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Sensitivity analysis and design of reinforced concrete cantilever retaining walls using bacterial foraging optimization algorithm

M. Ghazavi
Faculty of Civil Engineering, K. N. T. University of Technology, Tehran, Iran

V. Salavati
Islamic Azad University, Central Tehran branch, Tehran, Iran

ABSTRACT: This paper presents an economic optimization and sensitivity analysis for reinforced concrete cantilever (RCC) retaining walls using the bacterial foraging optimization algorithm (BFOA). For this purpose and to solve the optimization problem, BFOA is inspired by the social foraging behavior of *Escherichia coli*. The results of analyses based on the BFOA method have been compared with other available optimization data extracted from other optimization schemes. The results show that the BFOA method can be successfully applied to find the minimum cost design of RCC retaining walls, overcoming the difficulties associated with the practical and realistic assessment of the structural costs and their complex inter-relationship with the imposed constraints on the solution space. A detailed sensitivity analysis for selected design variables, parameters and related safety factors will be presented.

Keywords: retaining walls, reinforced concrete, bacterial foraging optimization algorithm, sensitivity analysis.

1 INTRODUCTION

Concrete cantilever retaining walls are one soil-structure system used to support earth backfills. The construction of concrete cantilever retaining walls is typically motivated by the need to eliminate slope failure and instability in road construction projects. They are also used to support bridges and similar overpass and underpass elements. Their design must satisfy two major requirements: internal stability, which is ensured by sufficient resistance against bending moments and shear forces, and external stability, which means that, except for small movements necessary to mobilize the earth pressures, the wall must be in equilibrium with respect to external forces.

Current design of concrete retaining walls is highly dependent on the experience of engineers. The structure is defined on a trial-and-error basis. Tentative design must satisfy the limit states prescribed by concrete codes. This process leads to safe designs, but the cost of the RCC walls is, consequently, highly dependent upon the experience of the designer. Structural optimization methods are good alternatives to designs based on experience.

Over the past years a number of optimization algorithms have been used extensively in structural optimization problems, from exact methods, to heuristic search methods widely applied for global optimization. The exact methods usually following iterative techniques of linear programming to find the optimal solution, and heuristic search methods usually used stochastic search algorithms to find the optimal solution. The first category is useful when the number of variables is limited and they require a small number of iterations. The second category involves simple algorithms such as genetic algorithm, particle swarm, ant colony, and so on (Perea et al., 2007). However, they also require a considerable computational effort, since they include a large number of iterations in which the objective function is evaluated and the structural constraints are checked.

Optimum design of retaining walls has been the subject of a number of studies. Saribas and Erbatur (1996) used exact method to solve seven design variable optimization problem. Ceranic et al. (2001) applied simulated annealing (SA) to minimum cost design of retaining walls. Yepes et al. (2008) implemented a parametric study of optimum earth-retaining walls by SA.

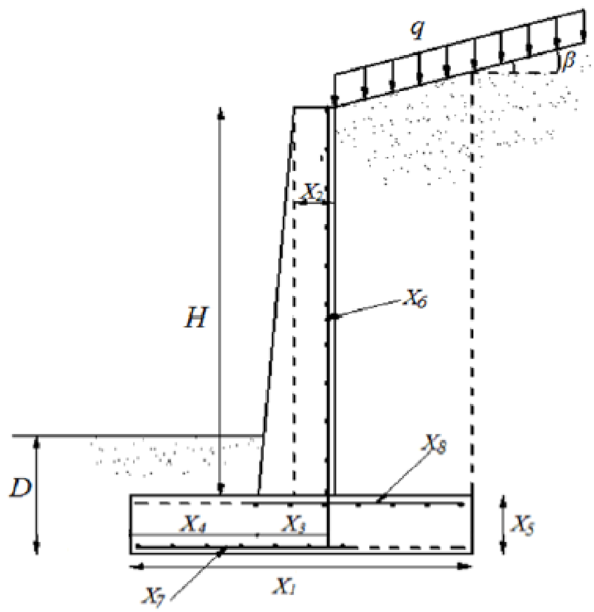

Figure 1. Cross section of the RCC retaining wall.

