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The water level as a time-variant parameter in reliability calculations of river flood embankments

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ABSTRACT: Different approaches exist for the determination of the failure probability of a river flood embankment. If the analysis relates to a annual maximum river discharge for a certain river gauge, coming from hydrologic evaluations, four constituents are needed in order to determine the reliability of a river embankment. These four constituents are the limit state equation, the extreme value distribution for the annual maximum river discharge, the exceedance duration line and the relationship between the river water level and the annual maximum river discharge. A fast-converging iteration cycle evaluates the four constituents and leads to the annual failure probability of the embankment. The concept is illustrated by a case study for an embankment at the Elbe river in Eastern Germany.

Keywords: Embankment, failure, hydraulics, statistical analysis, variability

1 INTRODUCTION

When discussing the reliability of river flood embankments, practitioners frequently ask what return period of the river water level is used for the design. Common international standards suggest a design based on specific return periods of the water level. Referring to a reliability analysis based on extreme value distributions, the answer is that all possible return periods are used to determine the annual probability of failure of a river embankment.

The probability of exceedance of the river water level can be described either directly by a probability density function of the annual maximum river water level or indirectly by the annual maximum river discharge. The river water level as the solicitation is therefore a common stochastic parameter differing from other geotechnical stochastic resistance parameters by the unit of its cumulative distribution on the vertical axes which is 1 / year.

2 THE WATER LEVEL AS STOCHASTIC PARAMETER WITH RESPECT TO A RETURN PERIOD

Common design standards suggest to use a single water level with a well-defined return period usually provided by the local authorities in order to design a river embankment. Based on the design standard but taking an uncertainty of the water level into account, van Gelder (2008) suggests a cumulative distribution for the load of a flood defence, i.e. the water level. Also Bachmann et al. (2007) provide a simple, regularly-shaped distribution function for the water level. The mathematical effort to determine the failure probability is quite low.

However, for the determination of the failure probability, the toe of the probability distribution may be of importance as it may largely influence the failure probability. Therefore, the fit of a normal distribution for the frequency of occurrence of a water level at a certain embankment section may not be accurate to model the extremes. In addition to that, the reference period of the failure probability directly corresponds to the return period of the water level which is a constraint for the shift of the annual failure probability to other reference periods.

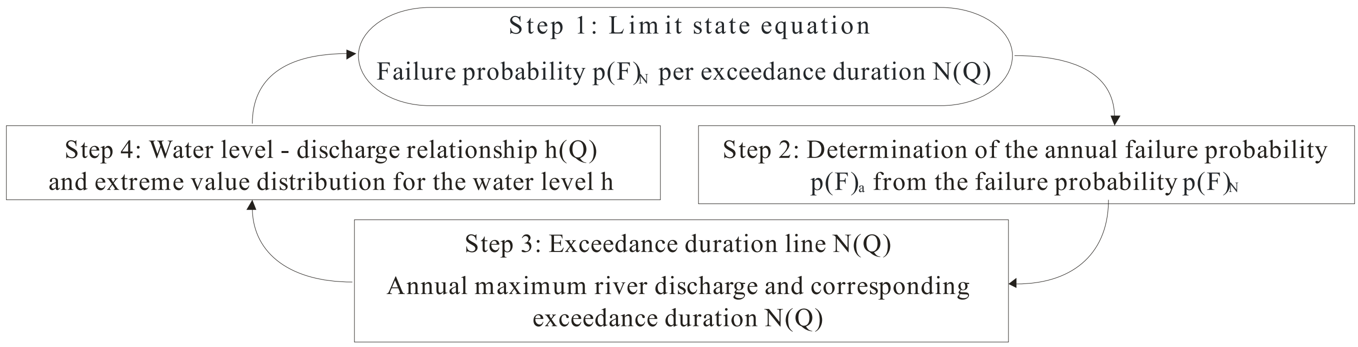

Figure 1. Iteration scheme for the determination of the annual failure probability of an embankment

The iteration scheme shown in Figure 1 is based on an extreme value distribution of the annual maximum river discharge. Four constituents are used in order to determine the reliability of a river embankment which are explained in the following sub-sections. The approach usually leads to an accurate result of the failure probability of an embankment which is shown by the case study in Section 3.

