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The Effect of Model Uncertainty on the Reliability of Spread Foundations

W. S. Forrest & T. L. L. Orr
Trinity College, University of Dublin, Ireland

ABSTRACT: In reliability analyses of the ultimate limit state design of a spread foundation, the probabilistic modelling of the calculation model is often ignored. However, as part of any reliability analysis, it is important to consider the uncertainty in the calculation model as well as the uncertainties in the soil strength parameters and applied loads. This paper investigates the model uncertainty by applying a random variable model factor, *M*, to the calculation model and examining what level of variation this random variable would need to have to affect the reliability of a foundation design. This is carried out by increasing the coefficient of variation of *M* and observing the effect this has on the reliability index, β and on the sensitivity factors, α, which represent the relative sensitivities of the basic random variables. A spread foundation subjected to different loads is examined at the ultimate limit state for drained and undrained conditions. This paper shows that a model factor to account for the model uncertainty is not required in the ultimate limit state design of a spread foundation since the uncertainties in the soil strength parameters or the loads in the case of an eccentrically loaded foundation are found to control the reliability of the designs.

Keywords: Spread foundation, bearing resistance, model factor, reliability analyses

1 INTRODUCTION

In reliability analyses of the ultimate limit state design of a spread foundation, the probabilistic modelling of the calculation model is often ignored. As part of any reliability analyses, it is important to consider the uncertainty in the calculation model as well as the uncertainties in the soil strength parameters and applied loads. For example, in the bearing resistance equation for a spread foundation for drained conditions there is some uncertainty in the equation itself, in particular in the value of the N_γ factor. Phoon (2005) suggested the model factor be considered as a random variable in reliability analyses and that approach is adopted in this analysis.

This paper investigates the uncertainty in the calculation model by applying a model factor, *M*, as a random variable in the calculation model. The coefficient of variation of this model factor, CoV_M, represents the uncertainty in the calculation model and the value of CoV_M is increased to examine the effect this has on the reliability index, β and on the sensitivity factors, α, which represent the relative sensitivities of β to the different soil strength and load random variables in the calculation model.

2 RELIABILITY THEORY

2.1 *Limit state design concept*

In the last four decades there has been increased interest in the application of reliability theory in civil engineering. Part of this application of reliability theory has been in the design of structures to ensure their safety and their ability to fulfil their design requirements. Modern geotechnical design codes, such as Eurocode 7 (2004), are based on the limit state design concept, the fundamental concept of which is that all possible limit states for a structure must be considered and their occurrence shown to be sufficiently unlikely to occur (Gulvanessian et al., 2002). In order to ensure that the occurrence of a limit state is sufficiently unlikely, a probabilistic or semi-probabilistic approach is adopted in the design process in order to achieve a certain target level of safety or β value.

2.2 *Bearing resistance calculation model*

Eurocode 7 gives in Annex D the following calculation model (equation) for the design drained bearing resistance, $R_{d,d}$, for a spread foundation:

$$R_{d,d} = A'(c'_d N_c s_c i_c + q' N_q s_q i_q + 0.5B' \gamma' N_\gamma s_\gamma i_\gamma) \tag{1}$$

where:

$$N_q = e^{\pi \tan \phi'_d} \tan^2(45 + \phi'_d/2) \tag{2}$$

$$N_c = (N_q - 1)\cot \phi'_d \tag{3}$$

$$N_\gamma = 2(N_q - 1)\tan \phi'_d \tag{4}$$

$$s_q = 1 + (B'/L')\sin \phi'_d \tag{5}$$

$$s_\gamma = 1 - 0.3(B'/L') \tag{6}$$

$$s_c = (s_q N_q - 1)/(N_q - 1) \tag{7}$$

$$i_c = i_q - (1 - i_q)/(N_c \cot \phi'_d) \tag{8}$$

$$i_q = [1 - H_d/(V_d + A' \cot \phi_{d'})]^m \tag{9}$$

$$i_\gamma = [1 - H_d/(V_d + A' \cot \phi_{d'})]^{m+1} \tag{10}$$

$m = [2 + (B'/L')]/[1 + B'/L']$ when H_d acts in the direction of B' (11)

where B' is the effective foundation breadth, L' is the effective foundation length, A' is the effective area (B' x L'), H_d is the design horizontal load and V_d is the design vertical load.

