

Resistance Factors for Design of Piles in Sand: Tools to Understand Design Reliability

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ABSTRACT: Load and resistance factor design (LRFD) of foundations is a popular design code format. Common methods used to establish resistance factors for geotechnical structures include calibration to assumed factors of safety and reliability analysis using field load test databases. Reliability analyses are the preferred tools for this work, but the needed probabilistic information regarding design method uncertainty is difficult to obtain. This paper illustrates an approach to uncertainty assessment that seeks to isolate the various sources of uncertainty. Using this approach, reliability analyses are used to develop resistance factors for the design of driven pipe piles in sand. The results illustrate how engineers with differing degrees of probability knowledge can use LRFD or the underlying reliability analyses to understand and manage risk. By attempting to isolate different sources of uncertainty, the paper suggests a methodology for quantitatively assessing the relative value of differing degrees and types of design knowledge.

Keywords: Deep Foundation, Cone Penetrometer Test, Reliability Analysis, Load and Resistance Factor Design

1 INTRODUCTION

Load and resistance factor design (LRFD), a specific format for limit states design (LSD), is a design framework that indirectly addresses the uncertainty in the methods of calculating a foundation's capacity (resistance) and estimating the demands (load) placed upon it. This indirect assessment affords the engineer a tool to manage risk in design decisions. LRFD factors for use in design practice can be determined through a number of methods, including calibration to customary factors of safety and probabilistic reliability analysis. Reliability analysis requires a probabilistic characterization of foundation analysis method uncertainty. For pile design, this characterization can include consideration of pile load test databases or studies of the individual contributing sources of uncertainty. This paper focuses on some interesting outcomes from applying a detailed analysis of the contributing sources of uncertainty.

1.1 Basic LRFD Formulation for Piles

In terms of LSD, pile foundations must be designed against any possible ultimate limit state (ULS). Axially loaded piles are often designed based exclusively on an ULS associated with either plunging or excessive settlement. The basic LRFD inequality for this ULS is

$$(RF)R_n \geq \sum (LF)_i Q_i \quad (1)$$

where RF is a resistance factor, R_n is the nominal design resistance, and $(LF)_i$ is a load factor for a particular load type Q_i . In pile design, both base and shaft resistance contribute to the overall load-carrying capacity of a pile. These two contributions suggest two possible LRFD inequalities:

$$(RF)(R_s + R_b) \geq \sum (LF)_i Q_i \quad (2)$$

or

$$(RF)_s R_s + (RF)_b R_b \geq \sum (LF)_i Q_i \quad (3)$$

where R_s and R_b are the shaft and base resistances, respectively, and $(RF)_s$ and $(RF)_b$ are the shaft and base resistance factors, respectively. Although shaft and base resistance depend on many of the same soil properties, they develop by different physical processes and are computed separately using equations with differing degrees of uncertainty. Therefore, it is more useful to apply $(RF)_s$ and $(RF)_b$ as separate resistance factors, as in Eq. (3).

1.2 Probabilistic Framework to Develop LRFD Factors

When using probability to develop LRFD factors, values of RF and LF are selected such that the resulting factored load $\sum(LF)_i Q_i$ and resistance $(RF)R_n$ satisfying Eq. (1) lead to designs with an acceptable probability of failure. The reliability index is a number that expresses the probability of failure (i.e., of achieving a limit state) relative to the uncertainty of the design variables. The simplest, first-order second-moment definition of reliability index β when the problem can be reduced to a single design variable X is

$$\beta = \frac{\mu_X - x_{LS}}{\sigma_X} \quad (4)$$

where μ_X is the mean (often, the design value) of X , x_{LS} is the limit state value of X , and σ_X is the standard deviation of X . For multi-variable equations, computing β involves an optimization process. In this paper, the spreadsheet solution method proposed by Low and Tang (1997) is used. Obtaining values of RF and LF for use in design therefore becomes a further process of optimization where β is fixed and RF and LF are computed for a range of design scenarios to obtain the most conservative values (lowest RF and highest LF). An assessment of the design method uncertainty (e.g., σ_X in the simple case exemplified by Eq. 4) is needed to perform this optimization.

One technique to quantify the uncertainty in design methods is to examine databases of predicted versus measured pile performance (e.g., Paikowsky 2004). An advantage of this technique is that a relatively large amount of data can be assembled on which to perform statistics. On the other hand, a disadvantage of this technique is that the method cannot discriminate between the various sources of uncertainty contributing to the observed scatter between predictions and measurements.