2.1 *Limit state equation*

The determination of the failure probability is based on a limit state equation Z of the following type:

$$Z = R - S \tag{1}$$

The resistance R against flooding as well as the solicitation S may be a function of any type. In many cases, the solicitation will consist of the water level h which plays a special role among the stochastic parameter as it relates to a return period finally leading to a failure probability with respect to a time unit.

2.2 *Extreme value distribution for the annual maximum river discharge*

There are two ways to formulate the frequency of occurrence of the annual maximum river discharge Q that are appropriate for the iteration scheme used in Figure 1. In the software PC-Ring developed by the Dutch Ministry of Transport, Public Works and Water Management (Steenbergen et al., 2004) for the regular flood risk report in the Netherlands, the relationship is defined as the so-called workline (2).

$$Q(T) = a \cdot \ln T + b \tag{2}$$

The workline approximates the hydrologic relationship between the annual maximum river discharge Q [in m^3/s] and its return period T [in years] for a certain river gauge by a simple logarithmic function. As the approximation may not always be accurate for all return periods, the river characteristics is approximated by three sections with different coefficients a and b. The workline is illustrated in Figure 2 while the return period is plotted in logarithmic scale. A shift of the reference period from the return period T [in years] to the number of exceedance durations T' [in days] can be illustrated by a vertical translation of the workline. In arithmetic terms, the workline can be shifted to the number of exceedance durations T' [in days] for which the discharge will be exceeded. The exceedance duration of a river flood is explained in Section 2.3.

$$T' = \frac{365}{N(Q)} \cdot T \tag{3}$$

The workline with respect to the number of exceedance durations T' may then be formulated as:

$$Q(T') = a \cdot \ln T' + a \cdot \ln N(Q) - a \cdot \ln 365 + b = a \cdot \ln T' + b' \text{ with } b' = a \cdot \ln N(Q) - a \cdot \ln 365 + b \tag{4}$$

The second way to formulate the frequency of occurrence of the annual maximum river discharge is an extreme value distribution in which the integral of the distribution from a certain discharge to infinity stands for the probability of exceedance $p(Q > Q^*)$ of the discharge Q^*. It can be derived that the above-mentioned logarithmic relationship between the return period and the discharge can be transformed into a Gumbel distribution (5) in which the coefficients a and b are adopted from the workline (2):

$$f(Q) = \frac{1}{a} \cdot EXP[-\frac{1}{a} \cdot (Q - b) - EXP(-\frac{1}{a} \cdot (Q - b)] \tag{5}$$

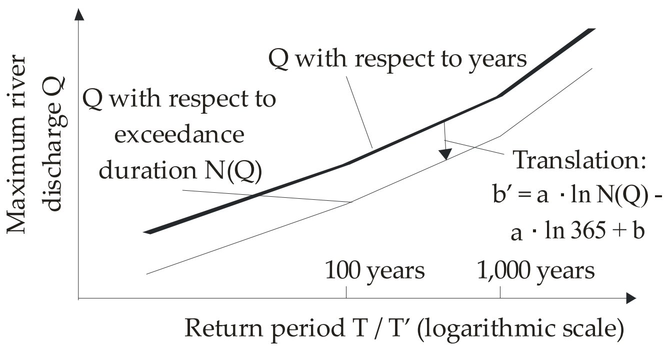

Figure 2. Workline consisting out of three sections as relationship between the annual maximum river discharge Q [in m³/s] and its return period T [in years] and shift of the reference period to the number of exceedance durations T'

