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Reliability analysis of a pile foundation in a residual soil: contribution of the uncertainties involved and partial factors

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ABSTRACT: Reliability evolved from other areas, such as structures, requiring special adaptation when applied to geotechnical engineering. This paper shows one way to treat geotechnical uncertainties in a simple way. A sensitivity analysis, based on a series of calculations of the probability of failure for a single pile foundation is done in order to investigate the influence of each uncertainty source in reliability index. This experimental pile was installed in a residual soil in Portugal and was designed to withstand a vertical axial load. It was found, for this experimental case study, that the most important uncertainty source comes from model error, and not from the soil's spatial variability and uncertainty. Finally, the procedure to evaluate the resistance and load partial safety factors is shown and, for the same pile foundation, the safety factors are calculated and compared to the ones recommended by the Eurocode 7. Both methodologies are based on Monte Carlo simulation technique.

Keywords: bearing capacity, Eurocode, pile foundation, reliability analysis, safety factors

1 INTRODUCTION

All civil engineers are aware of how uncertainties are important for the design. But in some areas, such as in geotechnics, the uncertainties are mostly unknown or really difficult to measure. That is why, unlike in structural design, the traditional way that geotechnical engineers introduce the uncertainties in the design is using high global safety factors (SF), based on past experience. However, this way of treating uncertainties does not give a rational basis to understand their influence on the design. Based on such background, this paper shows one way that geotechnical uncertainties can be treated in a simple way.

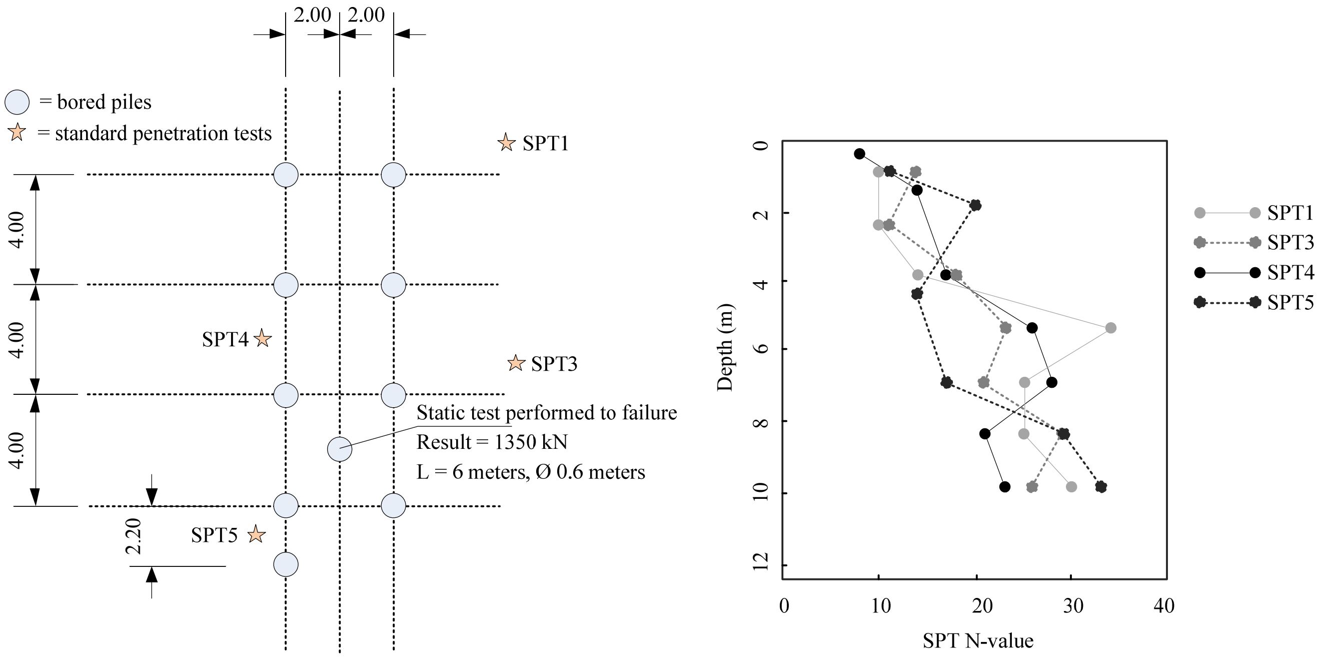
The reliability design has traditionally been classified into three levels:

