ISGSR 2011 - Vogt, Schuppener, Straub & Bräu (eds) - © 2011 Bundesanstalt für Wasserbau ISBN 978-3-939230-01-4

A Consistent Failure Model for Probabilistic Analysis of Shallow Foundations

A. Kisse
CDM Consult GmbH, Bochum, Germany

ABSTRACT: In today's codes of practice, e. g. Eurocode 7 different ultimate limit states are distinguished. To overcome these problems an alternative design approach has been established on the basis of a unique failure condition. This failure condition describes the ultimate limit state of shallow foundations over the whole loading range without distinguishing different failure modes. The failure condition spreads out a failure surface which represents the outer border of the permissible loading. Hence the distance of the actual loading from the failure surface describes the safety of the system. This safety can be determined easily using reliability analysis. Here, the Hasofer-Lind second moment reliability index $ß_{HL}$ will be evaluated. The reliability based design of the foundation for a vertical breakwater based on this model is presented. The influences of individual load combinations on the safety of the system taking into account scatter and correlations of the parameters are examined.

Keywords: Safety, Serviceability Limit State, Shallow foundation, Ultimate Limit State, Hasofer-Lind

1 INTRODUCTION

A thorough understanding of the structure-soil-interaction is the basis for a safe and economical design. In today's codes of practice, e. g. Eurocode 7 (2005) prescribe the limit state design (LSD). Within this design concept several ultimate limit states (ULS) and serviceability limit states (SLS) are investigated. Application of the LSD to shallow foundations includes the separate analysis of different failure modes, e. g. bearing resistance failure or sliding, which describe the complex behaviour of the foundation. This procedure has apparent disadvantages particularly in the design of foundations under complex loading such as coastal structures.

For foundations under complex loading different failure modes have to be examined for different load combinations within the LSD procedure. For example, the design of vertical caisson breakwaters on a feasibility level includes the investigation of loading under still-water level (SWL), wave crest and wave trough (Fig. 1). The limit states of uplift, rotation failure, sliding and bearing resistance failure in the rubble mound or in the subsoil are to be checked. Rotation failure, however, is often substituted by limiting the eccentricity of the resultant vertical loading to b/3 of the foundation width.

In contrast to this, with the failure condition of the Single Surface Hardening Model (Kisse, 2008) the isolated limit states are integrated in a consistent formulation, so that the distinction between different limit states is no longer necessary.

This concept allows for a clear definition of safety and provides a distinct basis for the application of probabilistic methods. The new generation of geotechnical design codes offers such methods. Since it make possible to regard e.g. the inherent uncertainty of the natural boundary conditions.

In this paper such a probabilistic design on basis of the very practicable Hasofer-Lind index $ß_{HL}$ is presented.

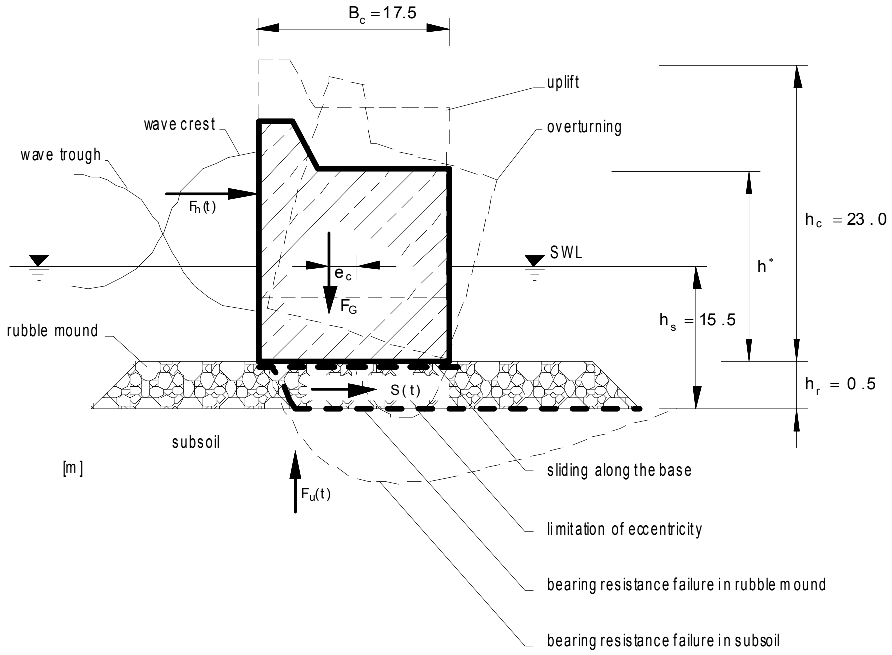


Figure 1. Failure modes for vertical caisson breakwaters (Lesny et al., 2000)