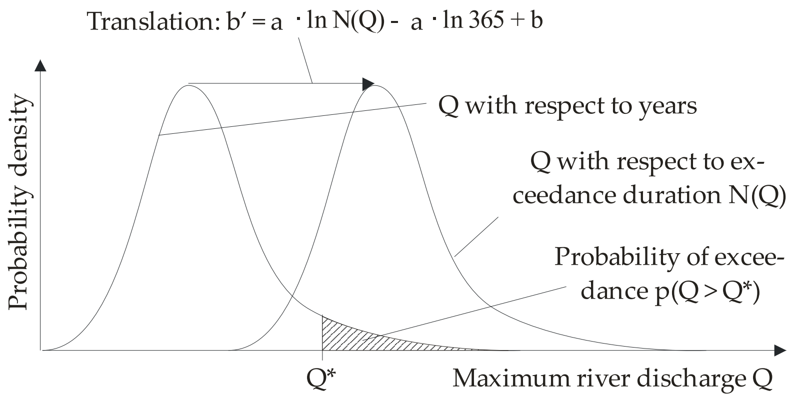

Figure 3. Gumbel distribution as relationship between the annual maximum river discharge Q and its frequency of occurrence p(Q > Q*) and shift of the reference period from the number of years (T) to the number of exceedance durations T'

Similar to the shift of the reference period of the workline (2), the reference period does not change the shape of the extreme value distribution. However, a horizontal translation illustrated in Figure 3 can be noticed for the extreme value distribution which analytically leads to the same transformation from T to T' from equation (3) when b' is used instead of b as mentioned in equation (4).

The return period T can be derived from the probability of exceedance p (Q > Q*) by equation (6):

$$T = -\frac{1}{\ln\left[1 - p\left(Q > Q^*\right)\right]} \qquad (6)$$

2.3 *Exceedance duration line*

The exceedance duration line correlates the duration of the flood wave N(Q) with the corresponding river discharge Q. It accounts for the adaptation of the reference period of the frequency of occurrence of the annual maximum river discharge to a shorter period for which flood events can be considered as independent. It originates from the capability of the software PC-Ring to couple high water levels coming from the river discharge with high water levels due to heavy storms which will have a different duration than the river flood.

The determination of the failure probability requires a shift of the reference period to an exceedance duration N(Q) because only for this reference period, flood events coming from heavy rainfall in the catchment area in combination with events coming from a heavy storms can be considered as independent and the extrapolation of the failure probability to one year may be performed taking a correlation of flood events into account.

The iteration scheme illustrated in Figure 1 determines a failure probability per exceedance duration $p(F)_N$. The extrapolation from the exceedance duration N(Q) [in days] to one year is done according to the model by Ferry-Borges and Castanheta (1971). It can be derived that if the water level has a high influence on the failure probability compared to other stochastic parameters, the annual failure probability $p(F)_a$ can be simply extrapolated from the failure probability per exceedance duration $p(F)_N$ as follows:

$$p(F)_a = p(F)_N \frac{365}{N(Q)} \qquad (7)$$

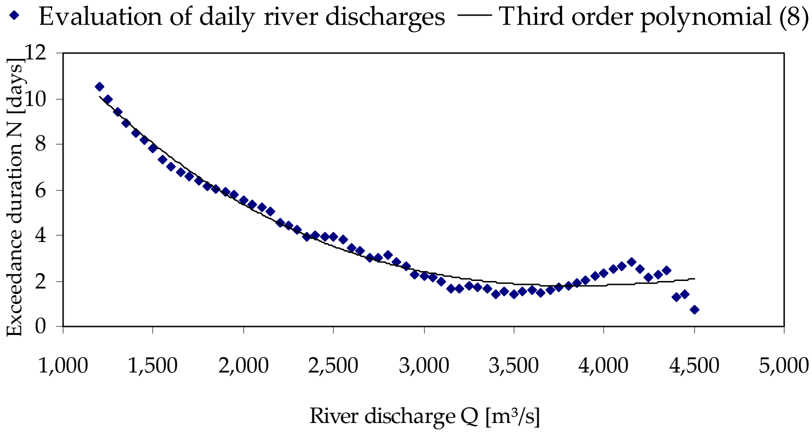

Figure 4. Exceedance duration coming from daily discharge statistics and approximation by a third order polynomial for the river gauge Dresden

If the influence of the water level on the failure probability is not so high, a reduction factor for the annual failure probability considering a correlation of dependent flood events in time must be taken into account.

