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# Reliability analysis of shallow foundations subjected to varied inclined loads

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ABSTRACT: Monte Carlo simulation was used to study the effect of inclination factor on the reliability of shallow foundations. The variation of inclination factor was inspected by considering the variation of horizontal and vertical loads. Meyerhof's bearing capacity equation was employed to formulate the performance function of bearing capacity of shallow foundations. A shallow foundation on cohesionless soils under various loading conditions was simulated. Friction angle of soil, horizontal and vertical loads were considered as non-correlated normally distributed variables in the study. The results showed that, probability of failure of the shallow foundation was less influenced by the variation of vertical load than that of friction angle and horizontal load. Reliability indexes of the foundation were derived with the probability of failure ($P_f$) using different methods. It was found that, when the limit equilibrium function is not normally distributed, for a given value of $P_f$, the value of reliability index varies with the method employed.

*Keywords: reliability, shallow foundation, inclination factor, Meyerhof's bearing capacity equation, Monte Carlo simulation*

## 1 INTRODUCTION

Shallow foundations are designed to bear loads from upper structures, including vertical and horizontal loads, which are combinations of dead and live loads. Live loads vary much more during the life of a foundation comparing to dead load. Live load can be vertically, such as machinery load, human weight and earthquake load, and horizontally, such as wind and earthquake load. Dead load can also be inclined such as load on shallow foundations of bridge abutments. The combination of live and dead loads, or horizontal and vertical loads, result in the variation of magnitude and direction of loading imposed on a foundation. To account for the variation of loading, a reliability analysis based on probabilistic theory is required.

Research has been done to study the influence of variation of soil properties on bearing capacity of shallow foundations "Cherubini (2000); Honjo *et al.* (2000); Phoon *et al.* (2003); Alawneh *et al.* (2006) ". Though load is one of the most variable parameters in shallow foundation design, not much discussion about the effect of variation of inclined load on the bearing capacity of foundations was available in publications. "Honjo *et al.* (2000)" used First Order Reliability Analysis (FORM) to study the variation of inclination factor on the reliability of shallow foundations with a modified Terzaghi equation. Load and resistance factor design (LRFD) method was introduced into shallow and deep foundation design "Paikowsky *et al.* (2004); Paikowsky *et al.* (2010) ". Partial factors were used to consider the variation of loads in Eurocode 7 "ENV(1997-1) (1997) ". For variable actions, a factor of 1.5 is applied for unfavorable / disturbance actions. Orr (2000) discussed the selection of partial factors and suggested that engineers should be careful in selecting these factors in terms of favorable or unfavorable actions. In many cases of shallow foundation design, either horizontal or vertical loads can be unfavorable. Applying same partial factors to these two actions might not be proper, as the variations of the two actions are not the same in many cases. Foye *et al.* (2006) stated that, the load and resistant factors used in current practice can not cover the wide range of problems in shallow foundation design. "Paikowsky *et al.*

(2010)" recommended to reduce the resistence factor of friction anlge of soils to 0.5 after the investigation of a large number of of shallow foundations for bridge abutment under inclined eccentric loads. The authors also suggested different loading factors for horizontal and vertical loads.

In bearing capacity analysis of shallow foundations, the ratio of the horizontal and vertical loading is described with an inclination factor. Different combination of horizontal and vertical loads will result in a variation of the magnitude of inclined load and the inclination factor, which in turn the bearing capacity. In this paper, the reliability of shallow footings is analysed considering the variation of the inclination factor and soil strength. A sensitive analysis was carried out to find out the variation of horizontal and vertical loads on the reliability of shallow foundations.

## 2 VARIATION OF SOIL AND LOADING PARAMETERS

### 2.1 *Variation of soil and loading parameters*

To carry out a reliability analysis, a thorough study of the related uncertainties is essential. In regarding to the bearing capacity of shallow foundations, there are many uncertainties involved, e.g. variation of soil properties with time and space, variation of magnitude and direction of loading, uncertainties in the bearing capacity equations or performance functions, distribution and correlation of the uncertainties. This research concentrates on the variation of soil strength and loads on the bearing capacity of shallow foundation.