In this paper, for the first time, authors proposed bacterial foraging optimization algorithm (BFOA) to minimum cost design of RCC retaining walls. BFOA is inspired by the social foraging behavior of *Escherichia coli*. BFOA has already drawn the attention of researchers because of its efficiency in solving real optimization problems arising in several application domains. The formulation of the problem includes 8 design variables: five variables define the geometry of the RCC walls and three variable deal with reinforcement set-up (Figure 1). For structural design details, the recommendations of the Building Code Requirements for Structural Concrete (ACI 318-08) are used. The effectiveness of the approach is illustrated by a common numerical example. As will be shown, BFOA can successfully be applied to minimum cost design of RCC retaining walls with respect to satisfy all geotechnical and structural constraints. To show the robustness of the BFOA, the authors compare the results driven from BFOA with Saribas and Erbatur (1996) for a numerical example. Finally, the solution approach allows to study sensitivity of optimum design.

2 FORMULATION OF THE OPTIMIZATION PROBLEM

The follow potential failure mechanisms considered in design of RCC walls: sliding, overturning, bearing capacity and foundation uplift. Additionally, each element of the wall must individually resist against the forces induced by the weight of backfill material. Thus, the optimization problem deals with the stability of the structure, design requirement, and geometrical constraint.

2.1 *Design variables*

The formulation of the problem includes 8 design variables: five geometrical ones dealing with the thickness of the stem at top and bottom, the thickness of the footing, as well as the toe and the heel lengths; and three variables for reinforcement set-up. Table 1 shows the 8 design variables. Figure 1 indicates the main variables for optimum design.

Table 1. Design variables definition

Symbol	Design variables
X_1	Total base width
X_2	Stem thickness at top
X_3	Stem thickness at bottom
X_4	Toe length
X_5	Thickness of the base slab
X_6	Vertical steel area of the stem
X_7	Horizontal steel area of the toe
X_8	Horizontal steel area of the heel

2.2 *Design parameters*

Parameters are pre-assigned data and they are kept constant in the optimization process. The height of the stem is the main parameter and considered fixed for calculations. Other design parameters include internal friction angle of the retained soil, internal friction angle of the base soil, slope of the retained backfill, backfill density, cohesion of the base soil, surcharge load, and the depth of the soil in front of the wall which are all constant during design process. The soil, structural, and other related design parameters pertinent are presented in Table 2.

Table 2. Input parameters for numerical example

Input parameters for numerical example	Unit	symbol	value
Height of stem	m	H	3
Yield strength of reinforcement steel	MPa	f_y	400
Compressive strength of concrete	MPa	f_c	21
Concrete cover	cm	c_c	7
Diameters of bars	cm	φ_{bar}	1.2
Surcharge load	kPa	q	20
Backfill slope	degree	β	10
Internal friction angle of backfill soil	degree	φ	36
Internal friction angle of base soil	degree	φ_{base}	0
Unit weight of backfill soil	kN/m^3	γ_s	17.5
Unit weight of base soil	kN/m^3	γ_{base}	18.5
Unit weight of concrete	kN/m^3	γ_c	23.5
Cohesion of base soil	kPa	c	125
Depth of soil in front of wall	m	D	0.5
Factor of safety for overturning stability	_	SF_o	1.5
Factor of safety against sliding	_	SF_s	1.5
Factor of safety for bearing capacity	_	SF_b	3
Wide beam shear strength of concrete	MPa	v	0.65
Maximum steel percentage	_	ρ_{max}	0.016
Minimum steel percentage	_	ρ_{min}	0.00333
Shrinkage reinforcement percent	_	ρ_{st}	0.002

2.3 *Constraints*

To insure the wall stability, 10 constraints are considered. They may be categorized as geotechnical or structural constraints. Geotechnical constraints involve overturning, sliding, ground stresses and no tension condition in foundation soil. Structural constraints involve toe shear, toe moment, heel shear, heel moment, shear at the bottom of the stem and moment at the bottom of the stem. These requirements the failure modes that are expressed as function of the design variables and correspond to 10 behavior constraints, defined as inequalities

$$g_i(x) \leq 0 \qquad i=1,\ldots,10 \tag{1}$$

where x is the vector of design variables. The basic expressions for geotechnical constraints are given in Eqs. (2) to (4) as:

$$M_R - SF_o M_o \geq 0 \tag{2}$$

$$\mu N_b - SF_s H_b \geq 0 \tag{3}$$

$$\sigma_{all} - \max|\sigma| \geq 0 \tag{4}$$

Eq. (2) corresponds to overturning stability of the wall, where M_R is the total favorable overturning moment; M_o is the total unfavorable overturning moment and SF_o is the overturning safety factor. Eq. (3) states the limit state of sliding. In Eq. (3), H_b is the total horizontal reaction at the base of the footing; N_b is the total vertical reaction of the base of the footing; μ is the base friction coefficient and SF_s is the sliding safety factor. In Eq. (4), σ is the pressure under the base slab; σ_{all} is the allowable bearing capacity.