The relationship of the duration of the flood wave N(Q) [in days] and the corresponding river discharge Q [in m³/s] is evaluated from daily measurements of the river discharge at a certain river gauge and approximated by a third order polynomial in which the coefficients c, d, e and f are fitted:

$$N(Q) = c \cdot Q^3 + d \cdot Q^2 + e \cdot Q + f \qquad (8)$$

2.4 *Relationship between the river discharge and the water level*

In order to evaluate the limit state equation (1) in which the water level h is a stochastic variable, the probability of exceedance of a certain water level is expressed in analytical terms using the extreme value distribution of the annual maximum river discharge. Coming from a hydrodynamic-numerical runoff-model evaluated for various river discharges, the relationship between the river discharge [in m³/s] and the water level [in mNN] at the embankment section can be approximated by a logarithmic function (9) in which the coefficients g and j are fitted. The approximation can be done only for part of the relationship in order to better fit the extremes. The inverse of the logarithmic relationship can be substituted in the extreme value distribution for the annual maximum river discharge (5) leading to a probability density function (10) for the annual maximum water level at the embankment section.

$$h(Q) = g \cdot \ln Q + j \qquad (9)$$

$$f(h) = \frac{1}{a} \cdot \mathrm{EXP}[-\frac{1}{a} \cdot (\mathrm{EXP}\left(\frac{1}{g}(h-j)\right) - b) - \mathrm{EXP}(-\frac{1}{a} \cdot (\mathrm{EXP}\left(\frac{1}{g}(h-j)\right) - b)] \qquad (10)$$

If the probability density function shall be set up not for the annual maximum water level but the water level with respect to the exceedance duration, the parameter b in equation (10) must be replaced by b’ according to equation (4).

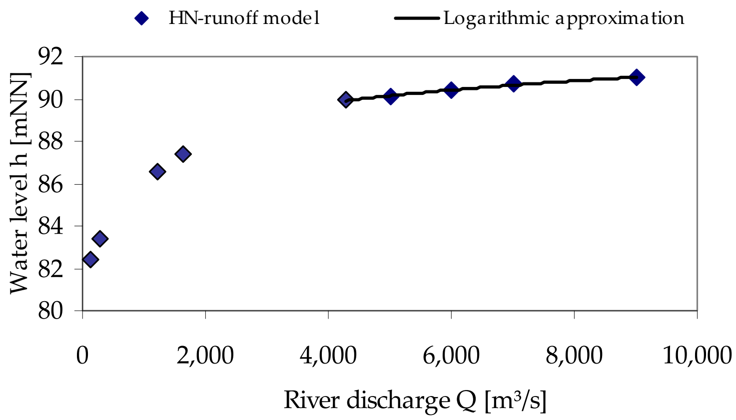

Figure 5. Relationship between the river discharge for the river gauge Dresden and the water level at the embankment section close to Torgau from a hydrodynamic-numerical (HN) runoff model and logarithmic approximation

3 CASE STUDY AT THE ELBE RIVER

3.1 *Hydrologic and hydraulic boundary conditions*

An embankment section at the Elbe river close to Torgau in Eastern Germany is used to illustrate the application of the iteration scheme in Figure 1 to determine the annual failure probability.

