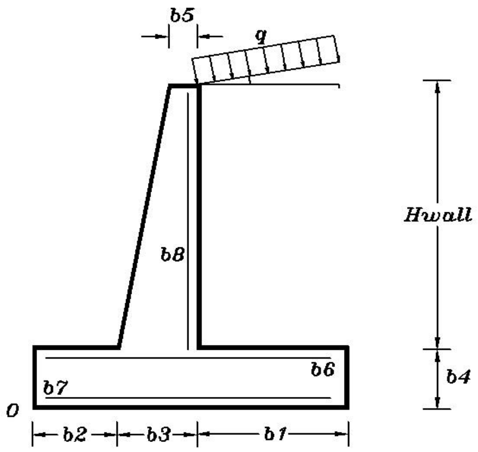

Figure 2. Concrete retaining wall section.

Rankine's earth-pressure theory corresponds to the stress and deformation conditions for the states of plastic equilibrium. The resultant active pressure on a vertical plane of height *H* through a semi-infinite mass of soil whose surface is inclined at an angle β to the horizontal is:

$$P_a = 0.5\gamma h^2\left(\cos\beta\frac{\cos\beta-\sqrt{\cos^2\beta-\cos^2\phi}}{\cos\beta+\sqrt{\cos^2\beta-\cos^2\phi}}\right) \tag{2}$$

where ϕ is the backfill friction angle, h is the wall height, b is the backfill surface with horizontal direction, and γ is the backfill unit weight.

It is usually required that the factor of safety against overturning be at least 1.5. However, the stability number for overturning is generally on the order of 1.5 to 2.0 depending on the importance of the wall. This is commonly determined by taking moments about the toe of all forces acting on the wall above the plane of base.

The factor of safety is the ratio of the moment of the forces resisting overturning to the moment of forces tending to cause overturning. Overturning about the toe can be computed by taking a moment summation about that point.
The sum of the moments of forces tending to resist overturning about point *O* (Fig. 2) can be expressed as:

$$\Sigma M_R = M_c + M_s + M_q + M_v \tag{3}$$

The sum of the moments of forces tending to overturning about point *O* is expressed as:

$$\Sigma M_0 = P_{ah}\bar{y} \tag{4}$$

where M_c, M_s and M_v are moments about the toe point *O* as shown in Fig. 2 due to weight W_c, W_s and P_{av} respectively. W_c = weight of the concrete; W_s = weight of the soil; P_{av} = vertical component. M_q is related to surcharge load. Various parameters are defined as: ϕ_1 = friction angle of the back fill soil, δ = wall friction angle $2\phi_1/3$, γ_1 = unit weight of the backfill soil (KN/m^3). $\bar{y}$ = moment arm. The factor of safety against overturning failure can be written as:

$$FS_{overturning} = \frac{\Sigma M_R}{\Sigma M_O} \tag{5}$$

The overall wall stability requires safety against sliding. The sum of the horizontal resisting forces can be written as:

$$\Sigma F_r = C_a B' + \Sigma W \tan\delta + P_D \tag{6}$$

The sum of the horizontal driving forces is given by:

$$\Sigma F_d = P_{ah} \tag{7}$$

where C_a = adhesion coefficient between base slab and base soil; γ_2 = unit weight of soil below the base slab of retaining wall (KN/m^3); ϕ_2 = friction angle of the soil below the base slab of the retaining wall; ΣW = sum of the vertical forces acting on retaining wall; P_D = any passive earth pressure developed by the soil in front of the wall. The factor of safety against sliding failure can be expressed as:

$$FS_{sliding} = \frac{\Sigma F_r}{\Sigma F_d} \tag{8}$$

The stability number is usually on the order of 1.5 to 2.0, again depending on the importance of the wall.

For stability, the line of action of the resultant force must lie within the middle third of the foundation base. The factor of safety against eccentricity failure is given by:

$$\frac{B}{6} > e \tag{9}$$

where B = base width of the wall and e=eccentricity of the result and force.

In many instances involving the construction of embankments, overpasses or bridge approaches, it is necessary to construct a retaining wall backfilled to a considerable elevation above the existing ground surface. In these circumstances, precaution must be taken to ensure that a base failure beneath the weight of the fill does not occur.