Among soil properties, unit weight, cohesion, and friction angle are the most frequently studied variables regarding to the reliability analysis of bearing capacity of shallow foundations. As these three parameters are most directly used to evaluate bearing capacity of shallow foundations in many available methods. The variation of a parameter is described with the coefficient of variation (COV) of its distribution. Research found that, unit weight varies in a relatively limited range with COV between 1-10%. COV values for friction angle are in a range of 5%-20 for sands and 7-56% for clays. The most highly varied and hardest to estimated parameter is the COV of shear strength of clays, especially that of undrained shear strength. For saturated clays, an increase of 1% of water content in saturated clay may cause a reduction of 20% of the soil's undrained shear strength "Muni (2000) ". In unsaturated soils, due to the appearance of suction, a decrease of water content will result in the increase of apparent cohesion in soils "Fredlund *et al.* (1978) ". Typical values of COV of soil properties taken from some publications are listed in Table 1.

Table 1. Coefficient of Variation of Soil Properties

| Parameter | | COV (%) | References |
|---|---|---|---|
| Unit weight ($\gamma$) | | 1-10 | Lee *et al.* (1983) |
| | | 5-10 | Lumb (1974) |
| | | 3-7 | Duncan (2000) |
| | | 2 | Christian *et al.* (1994) |
| Friction angle ($\phi'$) | Sands | 5-15 | Lee *et al.* (1983) |
| | Clay | 12-56 | Lee *et al.* (1983) |
| | Clay and sand | 5-15 | Phoon & Kulhawy (1999 a, b) |
| Cohesion ($c_u$) | Sandy soil | 25-30 | Lee *et al.* (1983) |
| | Clays | 20-50 | Lumb (1974); Lee *et al.* (1983) |
| Undrained strength ($S_u$) | unconfined | 20-55 | Phoon & Kulhawy (1999 a, b) |
| | UU test | 10-35 | Phoon & Kulhawy (1999 a, b) |
| | CU test | 20-55 | Phoon & Kulhawy (1999 a, b) |
| | Field vane shear | 10-40 | Phoon & Kulhawy (1999 a, b) |
| | | 10-20 | Duncan (2000) |
| | | 20-32 | Christian *et al.* (1994) |
| | $S_u/\sigma_{vo}'$ | 5-15 | Duncan (2000) |

A shallow foundation is designed to resist against loading from upper structures. The variation of loading needs to be considered in the reliability analysis of shallow foundations. The variation of loads can be narrow or wide, depending on the nature of the loads. COV of dead load, such as self weight of structures, normally varies within a range of 10%. While for variation of live loads, COV value can reach up to more than 100% for earth quake loads. Typical COVs for different types of loads are shown in Table 2. The table shows that, in a non-earthquake zone, wind load varies the most. Assuming that dead load, live load and snow load act vertically on a shallow foundation, the variation of vertical loads on a shallow foundation should be within a range of 25% considering the combination of above loading. The value varies from case to case.

Table 2. COV values for loads "EllingwoodGalambos (1982) "

| Load | COV |
|---|---|
| Dead load | 0.1 |
| Live load (50-year maximum) | 0.25 |
| Snow load (50-year maximum) | 0.26 |
| Wind load (50-year maximum) | 0.37 |
| Earthquake load (50-year maximum, Western and Eastern USA) | 1.38 |

## 3 PERFORMANCE FUNCTION

The performance function of shallow foundations can be obtained using a bearing capacity formula. One of the most commonly used equations for bearing capacity analysis is the Terzaghi equation:

$$q_u = cN_c + \gamma z(N_q - 1) + 0.5\gamma' BN_\gamma \qquad (1)$$

where, $q_u$ is the bearing capacity, $c$ is cohesion of soil, $\gamma$ is the unit weight of soil above ground water, $z$ is the embedment depth, $B$ is the width of the foundation, $\gamma'$ is the effective unit weight of soil, $N_c$, $N_q$ and $N_\gamma$ are the bearing capacity parameters. This equation was used by "Alawneh *et al.* (2006)" to analyse the reliability of shallow foundations by introducing a depth factor. The above equation has a limitation of not accounting for inclined load. "Honjo *et al.* (2000)" used a modified Terzaghi equation to calculate the bearing capacity of shallow foundations on cohesionless soils under inclined load:

$$q_u = 0.5 i_\gamma s_\gamma \gamma' BN_\gamma \qquad (2)$$

where $i_\gamma$ is the inclination factor and $s_\gamma$ is a shape factor of foundation.