The frequency of occurrence of a certain river discharge at the gauge Dresden is given by the workline (2) consisting of three different sections for the corresponding return periods:

$$Q(T) = \begin{cases} 751.24 \cdot \ln T + 895.23 \text{ for} & T < 100 \text{ years} \\ 839.97 \cdot \ln T + 492.26 \text{ for } 100 \text{ years} < T < 1{,}000 \text{ years} \\ 976.34 \cdot \ln T - 458.38 \text{ for} & T > 1{,}000 \text{ years} \end{cases} \quad (11)$$

It can also be formulated as an extreme value distribution according to equation (12) consisting out of three sections which are illustrated in Figure 7:

$$f(Q) = \begin{cases} \frac{1}{751.24} \cdot \text{EXP}[-\frac{1}{751.24} \cdot (Q - 895.23) - \text{EXP}(-\frac{1}{751.24} \cdot (Q - 895.23))] \text{ for} & Q < 4{,}355 \text{ m}^3/\text{s} \\ \frac{1}{839.97} \cdot \text{EXP}[-\frac{1}{839.97} \cdot (Q - 492.26) - \text{EXP}(-\frac{1}{839.97} \cdot (Q - 492.26))] \text{ for } 4{,}355 \text{m}^3/\text{s} < Q < 6{,}295 \text{ m}^3/\text{s} \\ \frac{1}{976.34} \cdot \text{EXP}[-\frac{1}{976.34} \cdot (Q + 458.38) - \text{EXP}(-\frac{1}{976.34} \cdot (Q + 458.38))] \text{ for} & Q > 6{,}295 \text{ m}^3/\text{s} \end{cases} \quad (12)$$

The exceedance duration line for the river gauge in Dresden and the relationship between the river discharge in Dresden and the water level at the considered embankment section are shown in Figures 4 and 5. They can be formulated analytically:

$$N(Q) = -1.682 \cdot 10^{-10} \cdot Q^3 + 2.691 \cdot 10^{-6} \cdot Q^2 - 0.132 \cdot Q + 22.35 \quad (13)$$

$$h(Q) = 1.4882 \cdot \ln Q + 77.475 \quad (14)$$

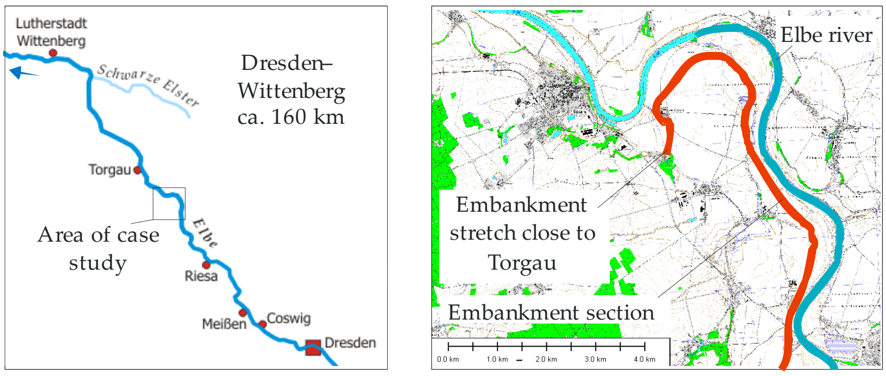

Figure 6. Area of case study at the Elbe river close to Torgau in Eastern Germany

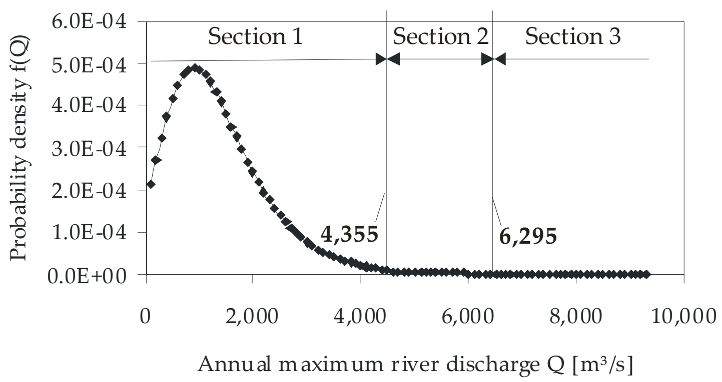

Figure 7. Extreme value distribution for the annual maximum river discharge at the river gauge in Dresden consisting out of three sections