If the subsoil consists of sand or gravel, there is no likelihood of such a failure. However, if the subsoil consists of clays or clayey slit, it is necessary to check their supporting capacity. The stability of the base against a bearing capacity failure is achieved by using a suitable safety factor with the computed ultimate bearing capacity where the safety factor is usually taken as 2 for granular soil and 3.0 for cohesive soil. The allowable soil pressure can be computed using the following bearing capacity equation:

$$q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5B_\gamma N_\gamma d_\gamma i_\gamma \tag{10}$$

where c=cohesion, d = depth factors; i = inclination factors, B width of the footing, and $\bar{q} = \gamma D$ in which D = depth of the wall base to the ground surface. In the above expression, N_c, N_q and N_γ are bearing capacity factors as functions of ϕ [5].

The maximum intensity of soil pressure at toe can be written as:

$$q_{max} = \frac{\sum W}{B'} \tag{11}$$

The factor of safety against bearing capacity failure can be defined as:

$$FS_q = \frac{q_{ult}}{q_{max}} \tag{12}$$

4 CONCRETE RETAINING WALL OPTIMIZATION

An optimal concrete retaining wall design is one with the minimal weight and cost that still allows the wall to satisfy given constraints. The basic stability requirements for a wall for all conditions of loading are overturning, sliding, and bearing capacity, and rotation and settlement [5].

The wall optimization problem can be expressed as:

$$g = Min W(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) \tag{13}$$

$$h = Min C(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) \tag{14}$$

while considering:

$$FS_q \geq 3 \tag{15}$$

$$FS_{over} \geq (FS_{over})_{all} \tag{16}$$

$$FS_{slid} \geq (FS_{slid})_{all} \tag{17}$$

where g = objective function; W = total weight; h = objective function; C = total price; b_1 = Heel projection; b_2 = Toe projection; b_3 = Stem thickness at bottom; b_4 = Thickness of base slab; b_5 = Stem thickness at top; b_6 = Horizontal steel area of the heel per unit length of the wall; b_7 = Horizontal steel area of the toe per unit length of the wall; b_8 = Vertical steel area of the stem per unit length of the wall; FS_q = factor of safety against bearing capacity failure; FS_{slid} = safety factor against sliding; FS_{over} = safety factor against overturning; $(FS_{slid})_{all}$ and $(FS_{over})_{all}$ = allowable values for FS_{slid} and FS_{over} respectively.

Two weight and cost objective functions have been chosen to optimize the wall from two viewpoints. In cost minimization, the objective function is defined as:

$$h(x) = C_s W_s + C_c V_c \tag{18}$$

where C_s = unit cost of steel; C_c = unit cost of concrete; W_s = weight of steel per unit length of the wall; and V_c = volume of concrete per unit length of the wall.

For weight optimization, the objective function is defined as:

$$g(x) = W_s + 100 V_c \gamma_c \tag{19}$$

where γ_c = unit weight of concrete, and 100 is used for consistency of units.
The ACO algorithm adapted for concrete retaining wall optimization is developed in the Fig. 3.