To consider the effect of inclined load, Meyerhof's bearing capacity equation was used to establish the performance function proposed by "Meyerhof (1951, 1953, 1963) ":

$$q_u = cN_c s_c i_c d_c + \gamma z N_q s_q i_q d_q + 0.5\gamma BN_\gamma s_\gamma i_\gamma d_\gamma \qquad (3)$$

where: $s_c$, $s_q$, $s_\gamma$ are shape factors, $i_c$, $i_q$, $i_\gamma$ are inclination factors, $d_c$, $d_q$, $d_\gamma$ are depth factors. The foundations discussed in this paper are well above ground water table unless specified. The following expressions are used for the coefficients in equation 3:

$$N_c = (N_q - 1)\cot(\phi)$$ "Meyerhof (1963) "

$$N_q = \tan^2(45° + \phi/2)e^{\pi \tan\phi}$$ "Meyerhof (1963) "

$$N_\gamma = 2(N_q + 1)\tan(\phi)$$ "Vesic (1973) "

$$s_c = 1 + BN_q / LN_c \qquad s_q = 1 + (B/L)\tan\phi \qquad s_\gamma = 1 - 0.4B/L$$ " De Beer (1970) "

$$d_c = 1 + 0.4z/B \quad \text{(for z<B)}$$
$$= 1 + 0.4\arctan(z/B) \quad \text{(for z} \geq \text{B)}$$ " Hansen (1970) "

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 z/B \quad \text{(for z<B)}$$ " Hansen (1970) "
$$= 1 + 2\tan\phi(1-\sin\phi)^2 \arctan(z/B) \quad \text{(for z} \geq \text{B)}$$

$$d_\gamma = 1$$ " Hansen (1970) "

$$i_c = i_q = 1 - (\alpha/90)^2 \qquad i_\gamma = (1 - \alpha/\phi)^2$$ "Meyerhof (1953) "

where $\phi$ is the friction angle of soil, $L$ is the length of the foundation, $\alpha$ is the inclination angle of the load. Assuming load $F$ is composed of a horizontal force $F_x$ and vertical force $F_y$, then

$$F = \sqrt{F_x^2 + F_y^2} \qquad (4)$$

The factor of safety (FOS) of the foundation can be expressed as:

$$FOS = \frac{q_u - \gamma z}{F / LB} \quad (5)$$

At limit state, set $FOS=1$, the performance function ($LSF$) can be expressed as:

$$LSF = (q_u - \gamma z)LB - F \quad (6)$$

and the inclination angle can be expressed as:

$$\alpha = \arctan(F_x / F_y) \quad (7)$$

## 4 PROBABILISTIC ANALYSIS

### 4.1 *General*

In this section the reliability of a few shallow foundations on cohesionless soils was studied using Monte Carlo simulation. The simulation was performed in a Matlab language environment. The distribution of performance function of a shallow foundation was plotted to look into the reliability index and probability of failure. The influence of COV values of the variables on probability of failure was studied.

### 4.2 *Random variables*

To carry out a reliability analysis, the first thing is to identify the random variables to be considered in the problem. In equations 4 and 5, the basic random variables are soil properties and external actions. For a shallow foundation, the width, length and embedment depth of the foundation can be treated as deterministic values. For soil properties, unit weight, cohesion and friction angle are normally considered as random variables. For forces, as expressed in equations 3, 4, 6, 7, and Table 2, inclination angle ($\alpha$) and total force $F$ are correlated random variables. When consider random variable, it is always better to use independent variables. So for external forces, horizontal and vertical forces ($F_x$, $F_y$) can be used as random variables instead of $F$ and $\alpha$.

