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Level III Reliability Based Design employing Numerical Analysis - Application of RBD to DEM -

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ABSTRACT: This study presents an example of reliability based design using Discrete Element Method. Rockfall retaining wall is employed as target structure. A series of rockfall simulations were carried out using DEM to obtain a response surface of the energy. Then the uncertainty of basic variables is quantified to conduct Monte Carlo Simulation. Finally, a relation between the exceedance provability and the energy was obtained.

Keywords: Reliability based design(RBD), DEM analysis, Rockfall

1 INTRODUCTION

In recent years, numerical analysis is beginning to be used in design of structures and ground. Discrete Element Method (DEM) is well known as one of the strong numerical tools. The method can express collision between solids such as rockfall problem. Because shape of rockfall and complex geometry can be expressed directly, the method can predict complex movement of rockfall. However, simulated results are highly sensitive to numerical parameters, and the results have large variation. Therefore, a framework in which quantitative results can be obtained from DEM analysis is required.

This study aims to show a framework of reliability based design (RBD) using DEM. Rockfall retaining wall is employed as a target structure. Based on results of this study, advantages of proposed framework are discussed.

2 A FRAMEWORK OF RELIABILITY DESIGN USING NUMERICAL ANALYSIS

Authors have proposed a framework of level III reliability based design (RBD) using numerical analysis (Honjo et al., 2010). Based on the framework, process of RBD of rockfall retaining wall using DEM is shown in this study.

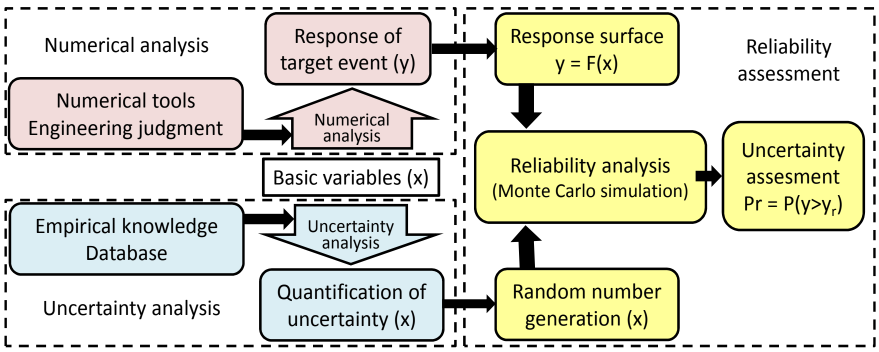

Figure 1. RBD framework using numerical analysis

The framework is separated into three parts: numerical analysis (I), the uncertainty analysis of basic variables (II) and the reliability analysis (III). In the numerical analysis (I), some cases are carried out under the different combination of basic variables (x), and the response of target event (y) is investigated. The energy of rockfall is focused on because that is quite important for design of the retaining wall. Thus, the energy (y) is calculated under the different combination of the parameters, such as the coefficient of restitution, friction angle and shape of rockfall.

The uncertainty analysis (II) is a process in which the uncertainty of basic variables is quantified. Database and empirical knowledge are used to obtain statistical information of basic variables, such as the mean value, the standard deviation and the distribution function. In the reliability assessment process (III), the response surface (RS) of target event is estimated from the results of numerical analysis. Then a simple Monte Carlo Simulation (MCS) is carried out using the RS and results of the uncertainty analysis. Finally, exceedance probability of occurrence of target event is quantified under the given conditions.

The advantages of the proposed framework are as follows,

- It is possible to respond immediately to development of numerical methods because the numerical analysis and the uncertainty analysis are separated.
- The relation between responses of target structures and basic variables provide useful information to designer.
- Designer can understand obtained results intuitively because MCS is used in the reliability assessment process.
- Numerical results are used just for estimating RS. Because MCS is carried out using RS, time and effort spent in the numerical analysis can be minimized.

3 FLOW OF PROPOSED RBM AND UNCERTAINTIES

Figure 2 shows proposed RBM of rockfall retaining wall. The uncertainties that should be considered are also described in the figure.

Like traditional design procedure, the field investigation is carried out. Some investigation items of rockfall such as position, size, shape and rock type is checked. The measuring error arises in this procedure.

After the field investigation, a numerical model and values of parameters are determined. The transformation error of parameters and the model error of DEM arise in this procedure. Then, a parametric study is carried out under the different combination of parameters. The energy of rockfall is obtained from each simulation cases.