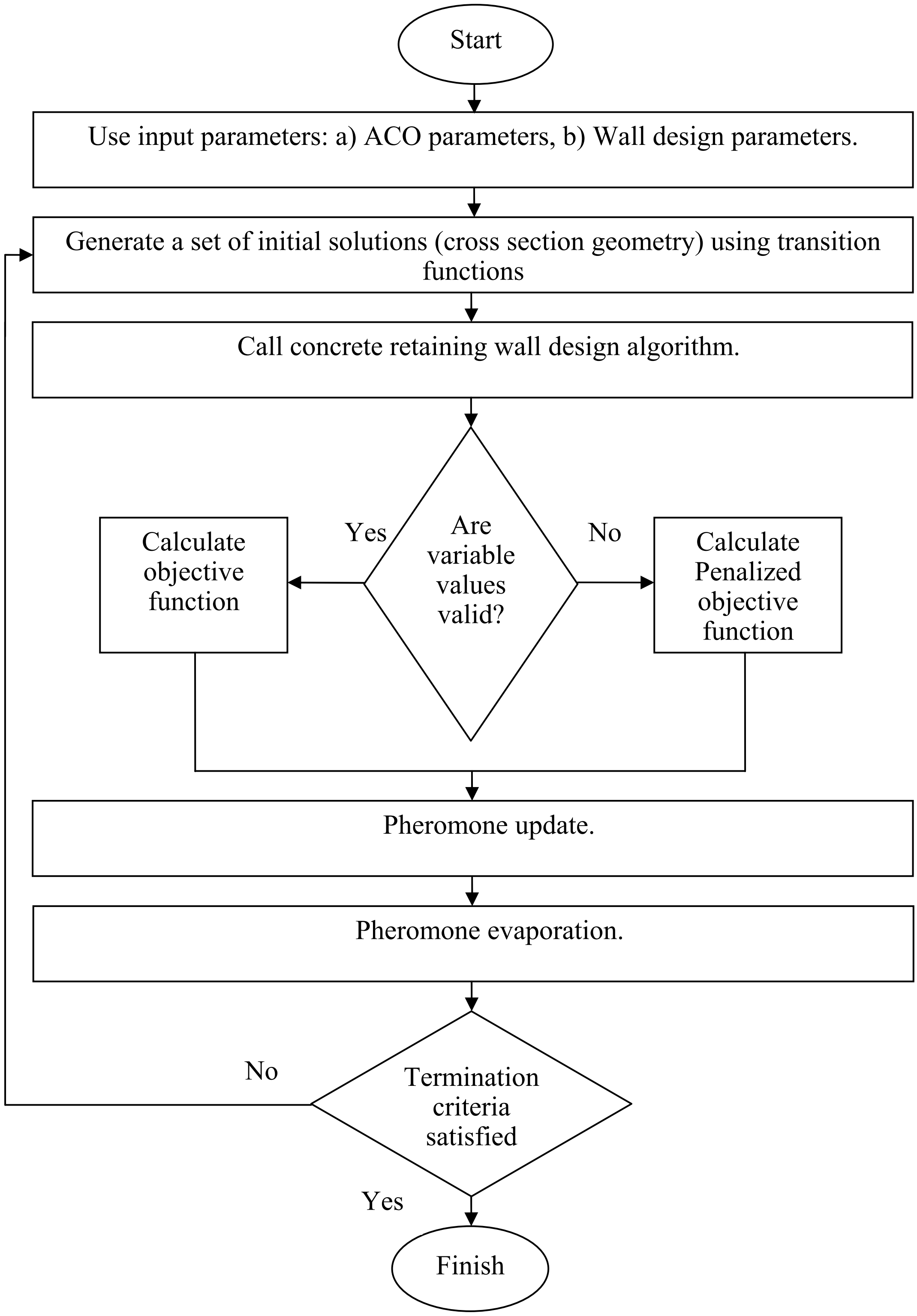

Figure 3. ACO application applied reinforced wall.

5 VERIFICATION

5.1 *Example 1*

To check the performance, robustness, and accuracy of the above algorithm, a retaining wall studied by Saribas and Erbatur [1] is considered. The details of this wall and other necessary input parameters are given in Table 1. It is noted that all the values given in this table are for a unit length of the wall. In the example problem, SI units are used.

Table 1. Input parameters

Input parameters	Unit	Symbol	Value
Height of stem	m	H	4.5
Yield strength of reinforcing steel	Mpa	f_y	400
Compressive strength of concrete	Mpa	f'_c	21
Surcharge load	kPa	q	30
Backfill slope	degree	β	15
Internal friction angle of retained soil	degree	ϕ_1	36
Internal friction angle of base soil	degree	ϕ_2	34
Unit weight of retained soil	kN/m^3	γ_1	17.5
Unit weight of base soil	kN/m^3	γ_2	18.5
Unit weight of concrete	kN/m^3	γ_c	23.5
Cohesion of base soil	kPa	C	100
Depth of soil in front of wall	m	D	0.75
Cost of steel	\$/kg	C_s	0.40
Cost of concrete	kg/m^3	C_c	40
Factor of safety for overturning stability	-	N_o	1.5
Factor of safety against sliding	-	N_s	1.5
Factor of safety for bearing capacity	-	SF_q	3.0

The optimum design results are shown in Tables 2 and 3. The optimum values of the design variables are tabulated together with suggested, upper and lower limits for easy interpretation (Table 2).