In order to simplify the problem, bearing capacity of shallow foundation on cohehionless soils is analyzed. As indicated in Table 1, unit weight of soil is less varied compare to friction angle. Considering that the aim of this paper is to study the influence of inclination factor on the reliability of bearing capacity of shallow foundations, the random variables considered in the following problems are friction angle ($\phi$), horizontal force ($F_x$) and vertical force ($F_y$). For simplification, these variables are considered as normally distributed.

### 4.3 *Probability of failure and reliability index*

A rectangular foundation, 2 m × 4 m was founded at a depth of 1 meter below ground surface. The soil is saturated clay ($\gamma=20\ kN/m^3$) with drained strength $c'=0$, $\phi'=25°$. The mean values of loadings on the foundation were: $F_x=80\ kN$, $F_y=400\ kN$. While calculated with these mean values, the factor of safety (FOS) of the foundation is 3.971. It is worthwhile to note that, in engineering design, FOS is normally calculated using the characteristic values instead of mean values of the parameters.

To carry out a reliability analysis, the COV values considered for $\phi'$, $F_x$ and $F_y$ were 0.1, 0.1 and 0.1. Ten thousand normally distributed random variables were created for each parameter to find out the probability of failure of the foundation with Monte Carlo simulation. The probability of failure was determined by:

$$P_f = (\text{number of cases } LSF<0) / (\text{total number of cases studied}) \quad (8)$$

Several simulations were run for this case considering that the size of random numbers created is not huge. The result showed that, the probability of failure ($P_f$) of the foundation was ranging between 0.04% and 0.06%, which is about 4 to 6 failure out 10,000 cases analyzed. The difference between the numbers is due to the limitation of the Monte Carlo simulation in producing consistence results when the size of random numbers used in the simulation is not large enough. By increasing the number of random variable to 100,000, a probability of failure of 0.046% can be obtained. It can be seen that, a size of 10,000 used in the Monte Carlo simulation gave reasonably close estimation. For probability of failure at 0.04%, a histogram of performance function was plotted in Figure 1. The figure shows that, the performance function is not normally distributed. This is due to the highly nonlinear expression of the performance function.

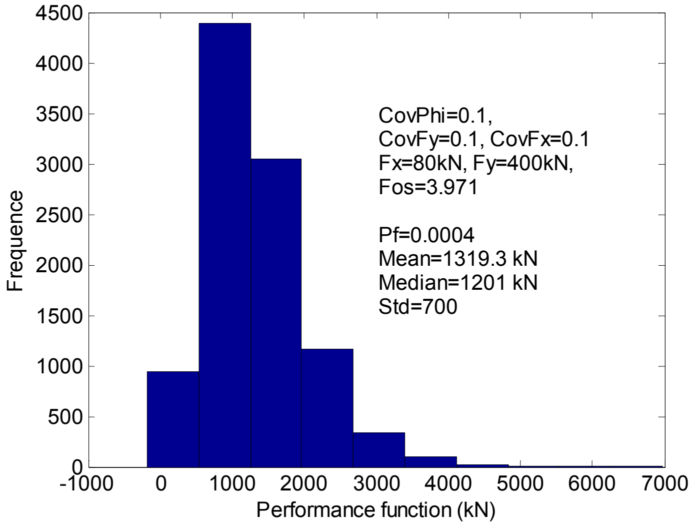


Figure 1. Histogram of performance function

Reliability index ($\beta$) is more familiar for engineers to evaluate the reliability of a structure. As shown in Figure 1, the performance function is more likely a shifted lognormal distribution. Using the equation proposed by Rosenbleuth and Esteva in 1972 to calculate reliability index of lognormal distribution "Paikowsky *et al.* (2004) ":

$$\beta = \ln(P_f / 460)/(-4.3) \qquad (9)$$

a reliability index of $\beta=3.245$ can be obtained with $P_f=0.0004$. By assuming normal distribution, the reliability index was 3.353.

The equation proposed by Cornell (1969) gave a reliability index of 1.885:

$$\beta = \frac{\mu(LES)}{\sigma(LES)} \qquad (10)$$

in which $\mu(LES)$ and $\sigma(LES)$ are the mean and standard deviation of performance function. The mean value and standard deviation of performance function were found at 1319.3 kN and 700 kN respectively by statistical analysis of the results.