- Level I: semi-probabilistic methods. Deterministic formulas are applied to the representative values (nominal or characteristic) multiplied by partial SF. The characteristic values are calculated based on statistical information, while partial SF are based on level II or III reliability methods.
- Level II: approximate probabilistic methods. The uncertainties are characterized by their mean, variance and covariance only (nonparametric). The probabilistic evaluation of safety is done by approximated numerical techniques, *i.e.* simplified hypothesis like first order reliability method (FORM).
- Level III: full probabilistic methods. Based on techniques that take into account all the variables' probabilistic characteristics, the probability of failure is analytically evaluated, but only when the problem is very simple. In more complex problems one needs to carry out simulations methods, for example, Monte Carlo simulations (MCS).

In this study, the level III methodology used (Honjo et al., 2010) aims to eliminate the possible confusions and difficulties that traditional reliability methodologies, applied in structures, can cause to geotechnical designers in practice. A series of calculations of the probability of failure for a case study were done, in order to investigate the influence of each uncertainty source. SF for the same case study (to be used in level I reliability design) were also evaluated based on design value method formulas and MCS (Kieu Le, 2008 and Honjo et al., 2009). The SF for resistance and load are then compared to the ones recommended by Eurocode 7 (CEN, 2007).

2 CASE STUDY

The case study presented in this paper is a single pile, vertically loaded, from an experimental site in Portugal (Figure 1.a). The pile was bored in residual soil and is 0.6m in diameter and 6m in depth. Different laboratory and *in situ* tests were performed in this experimental site, but only SPT (standard penetration tests) were considered in this paper (Figure 1.b) to evaluate the bearing capacity of the pile.



a) plan view b) SPT results

Figure 1. Experimental site - adapted from Fonseca and Santos (2008)

3 RELIABILITY ANALYSIS

3.1 *Methodology*

The methodology used for the reliability analyses has been adapted from previously published work by Honjo et al. (2010) and it is based on Monte Carlo simulations (MCS). This methodology differs from the typical employed in structural analysis. The goal is to remove the uncomfortable feelings that geotechnical engineers may have when using traditional reliability based design tools, like confusion and loss of perception of the results. Therefore, “Geotechnical Design Tools” and “Risk Assessment Tools” are separated as much as possible, allowing a better understanding of the different steps and responses obtained.

For a pile foundation and soil investigation with SPT, the process would be like shown in Figure 2, where the uncertainties are introduced in various stages. This process involves four steps:

1. Spatial variability and statistical estimation error are studied together. In many cases, it is very difficult or impossible to separate them. This step comprehends:
   - the calculation of a trend of *in situ* or laboratory tests (*e.g.* SPT – standard penetration test),
   - and analysis of residual errors, including estimation of autocorrelation distance (Vanmarcke, 1977).
2. Transformation error and modelling uncertainty are evaluated – these values are calculated based on documentation data, see for example Kulhawy and Mayne (1990), Okahara et al. (1991), Uzielli et al. (2006), AASHTO (2007) and Phoon (2008).
3. Resistances (R) and actions (E) are calculated and the performance function defined – Eq. (1):

$$M = g(R, E) = R - E \tag{1}$$

where M = safety margin, g = performance function, R = resistance and E = actions.

It should be noticed that the uncertainties of the actions are also obtained by bibliography and if the performance function is complex and/or requires quite amount of calculation efforts (like finite element method), the response surface method or neural networks can be used to find an approximate simpler function of the basic variables.

4. Finally, m MCS are performed in order to assess the probability of failure and reliability index of the problem by Eq. (2).

$$pf = \sum_{1}^{m} I, \quad I = \begin{cases} 0 \; if \;\; M \geq 0 \\ 1 \; if \;\; M < 0 \end{cases} \quad ; \; \beta = -\Phi^{-1}(pf) \tag{2}$$

where pf = probability of failure, m = number of MCS, I = failure indicator, M = safety margin, β = reliability index and Φ = is the normal cumulative density function with mean 0 and variance 1.