In the uncertainty analysis, the uncertainties are quantified. The measuring error is not taken into consideration in this study. The transformation error and the statistical estimation error are treated as variation of numerical parameters. The statistical estimation error is derived from spatial variation of strength and rock type. The variations of the parameters are estimated from literatures in this study. The model error is derived from the numerical modeling. In DEM analysis, rock body is assumed to be rigid body. Thus, it is impossible to reproduce actual phenomena perfectly. The model error includes such uncertainty. In this study, statistical values of the model error are assumed.

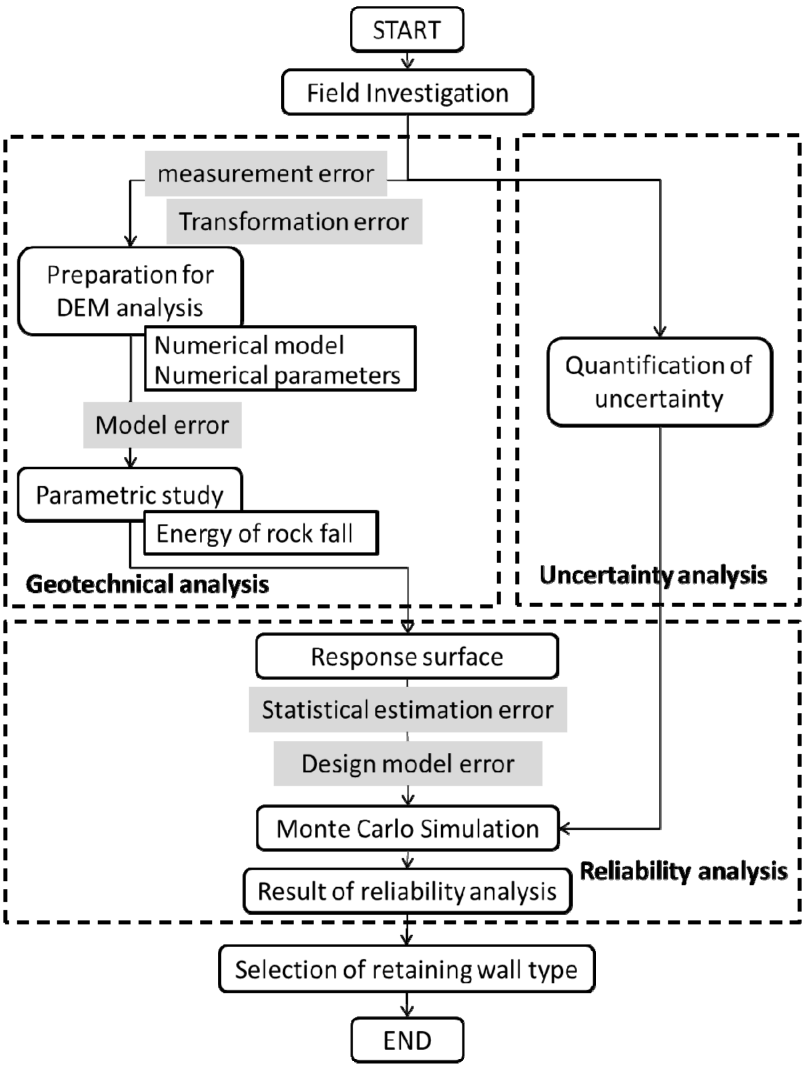

Figure 2. Proposed design scheme by response surface

In the reliability analysis, the RS is estimated. If there is large variation in the numerical results, the design model error should be considered. The error is treated as variation of the RS.

4 NUMERICAL ANALYSIS

4.1 *Numerical method and analysis condition*

2 dimensional DEM was adapted to rockfall simulation. In DEM analysis, as shown in Figure 3, an interparticle model is used to describe collision force. Interparticle force is calculated and movements of each particle are solved based on the equation of motion. Complex shape can be also express by connecting particles. Therefore, shape of rockfall and slope can be modeled directly.

Figure 4 shows schematic view of a numerical model used in this study. In a normal situation, although position and size of rock body and surface configuration of slope should be modeled based on the field investigation, virtual rock body and slope are used in this study. In the initial condition, the rock body is placed at the top of the slope and falls due to the gravity at the start of the simulation. The mass of the rock body is 400 kg. Although a retaining wall is drawn in Figure 4, it doesn't exist in the simulation. The velocity and the rotation rate of rockfall are checked when the rockfall pass thought in front of the retaining wall. The energy of rockfall is calculated from the velocity and the rotation rate.