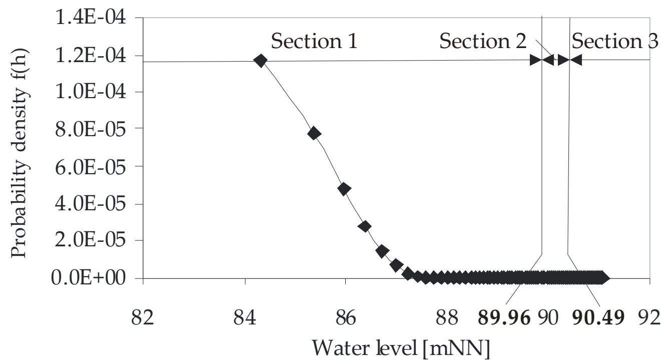

Figure 8. Extreme value distribution for the annual maximum water level at the considered embankment section consisting out of three sections

Using these expressions, a probability density function for the annual maximum water level can be generated which is only expressed in equation (15) for section 1 and which is illustrated in Figure 8:

$$f(Q) = \frac{1}{751.24} \cdot \mathrm{EXP}[-\frac{1}{751.24} \cdot (-\mathrm{EXP}((h - 77.475)/1.4482) - 895.23) \\ - \mathrm{EXP}(-\frac{1}{751.24} \cdot (-\mathrm{EXP}((h - 77.475)/1.4482) - 895.23))] \text{ for } h < 89.96 \text{ mNN} \quad (15)$$

3.2 *Validation of the 100-year water level*

The iteration scheme shown in Figure 1 is used to check whether the probability of failure due to overflow of the embankment section close to Torgau is equal to the known return period of the water level when all other uncertainties of the embankment section are neglected. The corresponding limit state equation subtracts the water level h as the only stochastic parameter from the water level of 89.96 mNN with a known return period of 100 years:

$$Z = 89.96 \text{ mNN} - h \quad (16)$$

The iteration starts assuming a exceedance duration of six days. The corresponding translation b' of the extreme value distribution of the water level h can be calculated according to:

$$b' = a \cdot \ln N(Q) - a \cdot \ln 365 + b = \begin{cases} 751.24 \cdot \ln 6 - 751.24 \cdot \ln 365 + 895.23 = -2{,}191 \\ 839.97 \cdot \ln 6 - 839.97 \cdot \ln 365 + 492.26 = -2{,}958 \\ 976.34 \cdot \ln 6 - 976.34 \cdot \ln 365 - 458.38 = -4{,}469 \end{cases} \quad (17)$$

The substitution of the parameters a, b', g and j from the equations (11), (17) and (14) into equation (10) leads to the probability density function of the water level h for the corresponding exceedance duration of 6 days for three sections for different return periods. For the Section 1 of the probability density function in equation (15), the coefficient b = 895.23 must be substituted by the coefficient b' = -2,191.

For the water level as the only one stochastic parameter, the limit state equation can be evaluated without using probabilistic calculation techniques but by simple integration of the probability density function from 89.96 mNN to infinity. The failure probability with respect to the exceedance duration of six days is $1.628 \cdot 10^{-4}$ 1 / 6 days. With the water level h being the only stochastic parameter, the failure probability can be simply extrapolated from a reference period of six days to one year by equation (7):

$$p(F)_a = 1.628 \cdot 10^{-4} \cdot \frac{365 \text{ days}/a}{6 \text{ days}} = 9.904 \cdot 10^{-3} / a \quad (18)$$

Using equation (6), the return period of the water level can be determined:

$$T = -\frac{1}{\ln\left[1 - 9.904 \cdot 10^{-3}\right]} = 100.5 \text{ a} \quad (19)$$