2 CONCEPT OF LIMIT STATES

Basis for the determination of the failure probability is the confrontation of effects S(X) and resistances R(X) in a limit state equation g(X):

$$g(X) = R(X) - S(X) \tag{1}$$

In which X is a vector of random variables describing the geometry of the foundation, the loads that are applied, the strength of materials etc. The probability of failure p_f is the probability p of ($R \leq S$) or in general

$$p_f = p(g(X) \leq 0) = \int_{g(X) \leq 0} f_X(X) dX \tag{2}$$

where $f_x(X)$ is the joint probability density function of the basic variables.

In general it is not possible to solve the integral analytically. Cornell (1969) introduced a method in which the difference of R - S is considered. So for a normal distribution of the value z it is possible to write

$$\mu_z = \mu_R - \mu_S, \quad \sigma_z = \sqrt{\sigma_R^{\ 2} + \sigma_S^{\ 2}} \tag{3}$$

This supplies the definition of the reliability index ß

$$\beta_z = \mu_z / \sigma_z \tag{4}$$

The failure probability is calculated then to

$$p_f = p(g(X) < 0) = \Phi(-\beta_z) \tag{5}$$

2.1 *Limit state equation*

The limit state equation g(X) divides the space in a safe region (g(X) > 0) and a failure region (g(X) ≤ 0). As mentioned before, for a vertical breakwater under complex loading a lot of limit states have to be checked. Desirably for the probability analysis it is to have a unique equation to describe the limit states, because with that we could consider the safety of the whole system at once not only for a single failure mode.

Kisse (2008) adopted such a failure condition proposed by Lesny et al. (2002) to calculate the failure of the system within the SSH-Model. Here the footing is loaded by a vertical load F_1, horizontal load components F_2 and F_3, a torsional moment M1 and bending moment components M_2 and M_3 (Fig. 2). The load components are summarized in the load vector:

$$\vec{Q}^T = \left[F_1\ F_2\ F_3\ M_1\ M_2\ M_3\right] \qquad (6)$$

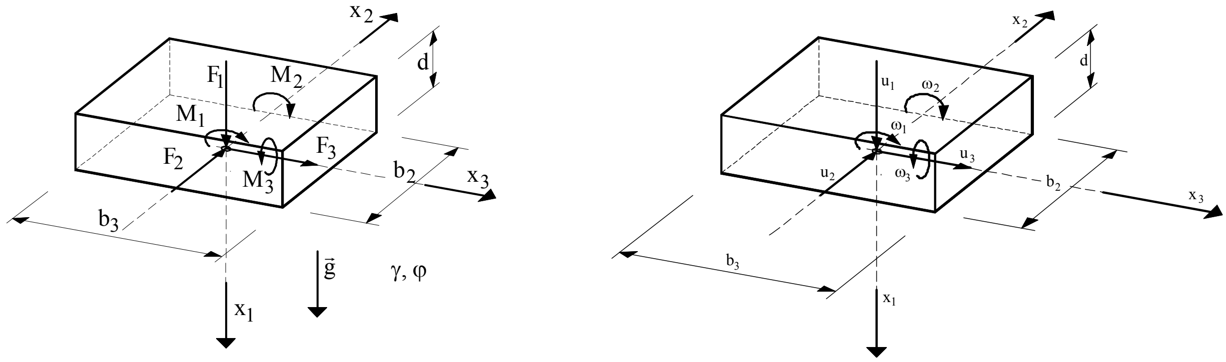


Figure 2. a) Geometry and loading, b) Corresponding displacements and rotations.