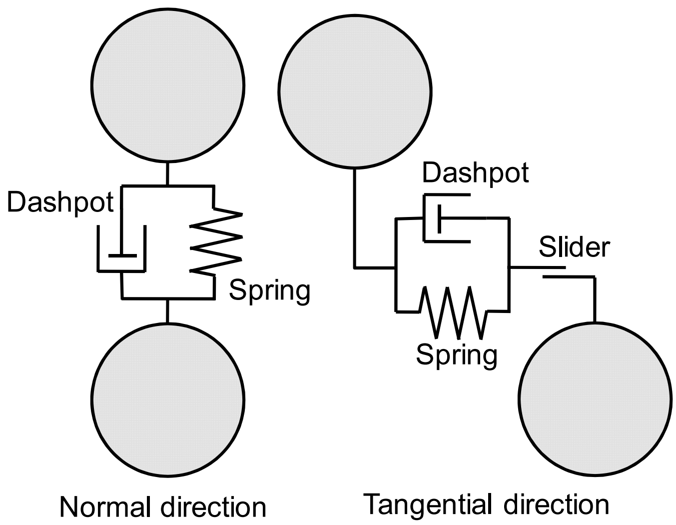

Figure 3. Interparticle force model of DEM

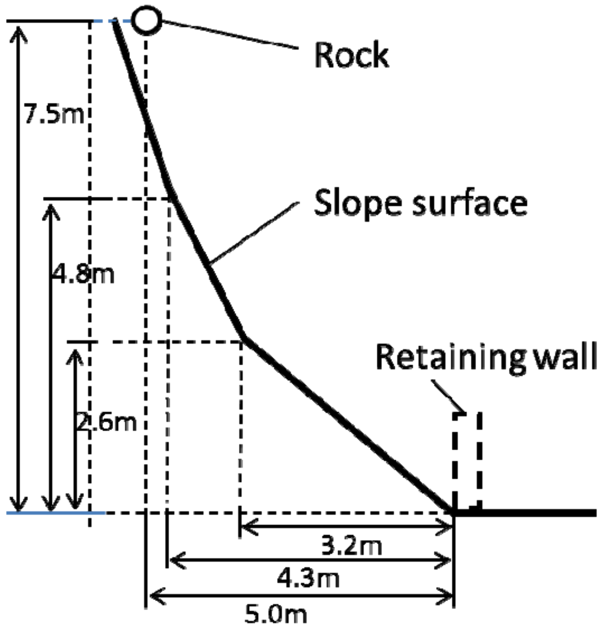

Figure 4. Numerical model

4.2 *Numerical parameters*

The interparticle model of DEM has the spring, the dashpot and the slider. Although many parameters should be determined, the key parameters are the coefficient of restitution and the friction angle. Therefore, these parameters were used as basic variable. In this simulation, shape of rock body is also unknown. Therefore the aspect ratio of the rock body is introduced as basic variable to investigate effect of the shape. The aspect ratio was changed under the constant volume of rock body as shown in Figure 5. We used 5 kinds of the coefficient of restitution (0.4-0.6), 5 kinds of the friction angle (20-40 degrees) and 8 kinds of the aspect ratio (1.083-1.940). A total of 200 cases were carried out under the different combination of the parameters.

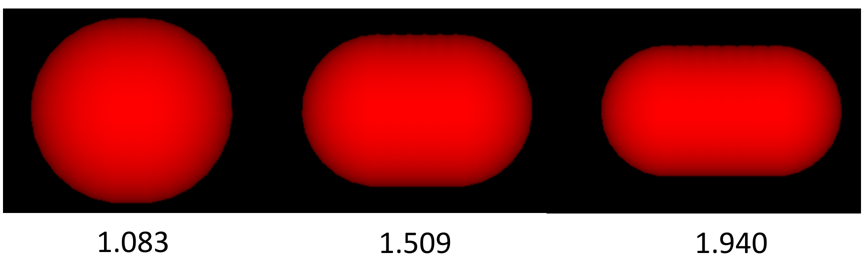

Figure 5. Shape of rock body on different aspect ratios

5 RESPONSE SURFACE

5.1 *Single regression analysis*

As explained previously, results obtained in DEM analysis are analyzed to assume RS. Firstly, we carried out single regression analysis to know correlation between the energy and each parameter. In figures 6, 7 and 8, the energy is plotted against each parameter. As shown in the figures, there is large variation in each result. In particular, strong correlation is not seen in the relation between the energy and the coefficient of restitution.