The distribution of the ground bearing pressure below the rigid base is assumed to be trapezoidal, that is, the effective eccentricity of the resultant vertical forces must lies within the middle third of the base.

This is the forth geotechnical constraints. Structural constraints such as toe shear, toe moment, heel shear, heel moment, shear at the bottom of the stem and moment at the bottom of the stem, must be satisfied according to Building Code Requirements for Structural Concrete (ACI 318-08). It is worth noting that some side constraints, like maximum or minimum percentage of steel in each section, must be satisfied, too.

2.4 *Cost function*

The problem of structural concrete optimization proposed in this study consists of an economic optimization. It deals with the minimization of the objective function F of Eq. (5), satisfying all the constraints discussed in the previous section.

$$F(x_1, x_2, ..., x_n) = \sum_{i=1}^{r} p_i * m_i(x_1, x_2, ..., x_n) \tag{5}$$

where x_i are design variables, p_i are the unit prices and m_i are the measurements of the seven units in which the construction of the RCC wall is split. The cost function is the value of materials (concrete and steel) and all the entries required to evaluate the entire cost of the wall per linear meter (formwork, excavation, fill, etc.), including, for example, the excavation of the foundation and the lateral fill of the walls.

3 PROPOSED BACTERIAL FORAGING STRATEGY

3.1 *Brief overview*

Bacterial foraging optimization algorithm (BFOA) is a new evolutionary computation technique which has been inspired by the foraging behavior of *Escherichia coli* and proposed by Passino (2002). Bacterial Foraging is an optimization technique based on population search and efficient for global search method. The idea of bacteria foraging principle is based on the fact that natural selection tends to eliminate animals with poor foraging strategies through methods for locating, handling, and ingesting food, and to favor the propagation of genes of those animals that have successful foraging strategies. They are more likely to apply reproductive success to have an optimal solution. After many generations, poor foraging strategies are either eliminated or shaped into good ones. These optimization models could provide a social foraging environment where groups of parameters communicate cooperatively for finding solutions to engineering problems like minimum cost design of structures. The *E. coli* bacteria that are present in our guts have a foraging strategy governed by four processes, namely, chemotaxis, swarming, reproduction, and elimination and dispersal (Passino, 2002). The BFOA parameters required for numerical application are presented in Table 3.

Table 3. BFOA parameters used for numerical example and sensitivity analysis

BFOA parameters	Symbols	value
Dimension of the search space	P	8
Total number of bacteria in the population	S	30
The number of chemotactic steps	N_c	4
The swimming length	N_s	4
The number of reproduction steps	N_{re}	2
The number of elimination-dispersal events	N_{ed}	2
Elimination-dispersal probability	P_{ed}	.2

3.2 *Chemotaxis*

An *E. coli* bacterium can move in two different ways: It can run (swim for a period of time) or it can tumble, and alternate between these two modes of operation in the entire lifetime. In the BFOA, a unit walk with random direction represents a tumble and a unit walk in the same direction indicates a run. In computational chemotaxis, the movement of the *i*th bacterium after one step is represented as:

$$\theta^i(j+1,k,l) = \theta^i(j,k,l) + C(i)\phi(j) \tag{6}$$

where $\theta^i(j,k,l)$ denotes the location of ith bacterium at jth chemotactic, kth reproductive and lth elimination and dispersal step. $C(i)$ is the length of unit walk, which is a constant in basic BFOA and $\phi(j)$ is the direction angle of the jth step. When its activity is run, $\phi(j)$ is same as $\phi(j-1)$, otherwise, $\phi(j)$ is a random angle directed within a range of $[0,2\pi]$. If the cost at $\theta^i(j+1,k,l)$ is better than the cost at $\theta^i(j,k,l)$ then the bacterium takes another step of size $C(i)$ in that direction otherwise it is allowed to tumble. This process is continued until the number of steps taken is greater than the number of chemotactic loop, N_c.