For the basic case of a footing on non-cohesive soil without embedment the geometry of the footing described by the side ratio b_2/b_3, weight γ, shear strength φ' of the soil and a quantity μ_S describing the roughness of the footing base have to be considered as well (Fig. 2).
With these input parameters the failure condition is defined by the following expression (Kisse, 2008):

$$g(\vec{Q},F_{10}) = \frac{F_1}{F_{10}} \cdot \left(1-\frac{F_1}{F_{10}}\right)^{\alpha} - \left(\sqrt{\frac{F_2^2+F_3^2}{(a_1 \cdot F_{10})^2} + \frac{M_1^2}{(a_2 \cdot (b_2+b_3) \cdot F_{10})^2} + \frac{M_2^2}{(a_3 \cdot b_3 \cdot F_{10})^2} + \frac{M_3^2}{(a_3 \cdot b_2 \cdot F_{10})^2}}\right) = 0 \quad (7)$$

In Eq. (7) all load components are referred to F_{10} which is the bearing resistance of a footing under vertical centric loading. This quantity is calculated using traditional bearing capacity formulae. The advantage of this formulation is that the complex mutual interaction of the load components is described directly without using reduction factors or the concept of the effective width. Other influences on the bearing capacity are included in F_{10}.

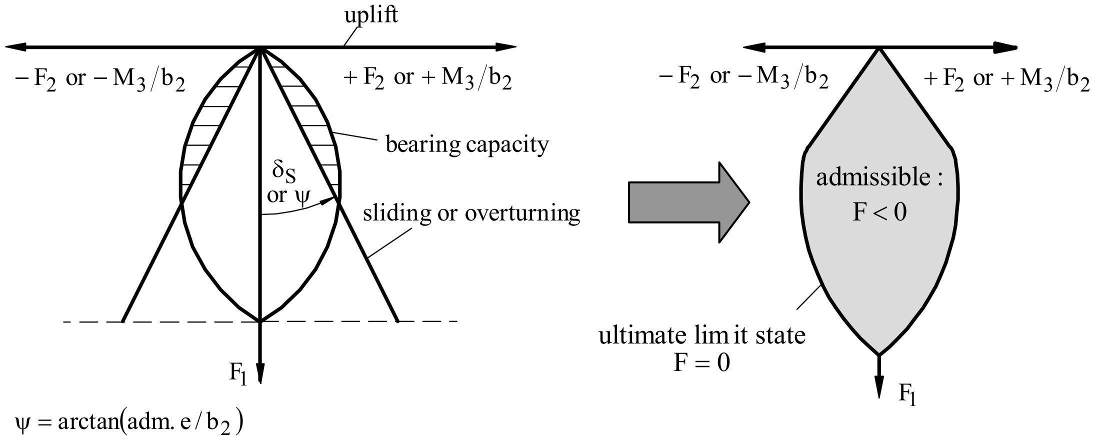


Figure 3. Isolated limit states (left) and resulting consistent failure condition (right).

In an interaction diagram (Fig. 3) the failure condition spans a failure surface, which is the outer boundary of the admissible loading. The parameters $a_{1,2,3}$ govern the inclination of this failure surface for small vertical loading where the limit states sliding and overturning have previously been relevant. These limit states are integrated by defining the parameters $a_{1,2,3}$ and α acc. to Eq. (8) (Lesny, 2001).

$$a_1 = \pi/2 \cdot \mu_S \cdot \tan\varphi' \cdot e^{-\pi/3 \cdot \tan\varphi'},\ a_2 = 0.098,\ a_3 = 0.42,\ \alpha = 1.3 \quad (8)$$

The limit state uplift is already included in Eq. (7), because only positive vertical loads are admissible. The parameters have been derived from an analysis of numerous small scale model tests (Lesny 2001, Lesny and Richwien, 2002).

2.2 *Hasofer-Lind index*

As shown before the failure condition of the model spreads out a failure surface which represents the outer border of the permissible loading. Hence the distance of the actual loading from the failure surface describes the safety of the system (in anticipation of the next chapter see Fig. 5).

This safety can be determined easily using reliability analysis. Here, the widely used Hasofer-Lind second moment reliability index $ß_{HL}$ will be evaluated (Hasofer and Lind, 1974). The classical approach for computing the index is based on the transformation of the limit state surface into the space of standard normal varieties

$$X'_i = \left(X_i - \mu_{X_i}\right) \big/ \sigma_{X_i} \qquad (9)$$

where μ_{Xi} and σ_{Xi} are the mean and standard deviation of variable X_i. The limit state equation g(X) is also transformed to the standard space (Fig. 4). The reliability index $ß_{HL}$ is defined as the distance from origin to the nearest point D of the limit state surface. This point D is called the design point.