The return period can then be written into the section of the workline (11) with the appropriate return period and the exceedance duration can be updated according to equation (13) which leads to the next iteration step.

$$Q(T) = 839.97 \cdot \ln 100.5 + 492.26 = 4{,}364 m^3/s \quad \text{for } 100 \text{ years} < T < 1{,}000 \text{ years} \tag{20}$$

$$N(Q) = -1.682 \cdot 10^{-10} \cdot (4{,}364 m^3/s)^3 + 2.691 \cdot 10^{-6} \cdot (4{,}364 m^3/s)^2 - 0.132 \cdot (4{,}364 m^3/s) + 22.35 = 2.015 \text{days} \tag{21}$$

The results are summarized in Table 1. After two steps, the iteration already converges and yields a return period of 100.5 years which confirms the return period of the water level of 100 years set in the limit state equation. It shall be demonstrated in the next example that the concept to determine the failure probability can also be applied for a reliability analysis for a regular failure mode of an embankment.

Table 1. Results of the validation of the 100-year-water level

Iteration step	N(Q)	$p(F)_{N(Q)}$	$p(F)_a$	T	Q(T)	N(Q)	
1	6 days	$1.628 \cdot 10^{-4}$		$9.904 \cdot 10^{-3}$	100.5 years	4,364 m³/s	2.015 days
2	2.015 days	$5.468 \cdot 10^{-5}$		$9.905 \cdot 10^{-3}$	100.5 years	4,364 m³/s	2.015 days

3.3 *Determination of the annual failure probability for overflow of the embankment*

The iteration scheme in Figure 1 is now applied to a limit state equation for overflow of the embankment which contains more than one stochastic parameter.

$$Z = h_d - h - \Delta h \tag{22}$$

The overflow of an embankment is not only dependent on the return period of the water level h but also on the uncertainty of the crest height h_d of the embankment and on the uncertainty Δh whether the true local water level in front of the embankment section will correspond to the water level at the river chainage in the hydrodynamic-numerical (HN) runoff model. The analysis is done for the same embankment section as in Section 3.2 and therefore the same hydrologic and hydraulic boundary conditions apply. For the crest height and the uncertainty of the local water level, a normal distribution is assumed with mean values and standard deviations shown in Figure 9. A reliability analysis for the embankment section with the software PC-Ring leads to an annual failure probability of $3.25 \cdot 10^{-4}$ 1/a (Moellmann, 2009).

As in Section 3.2, the iteration starts with an assumed exceedance duration of six days for which the translation b' of the extreme value distribution of the water level h can be calculated according to equation (17). As the return period T of the flood is not known, all three sections of the extreme value distribution of the water level need to be evaluated in order to get the annual failure probability p(F)a. In contrast to the simple integration of the probability density function in Section 3.2, the failure probability for overflow must be determined using probabilistic calculation techniques as there are three stochastic parameters. The software Probox (Courage and Steenbergen, 2005) is used to apply the First Order Reliability Method (FORM) (Waarts, 2000). In order to achieve convergence of this iterative calculation technique, the start vector of the stochastic parameters in standard-normalized space needs to be manipulated.

The analysis does not only lead to a failure probability per exceedance duration and a corresponding reliability index β, it also provides sensitivity factors α_i that indicate the sensitivity of the stochastic input parameters on the failure probability. The higher the value, the more the result is affected by the uncertainty of the parameter. In order to update the corresponding river discharge, not the return period T of the flood but the return period of the most probable standard normalized river discharge u_q is used according to equation (6) which corresponds to the probability of exceedance of the river discharge Q*:

$$u_q = -\alpha_q \cdot \beta \tag{23}$$

$$p(Q > Q^*) = \int_{-\infty}^{u_q} \frac{1}{\sqrt{2\pi}} \text{EXP}(-0.5 \cdot t^2) dt \tag{24}$$