The design undrained bearing resistance, $R_{u,d}$, was determined using the calculation model in Annex D of Eurocode 7 consisting of the following equation:

$$R_{u,d} = A'((\pi + 2)c_{u,d} s_c i_c + q) \tag{12}$$

where A' is the effective foundation base area, s_c is a shape factor equal to 1.2 for a square foundation, q is the overburden pressure at the foundation base and i_c is an inclination factor given as follows, where H_d is the horizontal load:

$$i_c = 0.5(1 + \sqrt{1 - H_d/(A' c_{u,d})}) \tag{13}$$

2.3 *First-Order Reliability Method*

The first-order reliability method may be used to determine the β values for the designs and the sensitivity factors $\alpha_{\tan\phi'}$, $\alpha_{c'}$, α_{cu}, α_γ, α_G, α_{Qv} and α_{Qh} for the random variables $\tan\phi'$, c', c_u, G_v, Q_v and Q_h. This method was originally proposed by Hasofer and Lind (1974) for normally distributed variables and was later extended for non-normal distributions by Rackwitz and Fiessler (1978). In accordance with the reliability analysis program STRUREL (2004), all the basic variables are normalised as follows:

$$Z_i = (X_i - \mu_i)/\sigma_{X_i} \qquad \text{(for i = 1, ……N)} \tag{14}$$

The reliability analyses were carried out using the following equations as the performance or limit state functions that define the limit state surfaces for drained and undrained bearing resistance failure:

$$Z_1 = M_1 A'\left(c'_d N_c s_c i_c + q' N_q s_q i_q + 0.5B'\gamma' N_\gamma s_\gamma i_\gamma\right) - \left(G_d + Q_{v,d}\right) \tag{15}$$

$$Z_2 = M_2 A'\left((\pi + 2)c_{u,d} s_c i_c + q\right) - \left(G_d + Q_{v,d}\right) \tag{16}$$

where M_1 and M_2 are the model factors for the drained and undrained equations respectively.

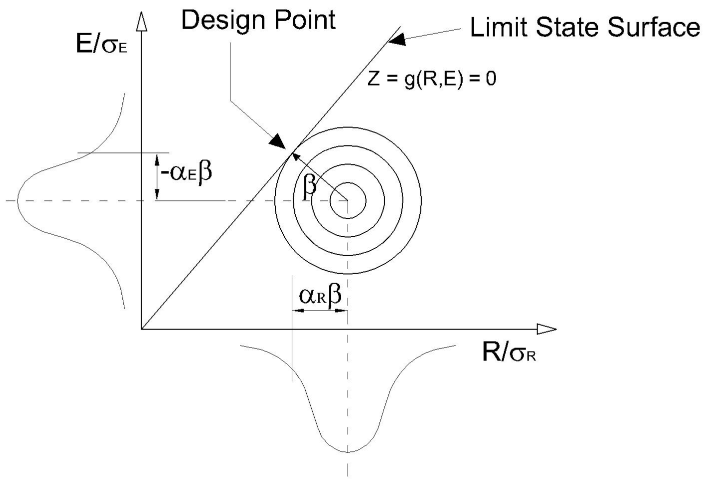

Figure 1. Reliability index and sensitivity factors in normalised space

The reliability index, β is defined as the minimal distance from the limit state surface to the origin in normalised space as shown in Figure 1. The sensitivity factors α_i, or cosine directors, are the components of the unit vector indicating the direction of the vector giving the minimal distance of the design point from the limit state surface (Honjo et al. , 2000). There is an α_i value for each random variable being considered in the reliability analysis and the α_i values are in the range -1 to +1. The closer a particular α_i value is to -1 or +1, the greater effect the random variable i has on the β value.

It was assumed that there is a positive correlation between the horizontal and vertical variable loads and a negative correlation between *tanϕ'* and *c' (Cherubini, 2000, Forrest and Orr, 2010b)*. All the other random variables were assumed to be independent. The assumed correlations between the random variables in this analysis are given in the correlation matrix in Table 1.

Table 1. Correlation matrix with correlation factors relating the random variables

	G	Q_v	Q_h	γ	$tan\phi'$	c'
G	1	0	0	0	0	0
Q_v	0	1	0.5	0	0	0
Q_h	0	0.5	1	0	0	0
γ	0	0	0	1	0	0
$tan\phi'$	0	0	0	0	1	-0.47
c'	0	0	0	0	-0.47	1

2.4 *Model factor*

A model factor to account for uncertainty in the bearing resistance equation is usually not included in the analysis of geotechnical design situations. However uncertainty in the calculation model may be significant and Rackwitz (2000) said it should be accounted for by including a quantity which captures the uncertainty in the calculation model. This is addressed in this study by applying a model factor, M, as a random variable with a mean value of unity to the calculation model, as shown in Equations 15 and 16, and examining what level of uncertainty in the calculation model, represented by the coefficient of variation of M (CoV_M), is necessary for this uncertainty to affect the reliability of the design. This is carried out by increasing the value of CoV_M and observing the effect this has on the β values and on the α_i values. The coefficients of variation (CoV) and probabilistic distributions for all the parameters are given in Table 2.