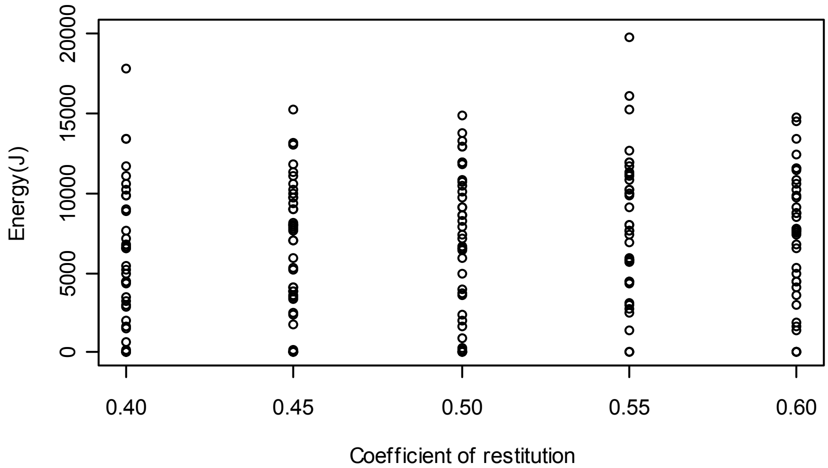

Figure 6. Relation between the energy and the coefficient of restitution

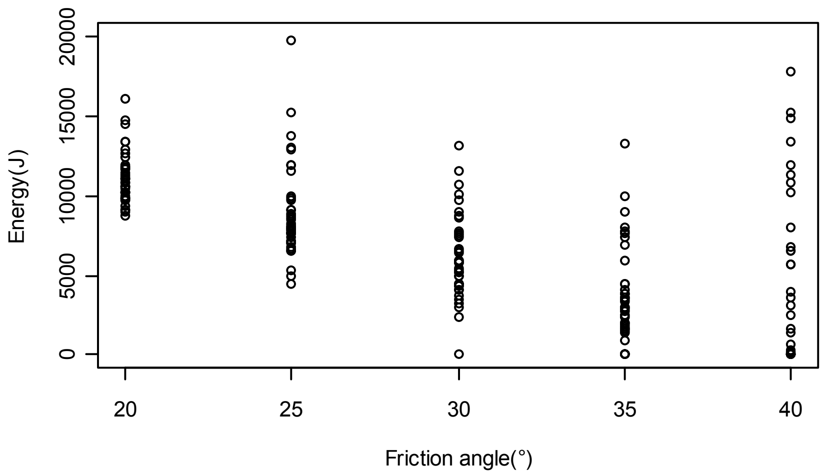

Figure 7. Relation between the energy and the friction angle

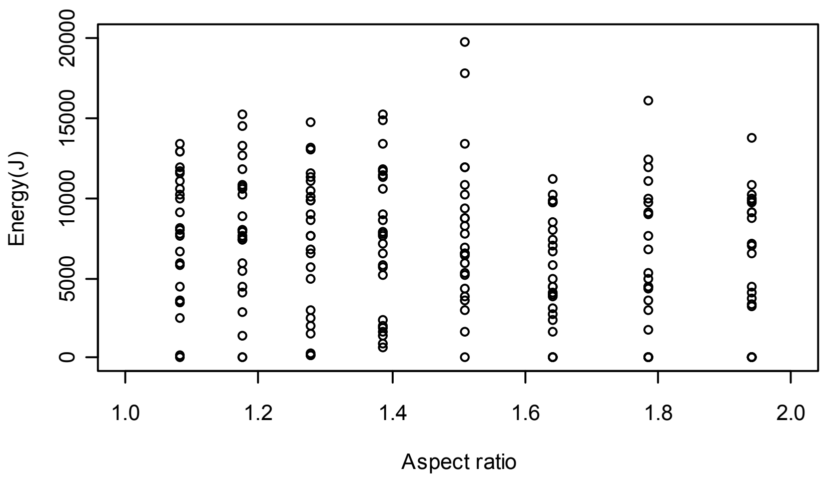

Figure 8. Relation between the energy and the aspect ratio

5.2 *Multiple regression analysis*

Based on results of the single regression analysis, RS was assumed as a function of the friction angle and the aspect ratio. Table 1 shows equations assumed in this analysis. The standard deviation, the residual and AIC value (Akamine, 1973) are also described in the table. Based on the results, No.6 was selected and following RS was obtained.

$$E = 22685 Asp - 7759 \log(F) Asp + 11864 \tag{1}$$

where, *E, Asp* and *F* are the energy, the aspect ratio and the friction angle, respectively. Figure 9 shows the obtained RS. As explained previously, the model error of DEM is one of the errors that should be taken into consideration. Furthermore, the design model error should be considered because there is large variation in numerical results. By considering the these errors, the RS is updated as follows,

$$E = \left(22685 Asp - 7759 \log(F) Asp + 11864\right) \cdot \delta_{DEM} \cdot \delta_{RS} \tag{2}$$

where δ_{DEM} is the coefficient of the model error and δ_{RS} is the coefficient of the design model error. The design model error was treated as the model error of the RS.