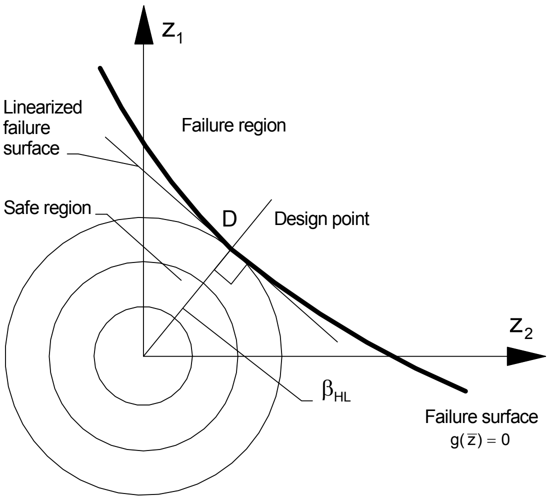


Figure 4. Illustration of reliability index ß in the plane (Burcharth, 1997)

For practical applications the methods proposed by Low and Phoon (2002) and Low (2005) are especially suitable for the determination of the index $ß_{HL}$. With this formulation it is possible to indicate the safety of the system not only for the mean values but also in dependence of correlations of the parameters. The matrix form of the Hasofer-Lind reliability index is (Low, 2005):

$$\beta_{HL} = \min_{X \in F} \sqrt{\left(\frac{x_i - \mu_i}{\sigma_i}\right)^T \cdot R^{-1} \cdot \left(\frac{x_i - \mu_i}{\sigma_i}\right)} \qquad (10)$$

where X is a vector representing the set of random variables X_i, μ_i are the mean values, R is the correlation matrix, σ_i is the standard deviation and F the failure domain.

3 NUMERICAL ANALYSIS

In the following the application of the new system law is shown using the example of a vertical breakwater which is placed on a thin rubble mound on sandy subsoil (Fig. 1). The geometry and soil conditions are taken from De Groot et al. (1996) and Lesny et al. (2000). The loads are calculated within the EU-MAST III PROVERBS project (Probabilistic design tools for vertical breakwaters, Oumeraci et al., 2001).

For the design the loading under still-water level (SWL), wave crest and wave trough are considered. For the case of simplification it is assumed that all wave loads followed a normal distribution. One refers to that with the method after Eq. (10) other distributions for the wave loads can also be considered. The extreme loads are listed in table 1.

Table 1. Extreme loads for vertical breakwaters after De Groot et al. (1996) and Oumeraci et al. (2001).

Still water level LC 1					
			F_1 [kN/m]	F_3 [kN/m]	M_2 [kNm/m]
			4375	0	5180
Wave trough LC 2					
case	H_{max} [m]	T_0 [sec]	F_1 [kN/m]	F_3 [kN/m]	M_2 [kNm/m]
A1	5	8.5	4183	581	706
A2	6.5	10	4061	918	-1697
A3	8	11	3943	1257	-4149
A4	10	12	3787	1725	-7520
Wave crest LC 3					
case	H_{max} [m]	T_0 [sec]	F_1 [kN/m]	F_3 [kN/m]	M_2 [kNm/m]
A1	5	8.5	4780	-645	7513
A2	6.5	10	4983	-836	8558
A3	8	11	5163	-986	9474
A4	10	12	5395	-1148	10550

3.1 *Ultimate Limit State and Failure Surface*

In Fig. 5 the failure surfaces after Eq. (7) in the F_1-F_3-plane F_1-M_2/b_3-plane for different friction angles are presented. It can be recognized that with increasing friction angle of the subsoil the failure curve expands and, hence, greater bending moments and horizontal loads can be applied.