3.3 *Swarming*

The bacteria in times of stresses release attractants to signal bacteria to swarm together. Each bacterium also releases a repellant to signal others to be at a minimum distance from it. Thus all of them will have a cell to cell attraction via attractant and cell to cell repulsion via repellant. The cell to cell signaling in *E. coli* swarm may be mathematically represented as:

$$j_{cc}(\theta, P(j,k,l)) = \sum_{i=1}^{s} j_{cc}(\theta, \theta^i(j,k,l)) =$$

$$\sum_{i=1}^{s}\left[-d_a \exp\left(-w_a \sum_{m=1}^{p}(\theta_m - \theta_m^i)^2\right)\right] + \sum_{i=1}^{s}\left[h_r \exp\left(-w_r \sum_{m=1}^{p}(\theta_m - \theta_m^i)^2\right)\right] \tag{7}$$

Where $j_{cc}(\theta,P(j,k,l))$ represents the objective function value to be added to the actual objective function, S is the total number of bacteria, p is the number of variables to be optimized and $\theta=[\theta_1, \theta_2, \ldots, \theta_p]^T$ is a point in the p-dimensional search domain. d_a, w_a, h_r and w_r are coefficients to be chosen properly.

3.4 *Reproduction*

The least healthy bacteria eventually die while each of the healthier bacteria (those yielding lower value of the objective function) asexually split into two bacteria, which are then placed in the same location. This keeps the swarm size constant.

3.5 *Elimination and dispersal*

In long term, motile behavior of bacteria involves that all the bacteria may be annihilated at once in the local environment. The life of a population of bacteria changes either gradually by consumption of nutrients or suddenly due to some other influences. Events can kill or disperse all the bacteria in a region. They have the effect of possibly destroying the chemotactic progress, but in contrast, they also assist it, since dispersal may place bacteria near good food sources. Elimination and dispersal helps in reducing the premature solution point or local optima (Ritanjali et al., 2009). The main parameters of BFOA for optimal design of RCC walls are shown in Table 3.

4 NUMERICAL EXAMPLE

The effectiveness of the implemented BFOA algorithm on structural optimization is shown through the use of numerical example based on Saribas and Erbatur (1996). For the sake of comparison, this example was solved again using presented methodology and for the same conditions. Input parameters for analysis and optimal design process are given in Table 2.

It is worth noting that they did not measure the cost of excavation, formwork and backfill. In order to optimum design of this case, the optimal design procedure is coded in MATLAB. This example involves a RCC retaining wall with 3 meter height of the stem. The latter pressure corresponds to the active state and agrees with Rankin's theory. For calculation of ultimate bearing capacity, Hansen method is used. As recommended in Bowles (1982) all design variables have practical minimum and maximum value. Hence these upper and lower bound constraints are presented in Table 4.

Table 4. Lower bounds and upper bounds for design variables

Design variables	Bounds	
	Lower bound	Upper bound
X_1	0.4H(12/11)	(0.7H)/0.9
X_2	0.2	0.5
X_3	0.2	(H/0.9)/10
X_4	[0.4H(12/11)]/3	[(0.7H)/0.9]/3
X_5	[H(12/11)]/12	(H/0.9)/10
X_6	$10000\rho_{min}$ (X_{3l} - 0.01 d)*	$10000\rho_{max}$ (X_{3u}- 0.01 d)*
X_7	$10000\rho_{min}$ (X_{4l} - 0.01 d)*	$10000\rho_{max}$ (X_{4u}- 0.01 d)*
X_8	$10000\rho_{min}$ (X_{4l} - 0.01 d)*	$10000\rho_{max}$ (X_{4u} - 0.01 d)*

*Note: ρ_{min} and ρ_{max} are minimum and maximum steel ratios respectively. X_{il} and X_{iu} are lower bound and upper bound for X_i variable, respectively and $d = \varphi_{bar} / 2 + C_c$.

Table 5 compares the optimization results obtained from the BFOA method and Saribas and Erbatur (1996) for the retaining wall considered. As seen, only X_1 variable had sensible change for both two methods, also the optimum price evaluated using BFOA was 80.53 $/m, which is lower than that evaluated by Saribas and Erbatur (1996). In both of methods the program used lower bounds for X_4, X_5, X_7, X_8 variables.