Table 2. CoVs and probability distributions for the parameters in the ULS sensitivity analyses

Parameter under analysis	Model factor		Other parameters	
	CoV range (%)	Distribution type	CoV range (%)	Distribution type
M	0 – 20	Normal	-	-
G	-	-	10	Normal
Q_v	-	-	20	Lognormal
Q_h	-	-	20	Lognormal
c_u	-	-	25	Normal
tanϕ’	-	-	10	Normal
c’	-	-	120	Gamma

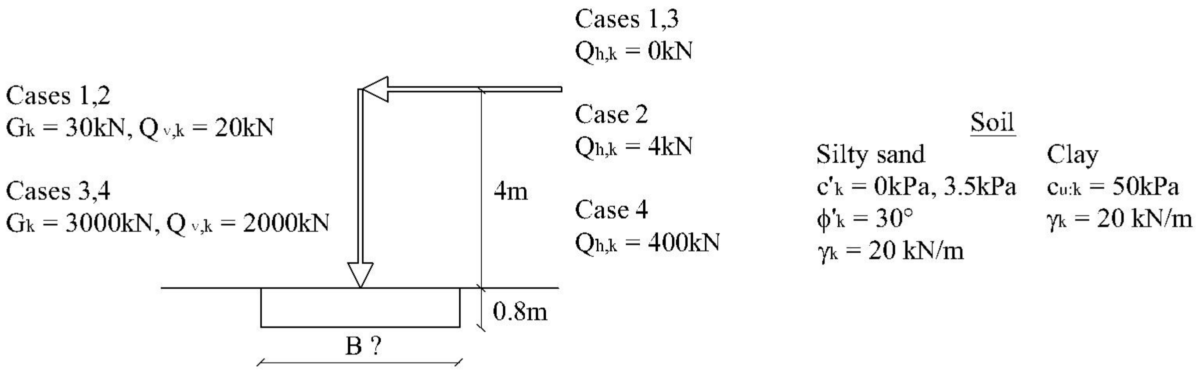

Figure 2. Square spread foundation

3 RELIABILITY ANALYSES

3.1 *Foundation design example*

To investigate the reliability of spread foundations designed to Eurocode 7, the following example shown in Figure 1 has been chosen, which is similar to an example for the International Workshop on the Evaluation of Eurocode 7 held in Dublin (Orr, 2005, Forrest and Orr, 2010a). This square pad foundation for a building is at 0.8m embedment depth, the groundwater at great depth, and two design situations, resting on a coarse-grained soil (silty sand) and on a fine-grained soil (clay), are considered. The first-order reliability method outlined above was used to design the foundation against ultimate limit state bearing resistance failure for the following four different load cases shown in Figure 1: Case 1 having small loads consisting of a characteristic vertical permanent load, G_k = 30kN, a characteristic vertical variable load, $Q_{v,k}$ = 20kN and no horizontal variable load, $Q_{h,k}$; Case 2 having the same small loads G_k = 30kN, $Q_{v,k}$ = 20kN but with $Q_{h,k}$ = 4kN; Case 3 having large loads consisting of G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 0; while Case 4 also having larger loads with G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 400kN.

Reliability analyses of the spread foundation were carried out assuming the dependencies between the random variables given in Table 2 for the four load cases listed above for drained conditions for the coarse-grained and fine-grained soils and for undrained conditions for the fine-grained soil. The results of these analyses are plotted in Figures 3 to 10 as graphs showing how the α values for the random variables in the analyses vary and the β values decreases as the value of CoV_M, increases. In the following discussion of the results of the analyses, a variable is only considered to have a significant influence on the β value if its α value exceeds 0.3.