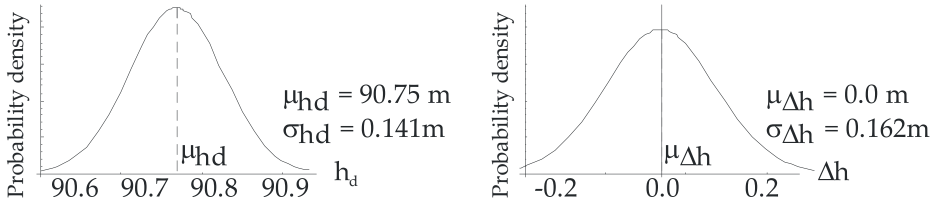

Figure 9. Normal distributions of the stochastic parameters crest height h_d (left) and local water level uncertainty Δh (right)

Table 2. Results of the determination of the annual failure probability for overflow of the embankment

Iteration step	N(Q)	$p(F)_{N(Q)}$	$p(F)_a$	T	α_q	u_q	$p(Q>Q^*)_a$	Q(T)	N(Q)
1	6 days	$7.994\odot10^{-6}$		$4.863\odot10^{-4}$	2056 years		-0.9745	3.214	$6.539\odot10^{-4}$
	6701 m³/s	4.121 days							
2	4.121 days	$5.496\odot10^{-6}$		$4.868\odot10^{-4}$	2054 years		-0.9752	3.216	$6.493\odot10^{-4}$
	6707 m³/s	4.122 days							
3	4.122 days	$5.498\odot10^{-6}$		$4.868\odot10^{-4}$	2054 years		-0.9752	3.216	$6.493\odot10^{-4}$
	6707 m³/s	4.122 days							

The results of the iteration steps are shown in Table 2. As the return period of the flood event is above 1,000 years, section 3 of the extreme value distribution of the water level needs to be evaluated. A convergence of the results can be noticed at least after three iteration steps. Table 3 compares the results of the concept presented in this paper with the results of PC-Ring (Moellmann, 2009). It can be noticed that there is a considerable difference between the results for the applied iteration scheme and the results with PC-Ring. This difference can be mainly explained by the different approximation of the relationship between the river discharge and the water level. In the applied concept, the relationship is approximated by a logarithmic function while it is assumed that a sectionwise linear approximation is used in PC-Ring. The difference becomes quite large as the annual failure probability is low and a slight shift of the probability density function has great influence on the failure probability.

Table 3. Comparison of the results of the applied iteration scheme in Figure 1 with the PC-Ring calculation

Analysis	$p(F)_a$	T	α_{hd}	α_q	$\alpha_{\Delta h}$
Iteration scheme	$4.868\odot10^{-4}$	2054 years	0.1453	-0.9752	-0.1669
PC-Ring	$3.252\odot10^{-4}$	3075 years	0.2264	-0.9522	-0.2028

4 CONCLUSIONS AND LIMITATIONS

It has been shown that it is possible to perform a plausibility check "by hand" of the computer-based calculation of the failure probability for an embankment. A probability density function for the water level with respect to an assumed exceedance duration needs to be set up. The application of the iteration scheme in Figure 1 leads to a convergence of the embankment reliability usually after three iteration steps. For a case study at an embankment at Elbe river in Eastern Germany with given hydrologic and hydraulic boundary conditions, the return period of the 100-year water level was confirmed by fully analytical calculations. The computer-aided determination of the annual failure probability for overflow of the embankment by the software PC-Ring was checked by the application of probabilistic calculation techniques. In contrast to an approach, in which a cumulative distribution for the annual maximum water level is used, the suggested method which sets up a probability density function with respect to the exceedance duration is more flexible and more accurate.

Limitations of the presented concept occur when the sensitivity factor of the annual maximum river discharge becomes smaller than 0.92. The extrapolation from the failure probability with respect to the exceedance duration to the annual failure probability according to equation (7) is not accurate and a correlation in time must be taken into account (Steenbergen and Vrouwenvelder, 2003).

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