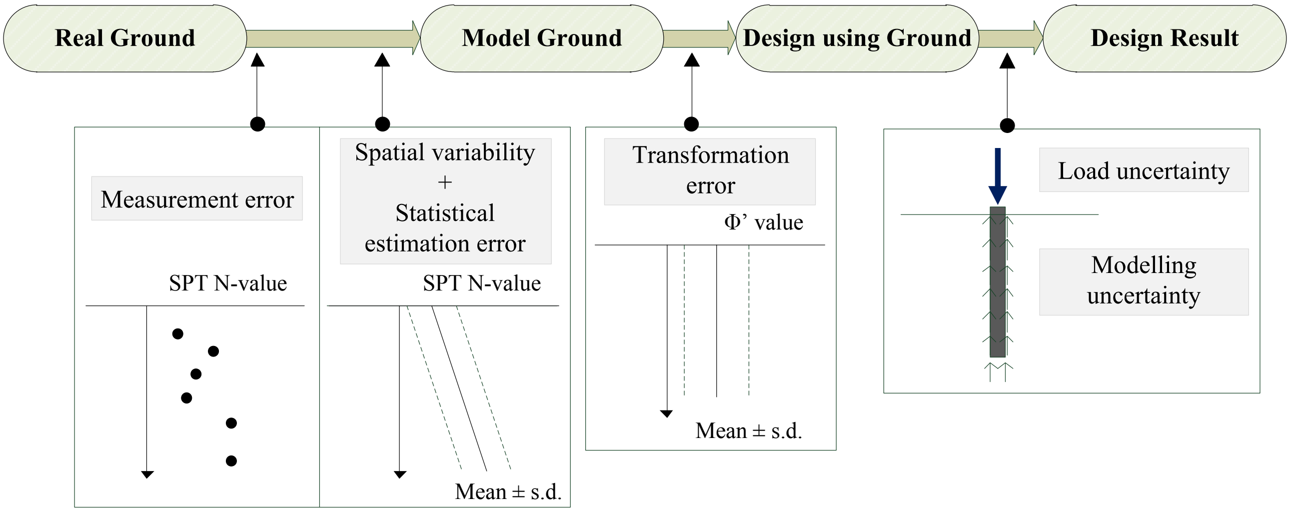


Figure 2. Proposed reliability analysis for a pile foundation

For this case study, where an empirical method was used to predict the bearing capacity of the pile, the performance function is given by Eq. (3).

$$M = \left(R_{tip} + R_{side}\right) - \left(G + Q\right) = \left(\delta_t \times Q_{tip} + \delta_f \times F_{side}\right) - \left(\delta_G \times G_k + \delta_Q \times Q_k\right) \tag{3}$$

where M = safety margin, R_{tip} = tip resistance of the pile, R_{side} = side resistance of the pile, G = permanent load, Q = variable load, δ_t = factor for model error uncertainty (tip resistance), Q_{tip} = predicted tip resistance, δ_f = factor for model error uncertainty (side resistance), F_{side} = predicted side resistance, G_k = permanent characteristic load and Q_k = variable characteristic load. The resistances are predicted by an empirical method, in this case, based on SPT (SHB, 2001 - Japanese method) and the actions were evaluated from the predicted load for a length of 6m and applying partial safety factors from Eurocode 7 (CEN, 2007) according to Eq. (4).

$$R_d \geq E_d \Leftrightarrow \frac{R_{predicted}}{1.15} \geq 1.35 \cdot Load + 1.50 \cdot Load \quad , \; considering \;\; G_k = Q_k = Load \leq 463\,kN \tag{4}$$

where R_d = design resistance value, E_d = design action value, $R_{predicted}$ = resistance predicted based on empirical method SHB (2001) (result: 1518 kN), $Load$ = value of load, [1.15, 1.35, 1.50] = partial safety factors (CEN,2007), G_k = permanent characteristic load and Q_k = variable characteristic load.

3.2 *Characterization and evaluation of uncertainties*

The uncertainties can be characterized as physical uncertainties (inherent uncertain nature of the parameter), modelling uncertainties (theoretical approaches and predictions), statistical uncertainties (finite size and fluctuations in the samples) and human errors (in the execution of multiple tasks). Human errors are a type of uncertainty that is not, in general, included in reliability analysis.