Table 5. Optimization result for retaining wall

Design variable	Units	Optimum values	
		Saribas and Erbatur	BFOA
X_1	m	1.578	1.507
X_2	m	0.2	0.2
X_3	m	0.258	0.282
X_4	m	0.436	0.436
X_5	m	0.273	0.273
X_6	cm^2	12.574	12.483
X_7	cm^2	6.551	6.551
X_8	cm^2	6.551	6.551
Minimum cost	$/m	82.474	80.53

5 SENSITIVITY ANALYSIS

A sensitivity analysis adds quality to a design and supplies very important information on the work being designed from the view point of cost and reliability. The sensitivity analysis is very useful to (a) the designer, who can know which data values are more influential on the design, (b) to the builder, who can know how changes in prices influence the total cost, and (c) to the code maker, who can know the costs and reliability changes associated with an increase or decrease in the required safety factors or failure probabilities. The basic parameters and prices considered for sensitivity analysis are given in Table 6 and Table 7, respectively. These prices were provided by Yepes et al. (2008). All other requirements for structural design are based on ACI 318-08.

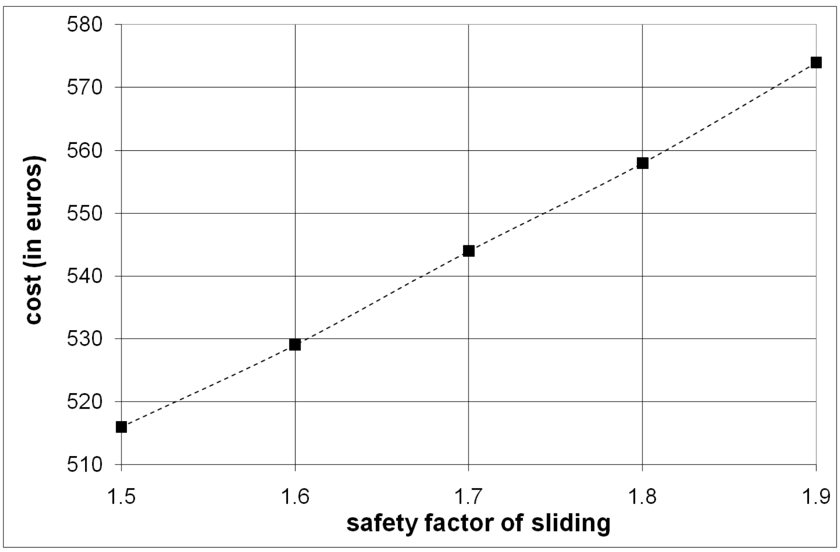

Figure 2. Cost variation against different safety factor of sliding for constant 5 meter height

Table 6. Input parameters for sensitivity analysis

Input parameters for sensitivity analysis	Unit	symbol	value
Yield strength of reinforcement steel	MPa	f_y	400
Compressive strength of concrete	MPa	f_c	21
Concrete cover	cm	c_c	7
Diameters of bars	cm	φ_{bar}	1.6
Surcharge load	kPa	q	20
Backfill slope	degree	β	10
Internal friction angle of backfill soil	degree	φ	36
Internal friction angle of base soil	degree	φ_{base}	20
Unit weight of backfill soil	kN/m^3	γ_s	17.5
Unit weight of base soil	kN/m^3	γ_{base}	18.5
Unit weight of concrete	kN/m^3	γ_c	23.5
Cohesion of base soil	kPa	c	50
Depth of soil in front of wall	m	D	0
Factor of safety for overturning stability	_	SF_o	1.5
Factor of safety against sliding	_	SF_s	1.5
Factor of safety for bearing capacity	_	SF_b	3

Table 7. Basic prices of the cost function of the walls

Units	Cost (euro)
m^3 of earth removal	3.01
m^2 of foundation formwork	18.03
m^2 of stem formwork	18.63
Kg of steel	.56
m^3 of concrete in foundations	50.65
$m^3$of concrete in stem	56.66
m^3 of earth fill-in	4.81

In this study, results concerned with sensitivity of optimum solutions with respect to height, the base friction coefficient, the type of fill as regards its angle of internal friction and safety factor for sliding are presented. In Figure 2 cost variation against safety factor of sliding is depicted. In this case the height of the wall is constant and is equal to 5 meter, the internal friction angle of the backfill soil is equal to 36°, base friction coefficient μ, is equal to .237($\mu=2/3\varphi_{base}$, where φ_{base} is 20°). A small coefficient for 1.5 causes an average decrease in cost of 11.24% as compared to a coefficient for 1.9.