3.2 *Results of drained reliability analyses*

The results of the drained reliability analyses of the vertically loaded foundation on coarse-grained soil plotted in Figures 3 and 4 show that, in both Case 1 and 3, $\tan\phi'$ is the only variable, apart from M, with an α value greater than ±0.3 and therefore this is the only variable which has a significant influence on the β value. When there is no model uncertainty (CoV_M = 0), $\alpha_{\tan\phi'}$ is close to one and therefore $\tan\phi'$ dominates the entire reliability analysis. The sensitivity factors for all the other random variables, α_G, α_{Q_v} and α_γ, are in the range -0.3 to 0.3 and therefore are not significant variables in these Cases. It can be seen that as CoV_M increases, α_M becomes the largest α value and M becomes the dominant variable when CoV_M >

17%. The magnitude of the loads has little effect on the reliability of foundation designs with vertical loads only since the α and β values in Figure 4 are similar to those in Figure 3.

For the vertically loaded foundations on fine-grained soil, the variations in the α values as CoV_M increases are more complex and are not the same as for Cases 1 and 3, as shown in Figures 5 and 6. In Case 1, $\alpha_{\tan\phi'}$ and $\alpha_{c'}$ are the leading random variables when CoV_M = 0%. However, as CoV_M increases, α_M becomes dominant while $\alpha_{\tan\phi'}$ reduces significantly and $\alpha_{c'}$ remains relatively unchanged. Interestingly, α_M has a larger value in Case 1 than in Case 3. Therefore the design on fine-grained soil is more sensitive to uncertainty in the calculation model when the loads are smaller and hence when the foundation breadth is smaller. When the load is larger, as in Case 3, Figures 5 and 6 show that the design is more sensitive to uncertainty in c' since the $\alpha_{c'}$ values are larger for Case 3 than for Case 1. The result is that, not only is $\alpha_{\tan\phi'}$ reduced significantly in the case of the larger loads, but α_M is also reduced so that β is not greatly affected by uncertainty in the calculation model, even when the model factor has a large coefficient of variation (e.g. $CoV_M \approx 20\%$).

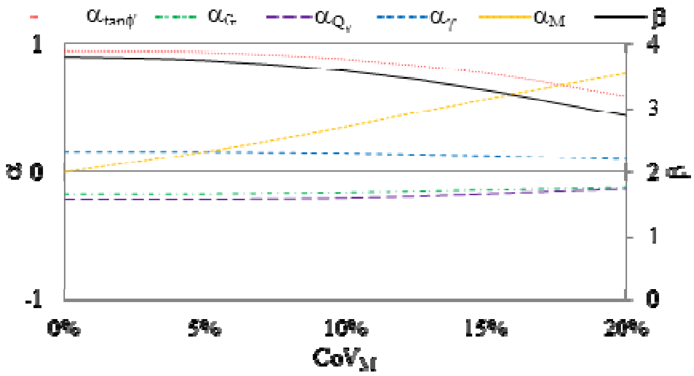

Figure 3. Sensitivity analysis of vertically loaded foundation with small loads on coarse-grained soil (Case 1)

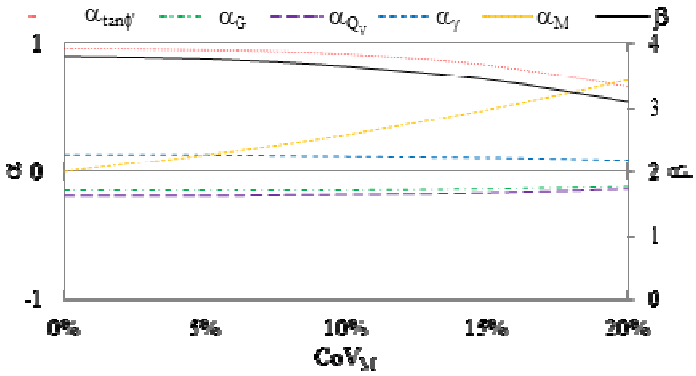

Figure 4. Sensitivity analysis of vertically loaded foundation with large loads on coarse-grained soil (Case 3)

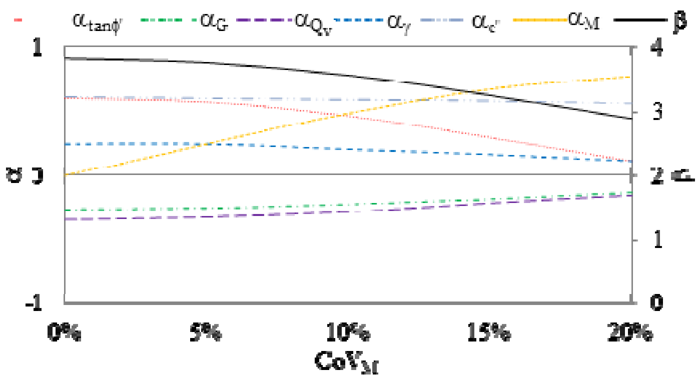

Figure 5. Sensitivity analysis of vertically loaded foundation with small loads on fine-grained soil (Case 1)