In this case study we have the physical uncertainties of actions (permanent and variable loads) and the inherent soil variability, as well as the modelling uncertainty (or model error) in the evaluation of resistance by an empirical method based on SPT. The Table 1 shows the values of the factors (δ) that take into account those uncertainties. The standard deviation of soil variability (value of N_{SPT}) can be reduced based on autocorrelation (Vanmarcke, 1977). Variables that vary continuously over a space or time are referred to as random fields (autocorrelation between variables). Normally values of a parameter measured at considerable distances are independent, but, if one measures the value of a parameter, the uncertainty in the value at a nearby point becomes less uncertain, because it is highly correlated to the first point value. That spatial autocorrelation allows the reduction of variances, but it is usually ignored due to difficulties in practical application.

Table 1. Uncertainties evaluation

	Soil variability		Modelling uncertainty		Actions' uncertainties	
	N_{SPT},tip	N_{SPT},side	tip	side	permanent	variable
Mean value	10.26+1.91z		1.12	1.07	1.0	0.6
Standard deviation	4.6*	4.6**	0.706	0.492	0.10	0.21
Distribution	Normal		Lognormal	Lognormal	Normal	Gumbel
Reference			Okahara et al. (1991)		Holicky et al. (2007)	

* reduced taking into account the influence zone on the pile tip (3×Diameter) as averaging over the thickness.
** reduced taking into account the length of the pile as averaging over the thickness.

3.3 *Evaluation of the reliability index and its sensitivity to uncertainties*

After quantifying the uncertainties, one can evaluate its impact on the performance of the structure. MCS (m=100,000) were done in order to evaluate the pile reliability, analysing different lengths [4, 5, 5.5, 6, 6.5, 7, 8, 9, 10] meters and different combinations of the uncertainties. The calculation of the probability of failure (*pf*) and reliability index (β) was repeated, considering only the uncertainties presented in Table 2 for each combination. The results are shown in Tables 3 and 4.

Table 2. Combinations of uncertainties studied

	Soil variability		Modelling uncertainty		Actions' uncertainties
Combination	N_{SPT},tip	N_{SPT},side	tip	side	permanent variable
1.1	√	√	√	√	√
1.2	√*	√*	√	√	√
2	√	√	-	-	√
3	-	-	√	√	√
4	√	-	√	-	√
5	-	√	-	√	√

√ means that the uncertainty was considered
* ignoring the reduction of variance based on autocorrelation (Vanmarcke, 1977).

Table 3. Results of probability of failure for different lengths and combinations

	Probability of failure								
Combination	4 m	5 m	5.5 m	6 m	6.5 m	7 m	8 m	9 m	10 m
1.1	0.31812	0.11033	0.05949	0.03036	0.01484	0.00758	0.00202	0.00094	0.00027
1.2	0.32912	0.12205	0.06791	0.03816	0.01948	0.01056	0.00323	0.00143	0.0005
2	0.09621	0.00317	0.00045	0.00007	0	0	0	0	0
3	0.29972	0.09474	0.04962	0.0243	0.01185	0.00541	0.00184	0.00064	0.00025
4	0.25511	0.02251	0.00422	0.00089	0.00019	0.00005	0	0	0
5	0.29029	0.04341	0.01137	0.00233	0.00047	0.00014	0.00002	0	0

Table 4. Results of reliability index for different lengths and combinations

	Reliability index								
Combination	4 m	5 m	5.5 m	6 m	6.5 m	7 m	8 m	9 m	10 m
1.1	0.47	1.22	1.56	1.88	2.17	2.43	2.88	3.11	3.46
1.2	0.44	1.16	1.49	1.77	2.06	2.31	2.72	2.98	3.29
2	1.3	2.73	3.32	3.81					
3	0.53	1.31	1.65	1.97	2.26	2.55	2.9	3.22	3.48
4	0.66	2	2.63	3.12	3.55	3.89			
5	0.55	1.71	2.28	2.83	3.31	3.63	4.11		