The results showed that, Cornell's method gave smallest number of reliability index. This makes sense considering that the histogram of the performance function is right skewed. On the other hand, the performance function is highly nonlinear, so the reliability index calculated with equation (10) might not reflect the reliability of the foundation is this case. It has been noticed that the method is highly influenced by the form of the performance function "USACE (1997) ". This result tells that when performance function is not normally distributed, one should be careful in selecting the methods to obtain reliability index from probability of failure or vice versa.

Since reliability index is not consistent when using different methods, probability of failure is used in the following sections for further studies.

### 4.4 *Variation of probability of failure with COVs*

To study the variation of probability of failure with COVs of the parameters, a set of Monte Carlo simulations were carried out. The above foundation was reanalyzed with varied COV values for the parameters. Eight COVs for each parameter were used. The COVs range from 0.025 to 0.2 with 0.025 intervals. The size of random number was 10,000. In total, $8^3$ Monte Carlo simulations were performed. The results were shown in Figure 2. In the figure, each curve represented the variation of $P_f$ with COV(Phi), (COV of $\phi$) for certain values of COV($F_x$) and COV($F_y$), (COV of $F_x$ and $F_y$). For example, the upmost curve is the probability of failure curve for (COV($F_x$), COV($F_y$)) at (0.2, 0.2) with COV(Phi) varying. The figure showed that, under such external forces, the probability of failure of the foundation is very sensitive to the COV of friction angle, COV(Phi). The probability of failure of the foundation increase dramatically when COV(Phi) exceeds the value of 0.1. With COV(Phi) increase from 0.1 to 0.2, the probability of failure increase dramatically from less than 0.5% to more than 4%. The figure also showed that, with the variation of the COVs of external forces, the probability of failure can vary up to a range of 200% or more for the same value of COV(Phi). The next section will discuss about the variation of probability of failure with COV($F_x$) and COV($F_y$) while COV(Phi) value is set.

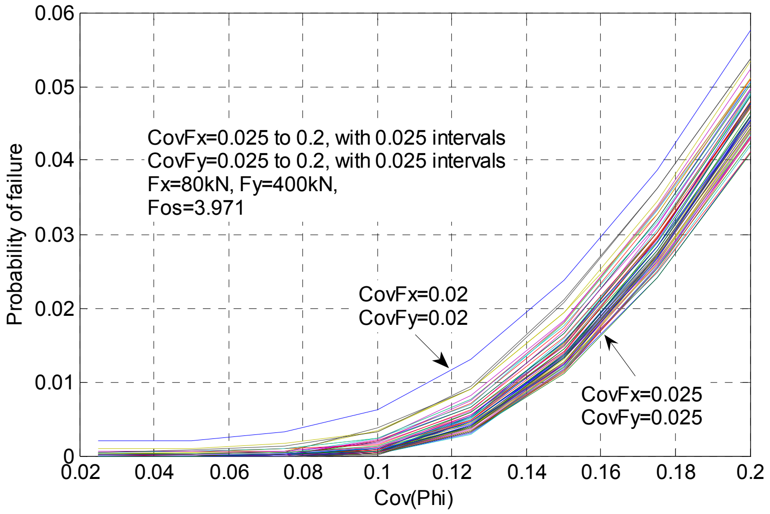


Figure 2. Variation of probability of failure with COVs

## 5 SENSITIVITY OF PROBABILY OF FAILURE TO INCLINED LOAD

As discussed above, COV values of external loads influence the reliability of shallow foundations. This section was to find out which one influences more on probability of failure: COV of horizontal load, $COV(F_x)$ or COV of vertical load, $COV(F_y)$.