Table 2. Optimum values of design variables

Design variable	unit	Lower bounds	Upper bounds	Optimum values minimum cost	Optimum values minimum weight
b_1	m	1.059	1.833	1.385	1.385
b_2	m	0.655	1.167	1.143	1.143
b_3	m	0.25	0.50	0.251	0.251
b_4	m	0.40	0.50	0.40	0.40
b_5	m	0.25	0.25	0.25	0.25
b_6	cm^2/m	11.059	67.68	14	14
b_7	cm^2/m	11.059	67.68	14	14
b_8	cm^2/m	5.761	67.68	59	59

Table 3. Optimum values of objective function

Objective function	Unit	Optimum value (Saribas)	Optimum value (AS)
Minimum cost	\$/m	189.546	201.185
Minimum weight	kg/m	5280.96	5540.3

As seen in Table 3, the results obtained from the present optimization analysis (AS) and those reported by Saribas and Erbatur are in close agreement. The deviations between two methods are 6.1% and 4.9% for cost and weight optimizations, respectively.

5.2 *Example 2*

For further validation of the developed optimization method, another example is considered and the results are compared with those given by Saribas and Erbatur [1], Sivakumar and Munwar [3], Bowles [5], and Das [11]. Three walls with heights of 3, 4, and 5 m are considered. Other specifications for the design of these retaining walls are presented in Table 4. To compare the results with Das and Bowles, a value of 0.3 m is assumed for b_5 for all walls.

Tables 5 to 8 compare optimum design results determined from the present method and those given by others as referenced. It is noted that in these tables, some fixed values are considered for b_1, b_2, and b_4. This stems from the fact that Das [11] and Bowles [5] do not optimize these values and they just recommend some experienced-based approximate values which are normally used in practice. As will be seen in these tables, these values can be easily optimized using the method described in this research or other optimization approaches.

Table 4. Input parameters [3]

Input parameters	Unit	Symbol	Value
Height of stem	m	H	3–4–5
Yield strength of reinforcing steel	Mpa	f_y	400
Compressive strength of concrete	Mpa	f'_c	21
Surcharge load	kPa	q	25
Backfill slope	degree	β	10
Internal friction angle of retained soil	degree	ϕ_1	36
Internal friction angle of base soil	degree	ϕ_2	0
Unit weight of retained soil	kN/m^3	γ_1	17.5
Unit weight of base soil	kN/m^3	γ_2	18.5
Unit weight of concrete	kN/m^3	γ_c	23.5
Cohesion of base soil	kPa	C	125
Depth of soil in front of wall	m	D	0.75
Cost of steel	\$/kg	C_s	0.40
Cost of concrete	kg/m^3	C_c	40
Factor of safety for overturning stability	-	N_o	1.5
Factor of safety against sliding	-	N_s	1.5
Factor of safety for bearing capacity	-	SF_q	3.0

Table 5. Comparative study for the projection of toe from the base of the stem b_2

Height of stem	3.0	4.0	5.0
Das $0.1H$	0.3	0.4	0.5
Bowles $0.233H$	0.7	0.933	1.167
Saribas and Erbatur for minimum cost	0.443	0.582	0.727
Saribas and Erbatur for minimum weight	0.436	0.603	0.789
Sivakumar and Munwar	0.72	0.96	1.20
Present study for minimum cost	0.555	0.726	0.939
Present study for minimum weight	0.629	0.842	1.013

Table 6. Comparative study for projection of heel from the base of the stem b_1

Height of stem	3.0	4.0	5.0
Das $0.1H$	1.5	2.0	2.5
Bowles $0.233H$	1.101	1.468	1.835
Saribas and Erbatur for minimum cost	0.864	1.161	1.411
Saribas and Erbatur for minimum weight	0.873	1.191	1.473
Sivakumar and Munwar	0.6	0.8	1.0
Present study for minimum cost	1.026	1.375	1.687
Present study for minimum weight	0.944	1.255	1.589