The value obtained for the actual pile length (Table 4 – length of 6m, β=1.88) is lower than the recommended by Eurocode for reliability class 2. The recommended values for the reliability index by Eurocode 0 (CEN, 2002) with a design of working life of 50 years for RC2 is 3.8. This can be justified by the fact that it is an experimental pile, so the consequences of failure are very low or even by the fact that the load predicted (1518 kN) is higher than the one actually used for the design of the pile, although it is not too far from the static load test (1350 kN). If the design of a pile is based on this type of soil, actions and this type of uncertainties, the length of the pile necessary to reach a reliability of 3.8 would have to be more than 10m. When comparing the results of reliability index considering all uncertainties with and without reduction of the variance (Tables 3 and 4 – combinations 1.1 and 1.2), it can be seen that the results are approximately the same. If one does not reduce the variance based on spatial autocorrelation, it is obviously a conservative action (although technically incorrect) as can be seen in the Tables 3 and 4 ($pf_{1.1} < pf_{1.2}$).

Taking into account the sensitivity analysis, the uncertainty that has more influence in the reliability for this case study is the modelling error (combination 2). The model uncertainty is much more important in the reliability of the pile. When removing the uncertainties of the soil variability, it can be seen that the results are almost the same as the ones obtained when considering all uncertainties (combinations 3 and 1.1). The results also show that the contribution of the side and tip uncertainties (combinations 4 and 5) are approximately the same, the side resistance is dominant (F_{side}/Q_{tip} around 2) but the uncertainties on the tip are higher.

4 PARTIAL SAFETY FACTORS

4.1 *Methodology*

In the geotechnical field, the design resistance of piles is very uncertain and the Eurocode 7 (CEN, 2007) establishes that the major uncertainty is not the strength of the *in situ* ground but the way the construction would interact with it. Therefore, the partial safety factor (SF) is essentially a factor of the resistance model, rather than on the strength of material. In such cases, it is appropriate to use resistance factor method rather than material strength method. The factors should be applied to the overall resistance given by a pile than the material strength of the ground.

The method used here, based on the work of Kieu Le (2008), attempts to combine design value method (DVM) and Monte Carlo simulations (MCS) to calculate load and resistance factors, that is believed to include the advantages of both methods, *i.e.* conceptual transparency, robustness, and flexibility of the calculation. DVM, based on FORM (first order reliability method), is one of the powerful methods to evaluate the partial SF (*e.g.* Thoft-Christensen and Baker, 1982; Honjo et al., 2009). However, if the performance function becomes complex, the application of the DVM using FORM for determination of load and resistance SF becomes very time-consuming or even impossible. The need to use other techniques to calculate load and resistance factors, based on the idea of DVM has been taken into consideration and its combination with MCS was the solution (Kieu Le, 2008).

Thus, the steps to evaluate the load and resistance factors are given as follows:

1. Gather probabilistic information and statistical parameters of the variables involved (Table 1).
2. Carry out MCS and evaluate resistances (R), actions (E) and R/E ratio.
3. Approximate a probability distribution to R and E results (here, normal and lognormal distributions are chosen, but other distributions can be also considered) (Figure 3).
4. Consider the linear function in Eq. (1) and R and E as two independent variables.
5. Select the points close to the limit state line, *i.e.* zone that satisfy the condition $R/E = 1\pm0.02$, and evaluate the likelihood of each point ($f_R(R)$ and $f_E(E)$, where f is the probability density function)
6. Compute approximate design point by two ways:
   a. maximum likelihood – $\max[f_R(R)\times f_E(E)]$
   b. normalizing the space by Eq. (5), then calculate the distance to the origin of each point, and the design point is the one with the shortest distance to the origin of the graph (Figure 4).
7. Calculate sensitivity factors (α_R and α_E) using:
   a. DVM formulas for normal fit or lognormal fit by Eq. (6),
   b. normalized space for normal fit or lognormal fit by Eq. (7).
8. And finally calculate the load and resistance factors ($\gamma_R < 1$and $\gamma_S > 1$) by normal fit or lognormal fit – Eq. (8). Both factors are multiplied by the characteristic values (different from Eurocodes approach, where the design resistance is the characteristic resistance divided by the partial SF).