Table 1. Functions used in multiple regression analysis

No.	Function	SD	R^2	AIC
1	$E = a \cdot Asp + b \cdot F + c$	3298	0.443	3811
2	$E = a \cdot Asp^2 + b \cdot F^2 + c$	3298	0.444	3811
3	$E = a \cdot Asp + b \cdot F + c \cdot Asp \cdot F + d$	3263	0.458	3806
4	$E = a \cdot Asp + b \cdot \log(F) + c$	3255	0.458	3806
5	$E = a \cdot Asp + b \cdot \log(F) + c \cdot \log(F) \cdot Asp + d$	3220	0.472	3801
6	$E = a \cdot Asp + b \cdot Asp \cdot \log(F) + c$	3212	0.472	3800

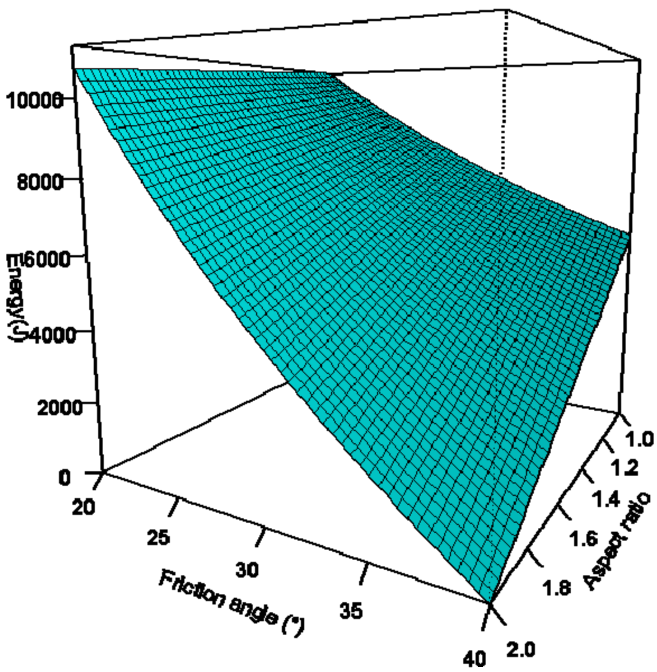

Figure 9. Response surface

6 UNCERTAINTY ANALYSIS

In the RS of energy, 4 kinds of basic variables are included, such as the friction angle, the aspect ratio and the model errors of DEM and RS. In the uncertainty analysis, variations of these basic variables are quantified. The mean value, the standard deviation and the type of distribution function are estimated based on results of the field investigation, database and empirical knowledge. In this study, however, the statistical values were assumed from common values of each basic variable, because virtual rockfall and slope are used in this study. Table 2 shows the statistical values of each basic variable. The variation of the model error of RS is calculated from results of numerical analysis.

Table 2. Statistical values of basic variables

Basic variable	Mean	SD	Distribution type
Friction angle (degree)	30	7	Normal
Aspect ratio	1.5	0.5	Normal
Model error (RS)	0.0	0.55	Normal
Model error (DEM)	1.0	0.2	Normal

7 RELIABILITY ANALYSIS

MCS was carried out using RS and quantified uncertainties of the basic variables. Figure 10 shows a histogram of calculated energy. Figure 11 shows the relation between the energy and the exceedance probability. Generally, type of rockfall retaining wall is selected based on the energy of rockfall. Therefore, the exceedance probability is very useful information for design of rockfall retaining wall. In addition, the energy of rockfall is calculated with consideration for the results of DEM analysis.