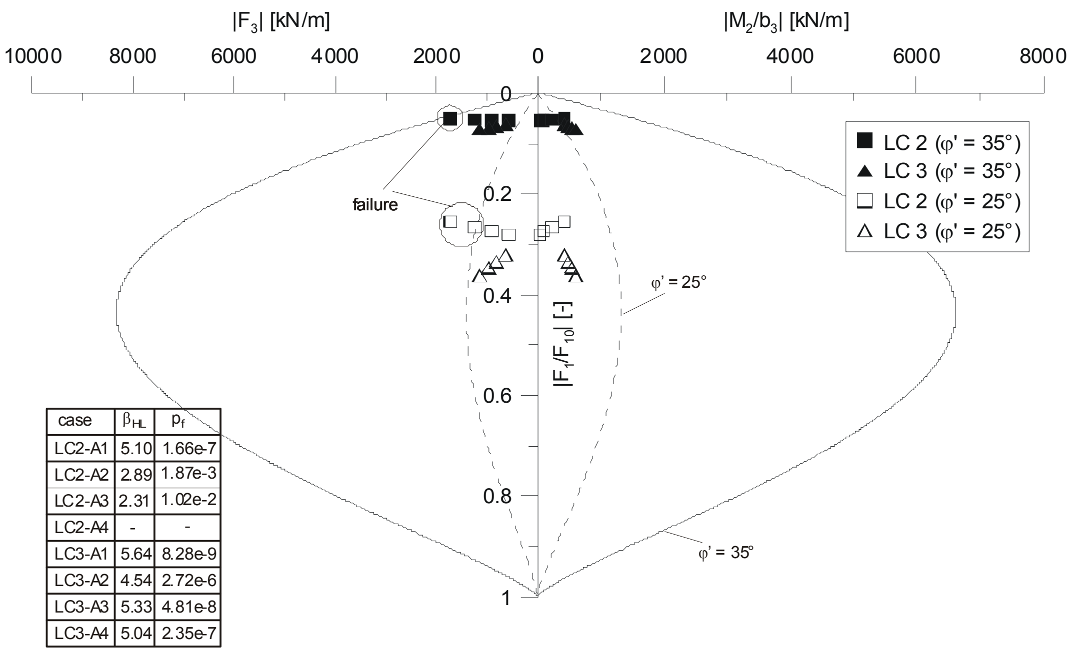


Figure 5. Intersection of failure curves and load points for two different friction angle φ’

Here the surfaces are plotted together with the loads applied the breakwater after table 1. It can be seen that for a friction angle of φ’ = 35° all cases lie inside the failure surface (filled out points). For the other case of a friction angle of φ’ = 25° some load points lie outside and some insight the failure surface. For the points (= load combinations) outside the body failure occurs. So the interaction diagram displays directly the interaction of the load components within the ULS.

3.2 *Reliability index*

With the load combinations in table 1 and the limit state surface formulated after Eq. (7) a reliability design for the foundation of a vertical caisson breakwater will be performed. For the following calculations

the Microsoft Excel software and its built-in optimization program Solver is used. The computations followed the spreadsheet formulations of Low and Phoon (2002) and Low (2005).
Beside the curves also the calculated values of the reliability index $ß_{HL}$ for a friction angle of φ' = 35° are specified in Fig. 6 (left side). Here the loads are uncorrelated and the coefficient of variation

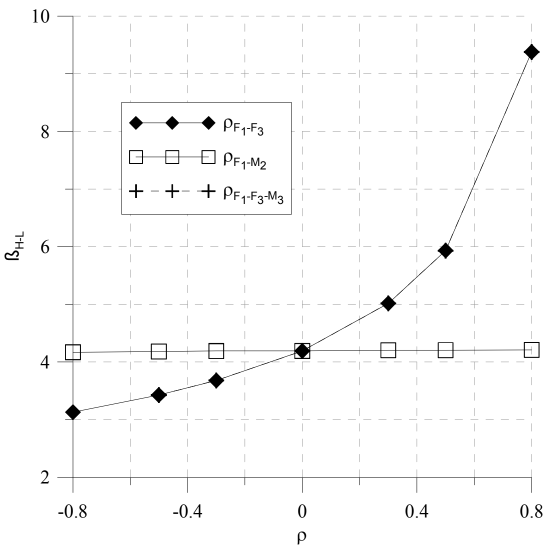
$$COV = {\sigma_{X_i}} \big/ {\mu_{X_i}} \qquad (11)$$

is taken as 20% for all load components. If we adopt a safety factor of 3.0 for the vertical breakwater under the observed loading conditions (for φ' = 35°), only for the cases LC 2-A2 and LC-A3 the ULS is not fulfilled and the foundation is not safe. The load point for LC-A4 lies on the failure surface and so failure occurs.