One of the advantages is that DVM implicitly assumes that sensitivity factors calculated in the current design may not be too different from the sensitivity factors of design that satisfies the target reliability index. Therefore, redesign of the structure is not required when the reliability index obtain is different from the target one.

$$Z = \frac{X - \mu_X}{\sigma_X} \tag{5}$$

$$\alpha_{1,R} = -\frac{\sigma_R}{\sqrt{{\sigma_R}^2 + {\sigma_E}^2}};\ \alpha_{1,E} = \frac{\sigma_E}{\sqrt{{\sigma_R}^2 + {\sigma_E}^2}};\ \alpha_{1,\ln(R)} = -\frac{V_R}{\sqrt{{V_R}^2 + {V_E}^2}};\ \alpha_{1,\ln(E)} = \frac{V_E}{\sqrt{{V_R}^2 + {V_E}^2}} \tag{6}$$

$$\begin{cases} \alpha_{2,R} = -\cos(angle) = -\dfrac{Z_R}{\sqrt{Z_R^{\,2} + Z_E^{\,2}}} \\ \alpha_{2,E} = \sin(angle) = \dfrac{Z_E}{\sqrt{Z_R^{\,2} + Z_E^{\,2}}} \end{cases} ; \begin{cases} \alpha_{2,\ln(R)} = -\dfrac{Z_{\ln(R)}}{\sqrt{Z_{\ln(R)}^{\,2} + Z_{\ln(E)}^{\,2}}} \\ \alpha_{2,\ln(E)} = -\dfrac{Z_{\ln(E)}}{\sqrt{Z_{\ln(R)}^{\,2} + Z_{\ln(E)}^{\,2}}} \end{cases} \quad (7)$$

$$\begin{cases} \gamma_R = \dfrac{\mu_R}{R_k} \cdot \left(1.0 + \beta_T \cdot \alpha_R \cdot V_R\right) \\ \gamma_E = \dfrac{\mu_E}{E_k} \cdot \left(1.0 + \beta_T \cdot \alpha_E \cdot V_E\right) \end{cases} ; \begin{cases} \gamma_{\ln(R)} = \dfrac{1}{\sqrt{1 + V_R^{\,2}}} \cdot \dfrac{\mu_R}{R_k} \cdot e^{(\beta_T \cdot \alpha_R \cdot V_R)} \\ \gamma_{\ln(E)} = \dfrac{1}{\sqrt{1 + V_E^{\,2}}} \cdot \dfrac{\mu_E}{E_k} \cdot e^{(\beta_T \cdot \alpha_E \cdot V_E)} \end{cases} \quad (8)$$

where Z = normalized variable ~N(0,1), X = normal random variable ~N(μ_X,σ^2_X), μ = mean value, σ = standard deviation, R = resistances, E = actions, α = sensitivity factor, V = coefficient of variation (σ/μ), *angle* = see Figure 4, R_k = resistance characteristic value, E_k = actions characteristic value, β_T = target reliability index.

4.2 *Evaluation of partial safety factors and comparison with Eurocode*

To evaluate the load and resistance partial SF, the same uncertainties shown in Table 1 were adopted for the single pile of the studied experimental site (bored in residual soil, 6m length and 0.6m of diameter). The MCS were carried out (m=1,000,000) and the histograms were obtained for the resistances (R) and actions (E), see Figure 3. The resistances distribution showed, as expected, that it has a higher dispersion than actions, and the lognormal distribution was the one that had the best fitting for both R and E. The probability of failure (pf = 0.03040 – Figure 4) corresponds to a reliability index of 1.875, that when compared with the one m=100,000 in previous calculations, has the same reliability index (Table 4 – length of 6m, β=1.88).

The characteristic values were assumed as the mean value for the resistance (1671.9 kN) and for the actions (loads) the mean value and the high fractile of 95% (E_{mean}=740.6 kN and $E_{95\%}$=924.8 kN). After the evaluation of all the necessary parameters shown in Table 5, the partial coefficients were calculated based on lognormal fit. The results can be consulted in Table 6.

