Level III Reliability Based Design employing Numerical Analysis - Application of RBD to FEM -

Y. Otake, Y. Honjo, T. Hara & S. Moriguchi
Department of Civil Engineerings, Gifu University, Gifu, Japan

ABSTRACT: A reliability based design scheme employing a response surface is applied to 25km long irrigation channel reliability assessment of vertical displacement by liquefaction. The problem includes very complex uncertainties such as statistical estimation error due to limited investigation, and model error involved in sophisticated FEM analysis. This scheme worked well to combine a sophisticated geotechnical analysis tool to the reliability analysis.

Keywords: Reliability based design (RBD), FEM analysis, liquefaction, irrigation channel

1 INTRODUCTION

Reliability based design (RBD) methods are attracting great interest of geotechnical engineers due to the introduction of Level I RBD in design codes development worldwide, e.g. the structural Eurocodes. On the other hand, the numerical methods, e.g. FEM analyses, based on rapid growth of computational capability and development of user friendly software, are frequently used in practical design of geotechnical structures. By these sophisticated methods, more accurate evaluation of performances of the structures are believed to become possible. However, methodologies for matching the RBD and these sophisticated numerical tools that takes into account the characteristics of geotechnical design are not sufficiently developed.

The authors are proposing a design scheme that separates the geotechnical analyses and the reliability analyses in the first stage, which are recombined in the final reliability assessment stage so as to realize level III RBD, i.e. the full probability RBD, that is more convenient for the practicing geotechnical engineers. In this paper, the scheme is applied to the liquefaction risk assessment of an existing 25 km long irrigation channel that includes many complex geotechnical factors. The liquefaction is evaluated by one of the state of the art FEM programs. The purpose is to demonstrate the effectiveness of the scheme proposed for a complex geotechnical reliability design problem. The way to handle the uncertainties involved in the design is described in detail.

Figure 1. Proposed design scheme by response surface

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A SCHEME FOR GEOTECHNICAL RBD

The scheme proposed here is illustrated in Figure 1. It is separated to the three parts: (I) geotechnical design, (II) uncertainty analysis of basic variables and (III) reliability assessment.

Geotechnical design, (I), is almost the same as usual design procedure for geotechnical structures. The response of the structure (bearing capacity, displacement at a certain point etc.), \( y \), is obtained from the basic variables, \( x \), by the design calculations.

In some cases \( y \) can be related to \( x \) by a simple performance function. In other cases, the response surface (RS) method can be used to relate \( x \) to \( y \) by a regression analysis (Box & Drepper, 1987).

The uncertainty analysis of basic variables, (II), is the main part of the scheme. Statistical analyses play the major role in this analysis. The reliability assessment, (III), is carried out by the results of the uncertainty analyses and the response surface by simple Monte Carlo simulation (MCS). The probability of failure, i.e. probability of the structure to exceed the limit state, is evaluated.

The expected benefits of the proposed scheme are considered to be as follows:

1. The scheme tries to separate the geotechnical design part and the uncertainty analysis part as much as possible in order for practicing geotechnical engineers to carry out RBD easier, and also make the best use of the numerical analysis tools in the design.
2. It would be understood by carrying out this scheme that estimating a response surface itself gives quite amount of useful design information. A RS gives impact of each basic variable to the performance of the structure near the limit state. The building of a RS requires mostly the good geotechnical design skill.
3. The reliability assessment is done by simple MCS, which is easy to understand intuitively. It does not require much knowledge of the probability theory.

OUTLINE OF THE FACILITY AND THE VERIFICATION METHOD

3.1 Outline of the Irrigation Channel

3.1.1 The characteristics of the irrigation channel

The irrigation channel under study is 25 km long and completed in 1970 (Figure 2). The geology under the channel can be divided into three parts, where 12 km long central part (STA 30 – 150) is described in the paper. It is an open channel RC frame structure and 90% is build in the embankment (Figure 2(a), embankment type), whereas 10% is excavated channel (embedded type) including siphons. The RC frame channel has width of about 10 m, height 5 m and 10 m long, i.e. each 10 m is an independent structure.