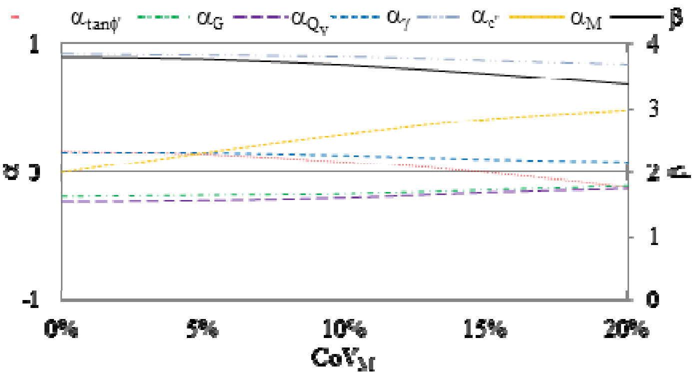

Figure 6. Sensitivity analysis of vertically loaded foundation with large loads on fine-grained soil (Case 3)

With regard to the two inclined-eccentrically vertically loaded foundations on coarse-grained soil, in Case 2, with the smaller vertical and horizontal loads, Q_h is the dominant variable and both G and tanϕ' (Figure 7) are significant as their α values exceed -0.3 and 0.3 respectively, whereas in Case 4, with the larger loads, tanϕ' is the dominant variable (Figure 8) since $\alpha_{tan\phi'} \geq 0.64$ while all the other α values are less than 0.3, This is due to the smaller loads in Case 2 requiring a smaller foundation width and hence the reliability of the designs is more sensitive to Q_h than to the soil strength parameters. In Case 2, the β values are only significantly affected by the model uncertainty, i.e. α_M is only > 0.3, when $\mathrm{CoV_M}$ > 17%. In Case 4, uncertainty in M has a greater effect since α_M becomes > 0.3 when $\mathrm{CoV_M}$ > 12%.

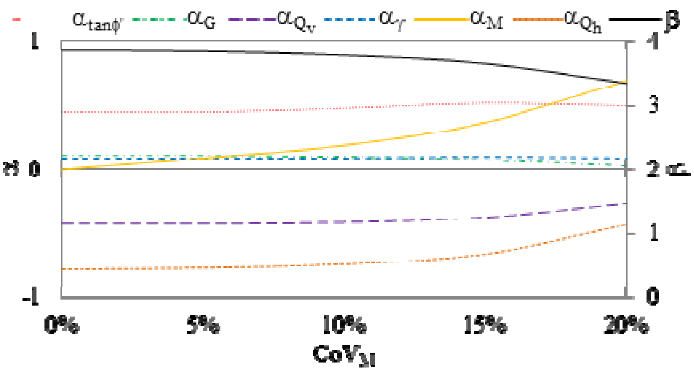

Figure 7. Sensitivity analysis of inclined-eccentrically loaded foundation with small loads on coarse-grained soil (Case 2)

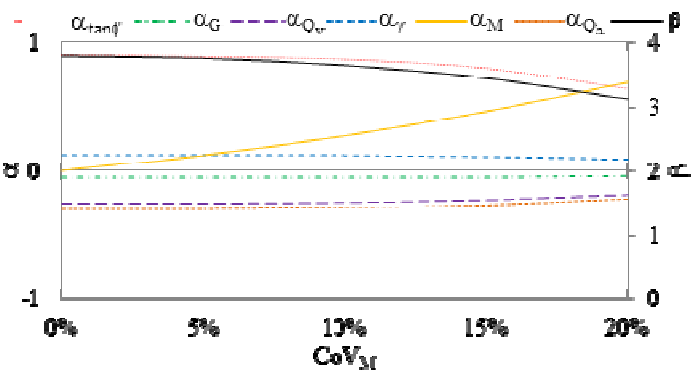

Figure 8. Sensitivity analysis of inclined-eccentrically loaded foundation with large loads in coarse-grained soil (Case 4)

For the two inclined-eccentrically loaded foundations on fine-grained soil, the graphs in Figures 9 for Case 2, the foundation with the small loads, show that α_M never exceeds 0.3 and therefore model uncertainty does not have a significant effect on the reliability, and, as for the inclined-eccentrically foundation on the coarse-grained soil, Q_h is the dominant variable. In Case 4, the foundation with the larger loads, the graphs in Figure 10 show that, while G is still significant with α_{Qv} > 0.3, c' dominates the reliability and uncertainty in the calculation model only becomes significant when $\mathrm{CoV_M}$ exceeds 15% and α_M exceeds 0.3.