Analysing the results based on lognormal formulas, the low values of the resistance factor, between 0.20 and 0.53, may result from the high number of points with a very high resistance, a thick tail (as one can see in Figure 3). Also, for load factors, that should be higher than 1, the values obtained are slightly lower than 1 (0.82 to 0.97) when the characteristic value adopted was the 95% fractile, according to the usual procedure. Only when using the mean value for the characteristic value of load, the load partial factors were between 1.03 and 1.21 that, although low, are higher than 1.

The values recommended by the Eurocode 7 (Annex A of CEN, 2007 – resistance factor between 0.67 and 1.00 and load factors between 1.00 and 1.50) are higher than the ones calculated here, the reason could be the fact that the reliability index (1.88) for this case study is far from the target one (3.8).

Table 5. Estimation and sensitivity factors based on lognormal distribution for R and E

	R (kN)	E (kN)
Mean values	1671.9	740.6
Standard deviation	659.4	107.6
Design point:		
- Max likelihood	787.8	772.4
- min(β)=1.37	782.8 (Z_{lnR} = -1.82)	767.6 (Z_{lnE} =0.31)
Sensitivity factors (1*)	-0.9383	0.3457
Sensitivity factors (2**)	-0.9845	0.1755

* calculation method: DVM Eq. (6)

** calculation method: normalized space Eq. (7)

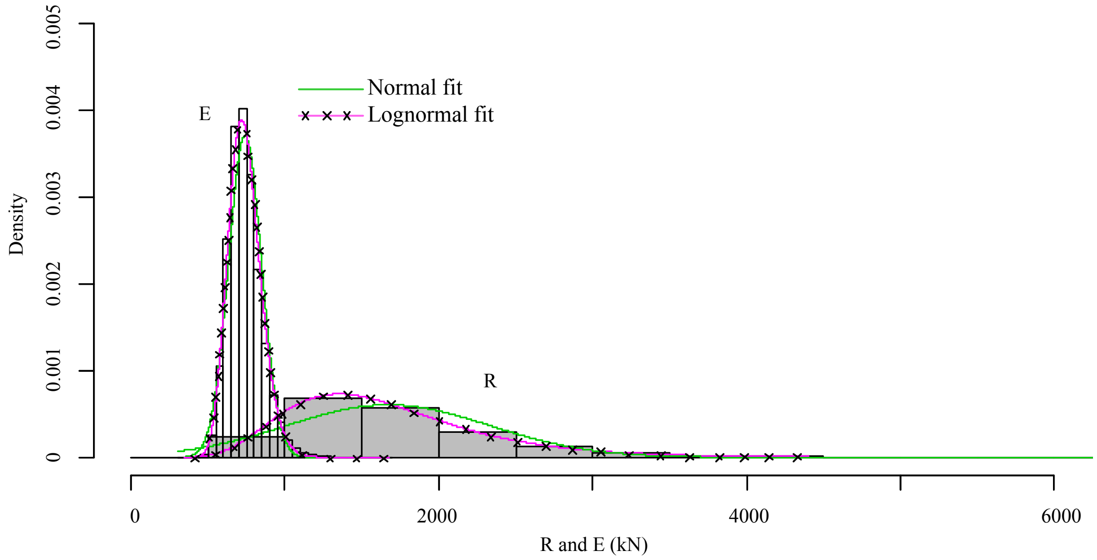


Figure 3. Distribution shape of resistance (R) and actions (E)