The embankment type is made of the RC frame channel and roads for maintenance on both sides of it 3 m wide and 4 m high embankment. The embedded type is embedded RC frame into the plane ground. At the crossings to the major roads, siphons are build using RC box type structure.

3.1.2 Ground characteristics

The channel is located on one of major Alluvial planes in Japan and geology is relatively homogeneous. There is a potentially liquefiable sand layer (As layer) of about 12 m thick whose SPT N-value is about 15 and the fine contents (Fc) less than 10%.

The soil investigation to measure SPT N-value had been carried out at the time of the construction at 32 locations about 450 m interval. However, 19 locations of them are only to 7 m deep with measurement interval of 3 m. The quantity of the investigation is far less compared to current practice. Besides these investigations, the dynamic triaxial tests to evaluate liquefaction resistance (\( R_{L20} \)) were carried out in more recent years for 2 samples taken at STA.50 and 145. \( R_{L20} \) were about 0.25 at STA.50 and 0.21 at STA.145. Note that length of each station is 10 m.

Under the liquefiable sand layer (As), there is a soft clay layer (Ac) of 25 m thick and SPT N-value of about 2, then relatively dense sand layer (Ds), which is underlaid by the bedrock of SPT N-value over 50.
3.1.3 Seismic characteristics of the site
The area is in the region where near future occurrence of Tokai-Tonankai earthquake is suspected. Model earthquake motion provided by the central disaster mitigation conference (2006) for the earthquake is employed in this study.

The nearer to the epicentre, the stronger the earthquake motion. Therefore, the downstream part is more susceptible to stronger earthquake motion. By the peak ground surface acceleration (PGA), it is 135gal at the most upstream point, 175gal at the middle point and 241gal at the most downstream point. The distinguished characteristics of this earthquake motion are its very long continuous time (about 120 sec) and dominance of the long period components (2 – 4 sec). As far as the continuous time and the spectral characteristics are concerned, there is no difference for the upstream and the downstream.

3.2 The Limit State and Evaluation of the Performance

3.2.1 The limit states
The performance requirements of this irrigation channel are to keep the water level that is sufficient for the natural distribution of water to the surrounding area and to provide sufficient quantity of water to the destinations. Since part of the water is used for urban water supply, it is necessary to keep the water level even right after the earthquake. Thus, to keep this water level after the earthquake is set as the performance requirement of the channel. To be more specific, a limit was set to the absolute settlement of the RC frame for maintaining the water level, and to the relative settlement of the adjacent frames to preserve necessary quantity of water flow. The limit state was set to 60 cm for the absolute settlement based on the free board of the channel, and to 60 cm for the relative settlement due to the frame base thickness.

3.2.2 Method to evaluate the seismic performance
The problem is to evaluate the residual settlement of the irrigation channel for the earthquake with considerably long duration and of long dominant period. The dynamic FEM based on the effective stress analysis, LIQCA2D07 (Oka et al. 1994), is employed in order to take into account of the mobilization and dissipation of the excess pore pressure. The effectiveness of the program was checked by analyzing shaking table test which had modeled the channel.

LIQCA2D07 has been used to analyze the liquefaction of ground induced by earthquakes, and has been employed in the actual design for several occasions already. If the modelling and soil parameter values are set appropriately, much accurate prediction of the performance is possible. However, there are quite many soil parameters some of which need to be set based on the dynamic triaxial test results. This process requires certain skill, experience and engineering judgement, whose result may not be the same from one
3.3 Procedure of the design and uncertainty

The design procedure and the uncertainties to be considered are presented in Figure 4. The settlement of the RC frame is predicted by LIQCA2D07 for various possible conditions. Based on this parametric study, a response surface (RS) is built which is to be used in the reliability assessment.