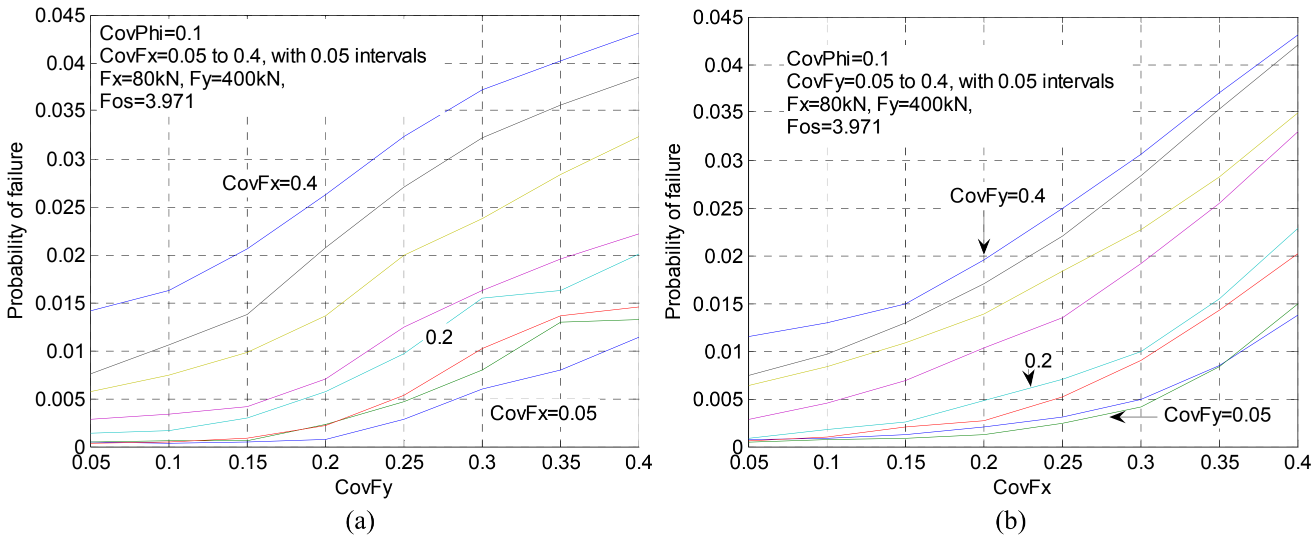


Figure 3. Probability of failure and COV of external forces.

Setting COV(Phi) at 0.1, by varying $COV(F_x)$ and $COV(F_y)$ from 0.05 to 0.4 with 0.05 intervals, the following plots can be obtained. From the figures, we can see that, $P_f$ values were more sensitive to the variation of $COV(F_x)$, as plots in Figure 3 (a) are more scattered than in (b), especially at higher values of $COV(F_x)$, e.g. $COV(F_x)>0.2$, which is mostly the case for wind load as shown in Table 2. In Figure 3 (b), at lower values of $COV(F_y)$, e.g. less than 0.2, the curves are more condensed. This suggested that with lower variation of vertical load, the probability of failure of a shallow foundation is more influenced by the variation of horizontal load. In Figure 3 (a), take $COV(F_y)=0.15$ for example, the probability of failure of the foundation increased from 0.0025 (β=2.807) to 0.021 (β=2.034) with the increase of $COV(F_x)$ from 0.2 to 0.4, noting that the reliability indexes were obtained from a normal distribution table.

## 6 CONCLUSION

Monte Carlo simulation was used to investigate the influence of the variation of inclined load on the reliability of shallow foundations. The performance function was built with a Meyerhof's equation. The simulation was performed in a Matlab language environment. A full analysis cost less than 1 minute to run on a HP Elitebook personal laptop for a simulation with $8^3 \times 10{,}000$ times of calculation on the performance function.

The results showed that, the reliability indexes exhibited huge difference using different methods when limit state function is not normally distributed. The authors recommended that, engineers should be careful in using the reliability index, especially when performance function is highly nonlinear and random variables are not normal distribution.

For foundations on cohesionless soils, the probability of failure of shallow foundation is most sensitive to the variation of friction angle of soils based on the simulations. In respect to loads, the variation of horizontal load has more influence on the probability of failure. In the choice of loading factors to carry out a LRFD, attention should be paid in choosing the loading factors, especially in cases when horizontal load is dominant and varies more. With the choice of loading factors, different factors should be considered for horizontal and vertical forces as recommended by "Paikowsky *et al.* (2010)".

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