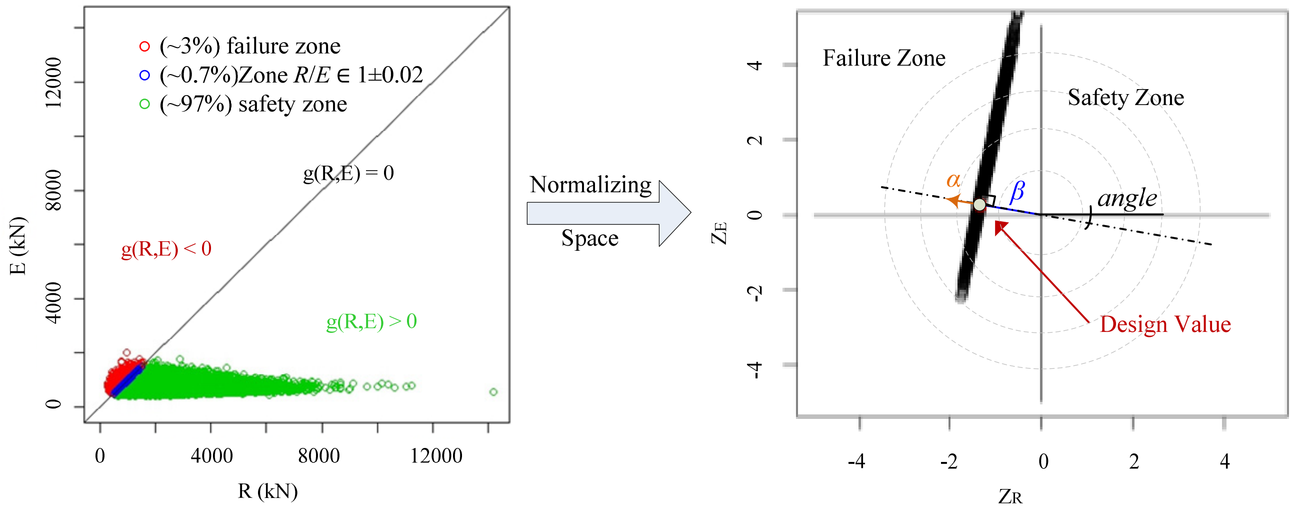


Figure 4. Graphical representation of m simulations and normalization of the space to evaluate the reliability index

Table 6. Partial factors for case study

	$\beta = 1.5$		$\beta = 2.0$		$\beta = 3.0$		$\beta = 4.0$	
	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
γ_R*	0.53	0.52	0.44	0.43	0.31	0.29	0.21	0.20
γ_E*	1.07	1.03	1.09	1.04	1.15	1.07	1.21	1.10
γ_E**	0.85	0.82	0.88	0.83	0.90	0.86	0.97	0.88

* R_k = mean, E_k = mean
** R_k = mean, E_k = high fractile 95%

5 CONCLUSIONS

In this paper, the reliability analysis of a pile foundation was performed based on the methodology proposed by Honjo et al. (2010). The uncertainties involved (actions, soil variability and model error) were evaluated and discussed for a specific case study. The soil variability and statistical error were evaluated by SPT. The proposed method is a user friendly reliability based design tool for geotechnical structures, for those who are not familiar with it. Monte Carlo simulations (100,000) were carried out for a pile installed in residual soil in an experimental site (Fonseca and Santos, 2008). Applying reliability analysis, the length that would give the proper security to the pile was calculated.

The results of this study showed that a length of approximately 10m (diameter 0.6m) was needed to obtain the reliability index required by the Eurocode (EC0 – RC2, β=3.8) and that the value obtained for the actual length of the pile installed (6m), β of 1.88, is lower than the recommended. This value can be justified by the fact that this is an experimental pile or the fact that the actions used for this problem were evaluated based on the prediction form SPT, and it might be different from the actually used to design the pile.

Also, for the studied case, it was concluded that it is not the soil's spatial variability that controls the major part of the uncertainty in geotechnical design of single pile foundations, that modelling uncertainty is the most important factor in reliability. It is believed that this happens for many other types of geotechnical problems, because the error in design equations, transformation of soil investigation results (*e.g.* SPT N values) to actual design parameters (*e.g.* cohesion, friction angle or even load capacity) is the most important factor in geotechnical reliability analysis (Hansen et al., 1995).

Finally, partial factors were evaluated by design value method formulas and Monte Carlo simulations, that is believed to include the advantages of both methods (Kieu Le, 2008). The lognormal distribution is the one that fits better the resistances and actions results for this case study, and analysing the outcomes: (1) the low values obtained for the resistance factor (0.20 to 0.53) may result from the high number of points with a very high resistance (thick tail of the distribution) and (2) the values obtained for the load factors, that should be higher than 1, resulted in between 0.82 and 0.97 for $E_k=E_{95\%}$ and 1.03 and 1.21 for $E_k=E_{mean}$. Both are very different form the recommendations of Eurocode 7, the reason could be the fact that the reliability index (1.88) for this case study is far from the target one (3.8) due to the value of load used (predicted by empirical SHB method) that might be very different from the actual load used in design.

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