Uncertainties considered in this study are model uncertainty of LIQCA2D07, spatial variability of soil parameter (i.e. SPT N-value), statistical estimation error and error associated to the approximation by RS. These uncertainties are quantitatively analysed by the statistical means and incorporated to the reliability assessment by MCS.

The settlement induced by the liquefaction is a complex phenomenon which is influenced by many factors. In stead of building a very complex RS, relatively simple RS was introduced in this study. The uncertainty associated to the RS, which is the residual of the regression analysis of the settlement by various factors are also introduced in the reliability assessment.

4 ANALYSIS OF UNCERTAINTIES

4.1 Spatial Variability and Statistical Estimation Error

4.1.1 Basic variables for the response surface

It is necessary to select a geotechnical parameter that is appropriate to represent ground characteristic in evaluating potential of liquefaction. Sn value proposed by Goto et al. (1982) is selected in this study to represent the strength of ground for liquefaction. This is weighted integration of adjusted SPT N-value, N1, over 20m depth. N1 is defined as 

$$\text{N1} = 170 \cdot N / \left( \sigma_v + 70 \right) $$

where \( \sigma_v \) is the effective overburden stress.

$$ S_n = 0.264 \cdot \int_0^{20} e^{-0.04N_1(x)-0.24x} dx - 0.885 $$

The characteristic of the sand layer is solely evaluated by N1 value in this index. This is justified in this case because as layer is very homogeneous and the grain size distribution is similar throughout the area, thus \( S_n \) is an effective index to evaluate the liquefaction strength of ground at least relatively. Furthermore, the liquefaction is not a phenomenon at a single point but for certain volume of soil mass. Therefore, it make sense to evaluate the ground property by some weighted averaging value over the depth like \( S_n \).

The irrigation channel is a very long continuous structure. The statistical estimation error of the ground should be different at the location where the soil investigation has been made and at other locations. This difference will be evaluated by distinguishing the general estimation and the local estimation problems: the relative location of investigation and construction is not taken into account in the former, whereas they are taken into account in the latter.
4.1.2 The general estimation of $S_n$

In the general estimation, the uncertainties of 12m thick As sand layer is treated in a unified way for 12km stretch. The trend of N1-value is modelled by a quadratic line as illustrated in Table 1. The residuals are also plotted against depth in Figure 5(a).

Table 1. Result of regression analysis on N1-value

<table>
<thead>
<tr>
<th>Models</th>
<th>Trend</th>
<th>SD</th>
<th>AIC</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>9.76+Z</td>
<td>7.38</td>
<td>1615</td>
<td>0.13</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.86+5.31Z-0.38Z^2</td>
<td>6.68</td>
<td>1569</td>
<td>0.29</td>
</tr>
<tr>
<td>Cubic</td>
<td>1.17+5.06Z-0.33Z^2-0.0027Z^3</td>
<td>6.60</td>
<td>1571</td>
<td>0.29</td>
</tr>
</tbody>
</table>

The fitness of the residuals to a normal distribution is checked by Q-Q plot presented in Figure 5(b). It is observed that the residuals are homogeneous over the depth and fit to a normal distribution. The vertical autocorrelation function is estimated by the moment method and the autocorrelation distance of an exponential autocorrelation function is estimated to be 0.80 m (Figure 5(c)).

4.1.3 The local estimation of $S_n$

The estimation error of $S_n$ is obtained by considering the investigation location and the estimation location. The method employed is Kriging and the conditional simulation based on it. The estimation is done in 2 steps as follows:

Step 1: estimation of $S_n$ at the investigation locations

There are two types of SPT investigations for this irrigation channel: N-value is measured at each 1 m interval in some locations (Sites-r1), whereas it is measured at each 3 m in other locations (Sites –r3). By definition, $S_n$ is calculated based on SPT N-value measured at 1 m interval. Thus, $S_n$ become a fixed value for Sites-r1. However, interpolation estimation error must be evaluated for Sites-r3. The conditional
simulation is employed to evaluate uncertainty of $S_n$ at Sites-r3: 1000 sets of N-values of 1m interval samples that pass through the measured values and yet have the same statistical characteristics as estimated above are generated to evaluate the uncertainty of $S_n$.