In general, the index $ß_{HL}$ and so the safety of the system depend on the correlation factor. Therefore the influence of the correlation of the different load components to each other was separately examined for the load case LC2-A2 (Fig. 6). The results for a correlation between F_1-F_3 and F_1-F_3-M_2 differs not, so only one curve can be seen. The reliability index $ß_{HL}$ is not affected by the correlation of F_1-M_2. This is due to the fact, that the dominate failure mode for this load case is sliding.

If the load components are correlated the ellipses are tilted. For positive correlation factors they are positivity tilted and for negative values they are negativity tilted. So in one case the distance from the point of view could be smaller than in the other case.

For the correlation F_1-F_3 and F_1-F_3-M_2 the reliability index $ß_{HL}$ is greater for positive correlations values as for negative ones. The reason for this is that the load point is located in the upper section of the interaction diagram (Fig. 5). Here the surface is curved to the right in the F_1-F_3-plane. If the vertical load component decreases the horizontal load component increases for a negative correlation and so the ellipsoid is negativity tilted. That mean, that the distance between the ellipse and the failure surface become smaller as for a positive correlation.



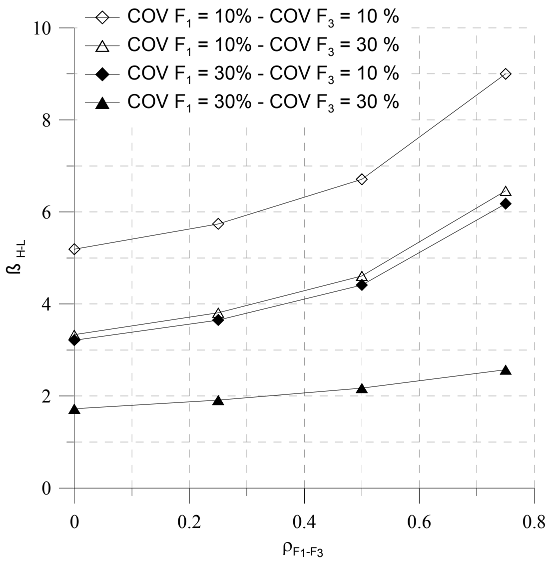


Figure 6. Effect of the correlation (left) and of the variability (right) of the applied loads and on the reliability index

To study the effect of the variability of the applied loads on the failure probability, Fig. 6 (right side) shows the reliability index versus the coefficient of variation of F_1 and F_3 and the correlation of these two loads. The results show that the failure probability is highly influenced by the coefficient of variation of the loads, the greater the scatter in F_3 the higher the failure probability of the foundation. Beyond that the COV affects also the dependence of the correlation. For a small coefficient of variation the influence of the correlation is more pronounced.

This means that the accurate determination of the distribution of this parameter is very important in obtaining reliable probabilistic results.

4 SERVICEABILITY

For the SLS it has to prove that the estimated displacements and rotations u_e are not greater than limiting tolerable displacements and rotations u_{tol}.

$$u_e \le u_{tol} \qquad (12)$$

Due to the presence of uncertainties the estimated and tolerable displacements and rotations are in fact random variables. So it seems to be preferable to use a reliability based approach to design for SLS.

A performance function g(X) for the reliability-based serviceability limit can be formulated in the following way (Zhang and Ng, 2005):

$$g(X) = u_{tol} - u_e \qquad (13)$$

Here g(X) > 0 defines a satisfactory performance region and g(X) ≤ 0 defines an unsatisfactory performance region like that one for the ULS. If the probability distributions of the displacements and rotations are known the reliability index $ß_{HL}$ can be calculated.

With the Single Surface Hardening Model after Kisse (2008) it is possible to determine the tolerable displacements and rotations over the whole loading range up to the ultimate limit state. So with the performance functions after Eq. (7) and (12) it could be possible to calculate the system failure probability.