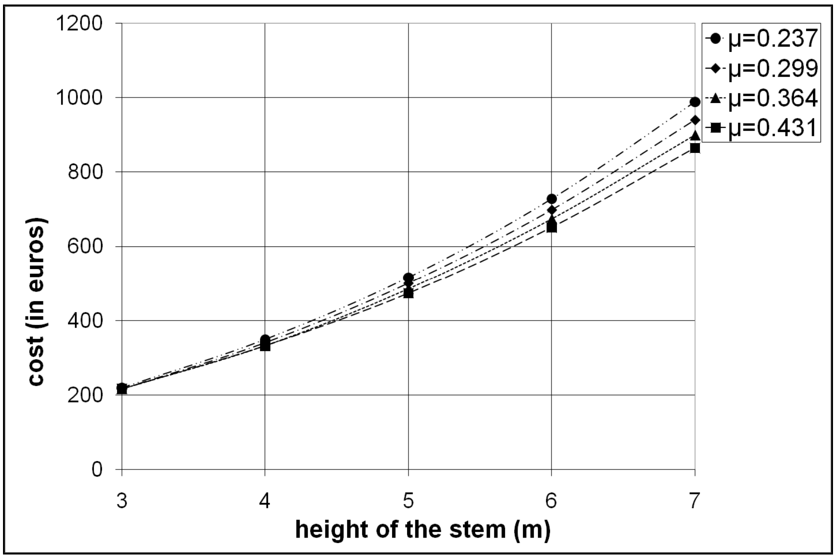

Figure 3. Cost variation for different base friction angle

Figure 3 illustrates the cost variation for different base friction angles. The internal friction angle of the base soil can vary from 20° to 35° with an increment of 5°. The results show that for higher height, optimum cost become more sensitive to internal friction angle of the base soil. For example, for a wall with 7 meter height, choosing μ=0.431, causes cost reduction of 14.33% in comparison with considering μ=0.273.

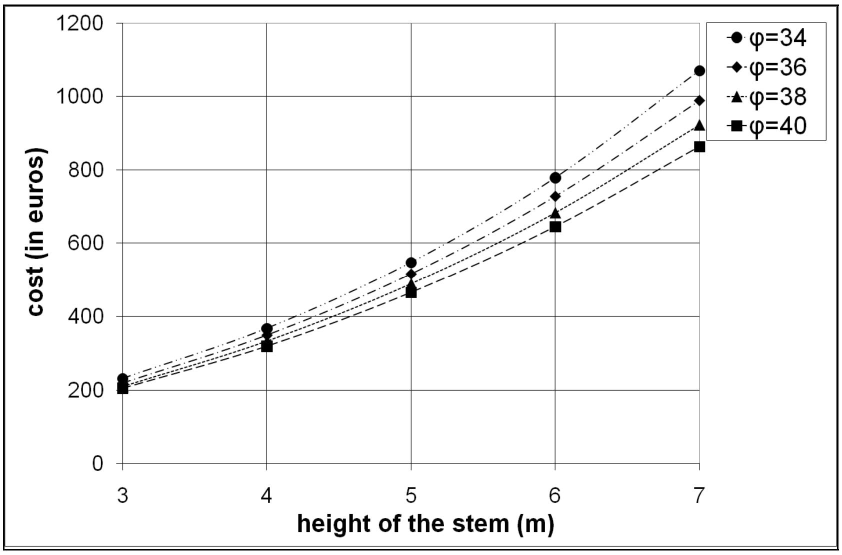

Figure 4. Cost variation for different backfills

Figure 4 shows cost variation against internal friction angle of backfill soil. The internal friction angle of the backfill soil can vary 34° to 40° with 2° increment. Figure 4 explains why it is beneficial to use more compacted soil offering greater internal friction angles.

6 CONCLUSIONS

This paper presents in detail the background and implementation of BFOA suitable for economic optimization and sensitivity analysis of RCC walls. BFOA is inspired by the pattern exhibited by bacterial foraging behavior. The bacterial foraging system primarily consists of four sequential mechanisms namely chemotaxis, swarming, reproduction and elimination-dispersal. The results from the considered numerical example based on using BFOA show the ability of the algorithm to find optimal results. The BFOA results are also comparable to other structural optimization methods and even offer better results. The simplicity of implementation of the BFOA makes it possible to apply for optimization of retaining walls.

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