Table 7. Comparative study for the thickness of base slab b_4

Height of stem	3.0	4.0	5.0
Das $0.1H$	0.3	0.4	0.5
Bowles $0.233H$	0.3	0.4	0.5
Saribas and Erbatur for minimum cost	0.273	0.364	0.455
Saribas and Erbatur for minimum weight	0.273	0.364	0.455
Sivakumar and Munwar	0.3	0.4	0.5
Present study for minimum cost	0.271	0.363	0.450
Present study for minimum weight	0.270	0.363	0.451

As seen in Tables 5 to 8, the current optimization method gives reasonable results. It is noted that the values obtained from the present developed optimization from viewpoints of weight and cost of retaining walls are relatively greater than those given by Saribas and Erbatur [1]. This could be attributed to the fact that the results of Saribas and Erbatur [1] do not account for uncertainties that exist in the soil, concrete, steel properties, and geometric properties of the wall [3]. In addition, both methods use different optimization algorithm.

Table 8. Comparative study for the cross sectional area of the retaining wall (m^2)

Height of stem	3.0	4.0	5.0
Das $0.1H$	1.440	2.380	3.550
Bowles $0.233H$	1.440	2.380	3.550
Saribas and Erbatur for minimum cost	1.340	2.071	3.037
Saribas and Erbatur for minimum weight	1.340	1.962	2.713
Sivakumar and Munwar	1.395	2.080	2.875
Present study for minimum cost	1.407	2.073	2.816
Present study for minimum weight	1.405	2.070	2.811

6 CONCLUSION

The present paper has shown how engineers can learn from ant colony for optimization of reinforced concrete retaining walls. By validation of the predicted results on optimizing the retaining wall that ant colony optimization (ACO) is a successful random search method that educates engineers to find global minimum in difficult combinational problems, which can hardly be attained by classical optimization methods. It has been demonstrated that the presented algorithm is able to find quickly the minimum weight and minimum cost justified geometry and specifications for reinforced concrete retaining walls.

REFERENCES

[1] Saribash, A., & Erbatur, F. 1996. Optimization and Sensitivity of Retaining Structurs. Journal of Geotechnical Engineering, ASCE, Vol. 122, No.8, pp. 649-656.
[2] Ceranic, B., & Fryer, C. 2001. An Application of Simulated Annealing to Optimum Design of Reinforced Concrete Retaining Structures. Journal of Computers and Structures, Vol. 79, pp. 1569-1581.
[3] Sivakumar Babu, G.L., & Munwar, Basha B. 2008. Optimum Design of Cantilever Retaining Walls Using Target Reliability Approach. International Journal of Geomechanics, ASCE, Vol. 8, No. 4, pp. 240-252.
[4] Ahmadi Nedushan, B., & Varaee, H. 2009. Optimal Design of Reinforced Concrete Retaining Walls Using a Swarm Intelligence Technique. The first International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering, UK.
[5] Bowles, J.E. 1996. Foundation Analysis and Design. The McGraw-Hill Companies, Inc. Fifth edition.
[6] Wahde, M. 2008. Biologically Inspired Optimization Methods: An Introduction. Boston: Wit Pr/Computational Mechanics.
[7] Bonabeau, E., & Dorigo, M., & Theraulaz, G. 1999. Swarm Intelligence: From Natural to Artificial Systems (Santa Fe Institute Studies on the Sciences of Complexity). New York: Oxford University Press, USA.
[8] Engelbrecht, A. 2007. Computational Intelligence: An Introduction. New York. NY: Wiley.
[9] Camp, C., & Bichon, B. 2004. Design of Space Trusses Using Ant Colony Optimization. Journal of Structural Engineering, ASCE ,Vol. 130, No. 5, pp. 741-751.
[10] Dorigo, M., & Maniezzo, V., & Colorni, A. 1996. The Ant System: Optimization by a Colony of Cooperating Agents. IEEE Transactions on Systems, Man, and Cybernetics–Part B, pp. 1-13.
[11] Das, B.M. 1999. Principles of Foundation Engineering. Fourth edition, Publishing Workflow System, Pacific Grove, Calif.