Step 2: the conditional simulation of $S_n$ over 12km stretch of the irrigation channel

$S_n$ values evaluated in Step 1 is used to estimate the correlation of $S_n$ for the horizontal direction. Then conditional simulation is used to generate $S_n$ over 12km stretch of the irrigation channel. In this conditional simulation, the estimation error of $S_n$ at Sites-r3 is also taken into account.

An exponential type autocorrelation function is fitted to describe the correlation of $S_n$ for the horizontal direction by the moment estimation method, whose results are smoothed for 50m as presented in Figure 7(a). The autocorrelation distance is estimated to be about 150m. The uncertainty involved in estimated $S_n$ is illustrated in Figure 7(b) by showing mean and mean +/- SD. The mean and SD obtained in the general estimation is also presented in the same figure.

$$\rho(\Delta x) = \exp(-\Delta x/150)$$

4.2 Model Error of FEM

The model error involved in estimating displacement of RC frame channel structure by LIQCA2D07 is evaluated here. The evaluation is done by correcting blind tests results for model tests on similar structures, i.e. embankments and embedded structures. The blind tests are type A prediction where predictions for the displacement is done without knowing the model test results. 17 blind test results are collected from the literature (JICE2002, Uzuok et al. (2003) and Yoshizawa at al. (2009)) and the ratio between “the predicted values” and “the true values” are obtained (Figure 8). The results lie between 0.4 and 1.6, where the mean 1.0 and SD 0.23. A Q-Q plot for a normal distribution is also presented to show that the results fit to a normal distribution.

4.3 Response Surface (RS) and Its Model Error

The vertical displacement (settlement for the embankment type and uplift for the embedded type) is related to $S_n$ and shear stress at the center part of liquefiable layer($\tau$) based on the 22 results of LIQCA2D07.

In order to evaluate the shear stress distribution along the depth, one dimensional linear equivalent response analysis by SHAKE (Schnable P.B. et al., 1972) is performed especially to take into account of the effect of the underneath soft clay layer.
The vertical displacement is related to \( S_n \) and \( \tau \) by a linear regression line:

\[
D = a \cdot S_n + b \cdot \tau + c + \varepsilon
\]  
(4)

Where \( D \): vertical displacement (cm) obtained by LIQCA2D07, \( \tau \): shear stress(kN/m\(^2\)) acting at the center part of liquefiable sand layer, \( a, b \) and \( c \): regression coefficients, and \( \varepsilon \): residual error.

Figure 9 presents fit of the model to the data, which exhibits reasonably good fit. The residuals are plotted on normal Q-Q plot. They follow a normal distribution well with mean 0.0 and SD 10.24 (cm) for the settlement and mean 0.0 and SD 2.83 (cm) for the uplift.

\[\begin{align*}
\text{Normal Q-Q plot} & \quad \text{Theoretical value} \\
\text{Observation} & \quad \text{Residual (cm)} \\
\end{align*}\]

Figure 9. Result of regression analysis

5 RELIABILITY ASSESSMENT

Finally, exceeding probability of the vertical displacement over the threshold value is evaluated by MCS based on obtained RS and the quantified uncertainty of various sources. The uncertainty considered are listed in Table-2.