3.3 *Results of undrained reliability analyses*

The reliability analyses of the foundations for undrained conditions were performed again for the CoVs and probabilistic distributions given in Table 2 and for the same four load cases. The results of these analyses showed that, for all the load cases, the α value for the undrained shear strength, c_u is close to 1.0 while the α values for the loads and the model factor are all close to zero and α_{cu} remains close to 1.0 and the α values for the loads and α_M remain close to zero as CoV_M increases from 0% to 20% so that the β value is relatively unchanged. Therefore uncertainty in the calculation model has little effect on the β value for these four load cases and the variation c_u dominates the reliability of the designs.

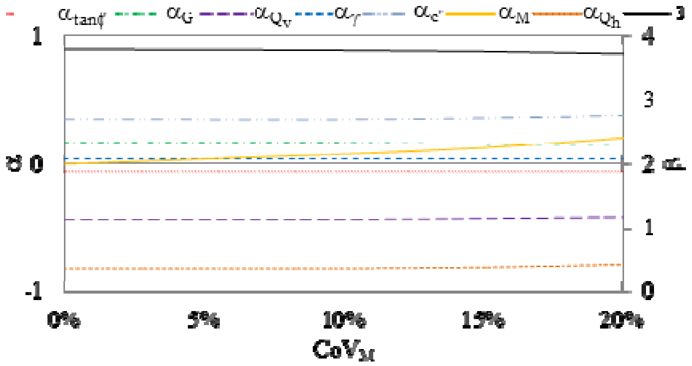

Figure 9. Sensitivity analysis of inclined-eccentrically loaded foundation with small loads on fine-grained soil (Case 2)

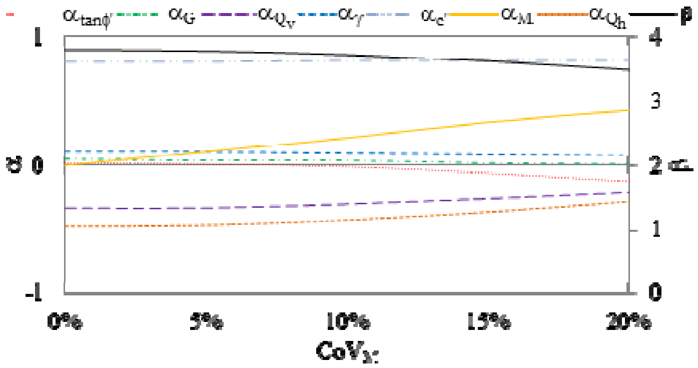

Figure 10. Sensitivity analysis of inclined-eccentrically loaded foundation with large loads on fine-grained soil (Case 4)

4 CONCLUSIONS

The effect of uncertainty in the calculation model for the bearing failure of a spread foundation has been investigated using reliability analyses for four load cases for both drained and undrained conditions. For all the cases examined, it has been found that the CoV_M needs to exceed about 15% before the uncertainty in the calculation model has any significant effect on the β value and hence on the reliability of a foundation design. Since in practice the CoV_M value will be very much less than 15% when using the bearing resistance equation for a spread foundation, the results of the analyses show that, for both drained and undrained conditions, it is not necessary to include a model factor in the design of a spread foundation subjected to either a vertical or an eccentric-inclined load because the uncertainties in the soil strength parameters or the loads will dominate the design. Uncertainty in the calculation model is more significant in the case of spread foundations for drained conditions than for undrained conditions due to the variation in the soil parameter values being less for the drained conditions. For undrained conditions, the uncertainty and variability of the undrained soil strength dominate the reliability of the design and the inclusion of a model factor has a negligible effect on the β values. Hence the inclusion of a model factor in the design of a spread foundation for undrained conditions will not significantly improve the reliability of the calculation. These findings justify the recommended value of 1.0 in Eurocode 7 for the partial model factor $\gamma_{R,d}$ to account for uncertainty in the calculation model for geotechnical designs.

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