4.1 *Displacement rule*

The displacements and rotations of the foundation due to arbitrary loading inside the failure surface are described by the displacement rule. The displacements u_i and rotations ω_i (Fig. 2) are summarized in a displacement vector (Kisse, 2008):

$$\vec{u}^T = [u_1\, u_2\, u_3\, \omega_1\, \omega_2\, \omega_3] \qquad (14)$$

Due to the complex interaction of load components, displacements and rotations the displacement rule is formulated using the well-known strain hardening plasticity theory with isotropic hardening. Hence, displacements and rotations are calculated according to Eq. (15), assuming that all deformations are plastic.

$$d\vec{u} = \frac{1}{H} \cdot \left(\frac{\partial F}{\partial \vec{Q}} \right)^T \cdot \frac{\partial G}{\partial \vec{Q}} \cdot d\vec{Q} \qquad (15)$$

The components of the displacement rule are a yield surface described by the yield condition F which is derived from the failure condition Eq. (7) with the parameter $a_{1,2,3}$ and α of Eq. (8):

$$F(\vec{Q}, F_a) = \frac{F_2^{\,2} + F_3^{\,2}}{(a_1 \cdot F_a)^2} + \frac{M_1^{\,2}}{(a_2 \cdot (b_2 + b_3) \cdot F_a)^2} + \frac{M_2^{\,2}}{(a_3 \cdot b_3 \cdot F_a)^2} + \frac{M_3^{\,2}}{(a_3 \cdot b_2 \cdot F_a)^2} - \left[\frac{F_1}{F_a} \cdot \left(1 - \frac{F_1}{F_a} \right)^{\alpha} \right]^2 = 0 \qquad (16)$$

a plastic potential G (in the same form) and a hardening function H:

$$H = -\frac{\partial F(\vec{Q}, F_a)}{\partial F_a} \cdot \frac{\partial F_a}{\partial \vec{u}} \cdot \frac{\partial G(\vec{Q}, F_b)}{\partial \vec{Q}} \qquad (17)$$

The yield surface acc. to Eq. (16) expands due to isotropic hardening until the failure surface defined by Eq. (7) is reached (Fig. 7). Hence, the parameters of the plastic potential G have to be determined as functions of a_i and α, respectively. The expansion of the yield surface depends mainly on the vertical displacement which itself depends on the degree of mobilization of the maximum resistance F_{10}. With that, it is sufficient enough to define the hardening parameter F_a in Eq. (16) as a function of these two quantities according to:

$$F_a = (F_{10} + k_f \cdot u_1) \cdot \left\{ 1 - \exp\left(\frac{-k_0 \cdot u_1}{F_{10} + k_f \cdot u_1} \right) \right\} \qquad (18)$$

Many hardening laws (e. g. Nova et al., 1991) require small scale model tests under centric vertical loading to determine the hardening parameter. Since this is not convenient for practical applications, the initial and final stiffness of the corresponding load-displacement curve, k_0 and k_f respectively, may be determined using a method proposed by Mayne and Poulos (2001) in which the soil stiffness can be determined by any standard procedure.

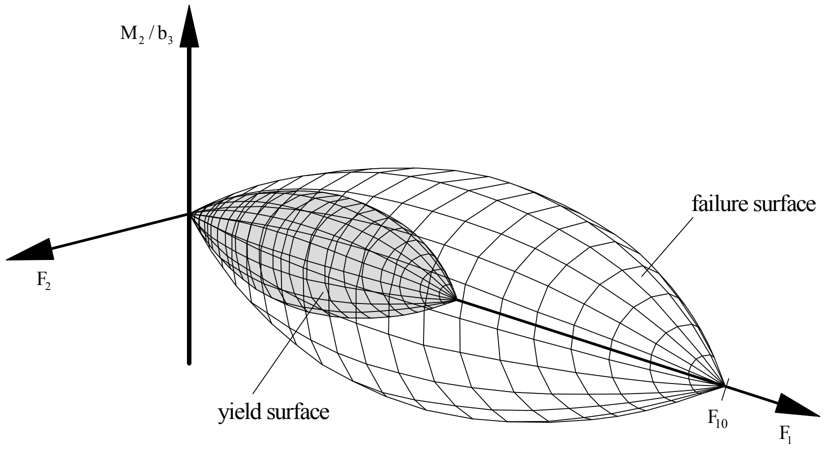


Figure 7. Isotropic expansion of the yield surface in the loading space.