The performance functions for the embankment type and the embedded type are respectively given as follows:

\[
D_{embk} = (-212 \cdot S_n - 18.8 \cdot \tau + 120) \cdot \delta_{RS} \cdot \delta_{FEM}
\]  
(5)

\[
D_{embd} = (100 \cdot S_n + 1.97 \cdot \tau + 51) \cdot \delta_{RS} \cdot \delta_{FEM}
\]  
(6)

Table 2. Input to reliability analysis

<table>
<thead>
<tr>
<th>Uncertain sources</th>
<th>Notation</th>
<th>mean</th>
<th>SD</th>
<th>Distribution type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_n )-value</td>
<td>( S_n )</td>
<td>-0.34(^{0(1)})</td>
<td>0.85(^{0(1)})</td>
<td>Normal</td>
</tr>
<tr>
<td>Earthquake shear stress</td>
<td>( \tau )</td>
<td>[12-17.5]</td>
<td>0</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Model error of RS</td>
<td>( \delta_{RS} )</td>
<td>1.0</td>
<td>0.09(^{0(2)})</td>
<td>Normal</td>
</tr>
<tr>
<td>Model error of LIQCA2D07</td>
<td>( \delta_{FEM} )</td>
<td>1.0</td>
<td>0.23</td>
<td>Normal</td>
</tr>
</tbody>
</table>

*1: values by the General estimation
*2: COV=10.24/110=0.09(embankment type) 2.83/48=0.06(embedded type)
The results are presented in Figure 10, 11 and 12. Figure 10 and 11 show the mean elevation after shaking of each RC frame (10 m long) for the general estimation and the local estimation of $S_n$ value respectively. It can be seen, in both cases, the displacement is larger in the downstream because of the stronger earthquake motion. In the downstream part, the mean settlement exceeds the threshold value of 60 (cm). The larger relative displacement occurs at location where the embankment type switches to the embedded type, which implies danger of leakage of water from the channel.

Although the general feature of the vertical displacement is similar for the general and local estimation of $S_n$, one can see more detailed behavior of each RC frame in the local estimation. For example, there is location where the mean settlement exceed 60 (cm) near STA90 in the local estimation.

Figure 10 presents the mean vertical displacement and the exceeding probability of it over the threshold values (i.e. 60 cm) are presented for the general and local estimation of $S_n$. The two cases are superposed in these figures for the comparison. The prediction based on the local estimation generally gives smaller exceeding probability, however there are several locations where this relationship is reversed. These probability can be used to determine the optimum enforcement plan of this irrigation channel.

Figure 10. Mean elevation after shaking (general estimation of $S_n$-value)

Figure 11. Mean elevation after shaking (local estimation of $S_n$-value)

Figure 12. Result of Reliability analysis
CONCLUSION

The results of the analysis are believed to provide useful information to designer and the owner of the structure, some of which can be listed as follows:

- The long stretching structure like this irrigation channel, quite amount of time and cost is necessary to conduct an enforcement construction. The information provided here is very useful in making plan for this enforcement construction and determine sequence of the construction.
- The contribution of each uncertain source to the final result can be calculated by an approximate means proposed by Honjo et al. (2011), which the result is shown in Table-3. It should be pointed out that the owner has an alternative to obtain more information on the soil property by adding soil investigation. The result of the local estimation gives very variable information concerning this aspect.
- The response surface obtained itself useful information for the designer. Furthermore, it can be used when additional information on ground property is given to reevaluate the reliability.

<table>
<thead>
<tr>
<th>Uncertainty sources</th>
<th>All uncertainty</th>
<th>S_r-value</th>
<th>Model error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>FEM</td>
</tr>
<tr>
<td>Site-r1 (STA63)</td>
<td>1.87</td>
<td>1.88</td>
<td>8.49</td>
</tr>
<tr>
<td>Site-r3 (STA56)</td>
<td>1.58</td>
<td>2.32</td>
<td>2.05</td>
</tr>
<tr>
<td>Site-nr (STA60)</td>
<td>1.02</td>
<td>1.42</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Note: Site-nr: no investigation at the site

REFERENCES

Y.Honjo, T.Hara, Y.Otake and T.T.Kieu Le (2011) : Reliability based design of Examples set by ETC10,Geotechnique (submitted)