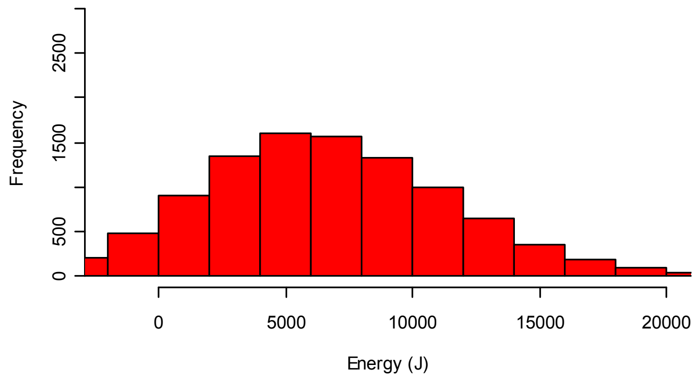

Figure 10. Histogram of energy obtained from MCS

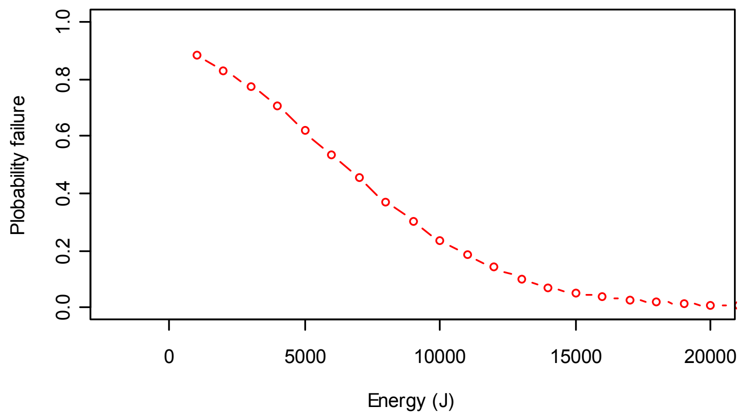

Figure 11. Relation between the energy and the exceedance probability

8 EFFECT OF NUMBER OF CALCULATION CASES

It is well know that calculation cost is one of the disadvantages of DEM. In particular, when we use 3 dimensional DEM, it requires an immense amount of time. However, there is a possibility to reduce effort and time of numerical analysis by using proposed method. Although 200 cases of rockfall simulation were carried out in this study to get relation between the energy of rockfall and numerical parameters, smaller number of calculation cases might be enough. Therefore effect of number of calculation cases is investigated. The number of calculation cases was decreased to 45 cases by reducing number of kinds of the aspect ratio. Base on the results of 45 cases, RS was assumed and the relation between the energy and the exceedance probability was calculated. Figure 12 shows obtained exceedance probability. Blue line is result obtained from 45 calculation cases and red line indicates the results obtained from 200 calculation cases. As shown in the figure, there is not big difference between the results. This indicates that 45 calculation cases are enough for the problem considered in this study.

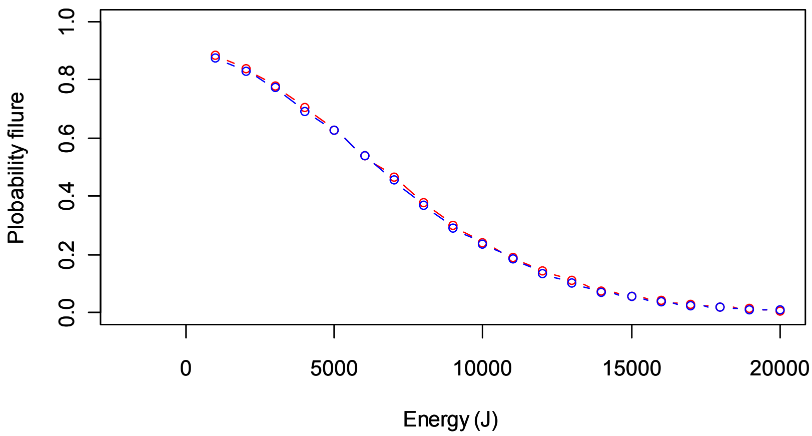

Figure 12. Response surface

9 CONCLUSION

The framework of RBD based on DEM was shown in this study. The rockfall retaining wall was selected as target structure and the exceedance probability was calculated at each value of the energy. By combining RBD and DEM, variation of results of DEM can be quantified and the exceedance provability of rockfall energy can be obtained from the results of MCS. It can be summarized the proposed framework is quiet useful for the design of rockfall retaining wall. In addition, the proposed framework can reduce effort and time of numerical analysis.

This study presents just procedures of the proposed framework. In order to figure out an effectiveness of the proposed framework, more validations are required, such as reproduction of reported real rockfall. In addition, fundamental studies such as qualification of the model error of DEM are required.

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