5 CONCLUSION

A design method has been presented which describes the complex behavior of shallow foundations under loading up to failure. The model includes a failure condition defining the ultimate bearing capacity. Hence, the separate analysis of different failure modes is no longer necessary. Together with the methods proposed by Low and Phoon (2002) and Low (2005) a practical application for the determination of the Hasofer-Lind index $ß_{HL}$ is formulated. With this formulation it is possible to indicate the safety of the system not only for the mean values but also in dependence of scatter and correlations of the parameters. The ability of the method was presented using an example of a vertical breakwater.

REFERENCES

Burcharth, H.F. 1997. Reliability based design of Coastal Structures. Advances in Coastal and Ocean Engineering, Vol. 3 Ed.: Liu, P. L.-F., World Scientific Publishing.

Cornell, C. 1969. A Probabilistic Structural Code. ACI Journal Vol. 66, pp. 974-985.

De Groot, M. B., Andersen, K. H., Burcharth, H. F., Ibsen, L. B., Kortenhaus, A., Lundgren, H., et al. 1996. Foundation Design of Caisson Breakwaters. Publ. No. 198 (Vol. 1).Oslo, Norway: Norwegian Geotechnical Institute.

DIN EN 1997-1 2005. Eurocode 7: Entwurf, Berechnung und Bemessung in der Geotechnik – Teil1: Allgemeine Regeln. Normenausschuss Bauwesen im Deutschen Institut fuer Normung e. V., Berlin (German version).

Hasofer, A. M. & Lind, N. C. 1974. An exact and invariant first order reliability format. Journal of Engineering Mechanics Division, ASCE 100, pp. 111-121.

Kisse, A. 2008. Entwicklung eines Systemgesetzes zur Beschreibung der Boden- Bauwerk-Interaktion flachgegruendeter Fundamente auf Sand. Mitteilungen aus dem Fachgebiet Grundbau und Bodenmechanik der Universität Duisburg-Essen, Heft 34, VGE Verlag, Essen. (in German)

Lesny K. & Richwien W. A 2002. Consistent Failure Model for Single Footings embedded in Sand. Proceedings of the International Workshop on Foundation Design Codes and Soil Investigation in View of International Harmonization and Performance Based Design, Kamakura, April 2002. Balkema, Lisse.

Lesny, K. 2001. Entwicklung eines konsistenten Versagensmodells zum Nachweis der Standsicherheit flachgegruendeter Fundamente. Mitteilungen aus dem Fachgebiet Grundbau und Bodenmechanik der Universität Essen, Heft 27, Verlag Glückauf GmbH, Essen. (in German)

Lesny, K., Kisse, A. & Richwien, W. 2002. Proof of Foundation Stability Using a Consistent Failure Model. Proc. of the Int. Conf. on Probabilistics in Geotechnics -Technical and Economic Risk Estimation, Graz, Austria, pp. 95-103.

Lesny, K., Perau, E., Richwien W.; Wang, Z. 2000. Some Aspects on Subsoil Failure of Vertical Breakwaters. Forschungsbericht aus dem Fachbereich Bauwesen, Heft 83, Universitaet Essen.

Low, B. & Phoon, K. K. 2002. Practical first-order reliability computations using spreadsheet. Proc. of the Int. Conf. on Probabilistics in Geotechnics: Technical and Economic Risk Estimation, Graz, Austria, pp. 93-46.

Low, B. 2005. Reliability-based design applied to retaining walls. Geotechnique 55, No. 1: 63-75.

Nova R & Montrasio L. 1991. Settlements of Shallow Foundations on Sand. Géotechnique, Vol. 41, pp. 243-256.

Mayne P W & Poulos H G. 2001. Discussion: Approximate displacement influence factors for elastic shallow foundations. Journal of Geotechnical and Geoenvironmental Engineering, Vol. 127, pp. 100-102.

Oumeraci, H.; Kortenhaus, A.; Allsop, W.; de Groot, M.; Crouch, R.; Vrijling, H.; Voortman, H. 2001. Probabilistic Design Tools of Vertical Breakwaters, A. A. Balkema, Lisse.

Zhang, L. M. & Ng, A. M. Y. 2005. Probabilistic limiting tol-erable displacements for serviceability limit state design of foundations. Geotechnique 55, No. 2, pp. 151-161.