An alternative approach, adapted from Ellingwood (1980) in Foye et al. (2006a,b), is to identify the different sources of uncertainty separately, assign probabilistic models to each, then combine them in the final analysis. This approach is conceptually illustrated in Figure 1. Each contributing source of uncertainty results in increased overall uncertainty as their contributions are aggregated. Thus, this approach requires data quantifying each of the component uncertainties – for example, data that only contain error introduced by *in situ* testing. In Figure 1, we show that an *in situ* test measurement results from soil variables (soil state and soil intrinsic variables), and from this *in situ* test measurement a pile resistance can be calculated. In the component approach, the variability of these individual transformations would be assessed. The difference between the database approach and the component approach is that the database approach only allows an examination of the final, actual versus predicted data (lower-left graph), whereas the component approach allows an explicit quantification of the individual analysis steps (all graphs) and their impact on the final, aggregated uncertainty.

2 EXAMPLE RESISTANCE FACTORS FOR PIPE PILES IN SAND

Pile design methods are either direct or property-based (Salgado 2008). Direct design methods rely on direct correlations between *in situ* tests performed prior to pile installation and measured pile capacity following driving. Thus, direct design methods omit the uncertainty from soil property correlations (upper right in Figure 1). Property-based design methods compute pile capacity using various soil parameters as input. These parameters are computed from *in situ* and/or laboratory tests performed on the soil prior to pile installation.

Property-based design methods present a greater challenge to designers because the underlying analyses typically fail to capture the physics of the problem and, in addition, the design variables in these

methods are more difficult to obtain reliably in the field. This condition results in property-based design methods that are significantly less reliable than comparable direct design methods.

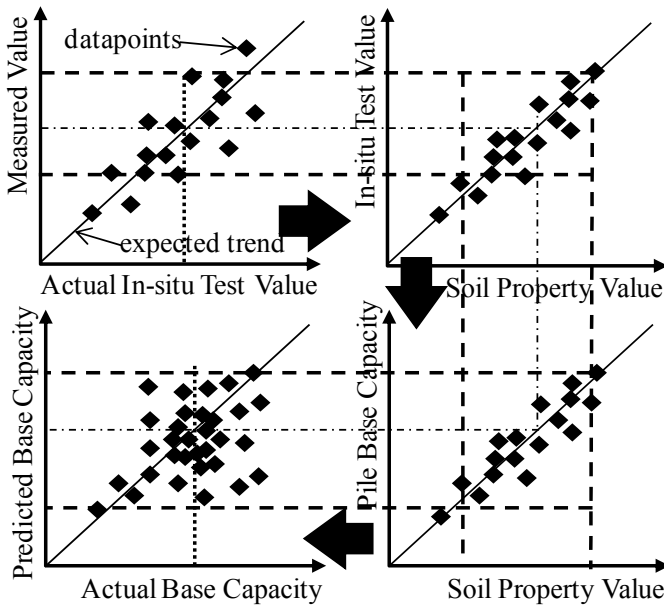


Figure 1. Conceptual representation of accounting for individual sources of uncertainty (clockwise from upper left): measurement, property correlation, capacity model, and final, aggregated uncertainty. The dashed line bounds qualitatively express the cumulative uncertainty at each step in pile capacity analysis. The alternating broken line represents the expected value.

2.1 Direct Estimation of Pile Shaft Capacity

The shaft capacity of a pile is computed by summing up the contributions of imaginary segments along the length of the pile. The total shaft capacity of a pile is expressed as

$$R_s = \sum_{i=1}^n q_{sLi} A_{si} \quad (5)$$

where q_{sLi} is the unit limit shaft resistance (force/unit area) for each segment i and A_{si} is the shaft area for that segment. Segment shaft area A_{si} is computed as $A_{si} = a_s dL$, where a_s is the perimeter of the cross section and dL is the segment length. Based on results by Lee et al. (2003) for open-ended piles, q_{sL} from Eq. (5) is defined as

$$q_{sL} = \left(\frac{q_{sL}}{q_c} \right) q_c = 0.002 q_c \quad (6)$$

Design equation (6) represents the model relationship between pre-driving CPT measurements and post-driving pile shaft resistance for open-ended pipe piles. To assess the uncertainty in this model relationship, the calibration chamber data from Paik and Salgado (2003) for shaft resistance was considered. The probabilistic model of this uncertainty was also aggregated with the uncertainty introduced by the equations used to estimate q_c within the calibration chamber. The corresponding RF value, determined for reliability index $\beta = 3.0$, was computed as 0.41 when used in conjunction with ASCE-7 (ASCE 2000) load factors. This value is repeated in Table 1 for comparison with other results given below.

2.2 Direct Estimation of Pile Base Capacity

The base capacity of a pile is computed by

$$R_b = q_{b,10\%} A_b \quad (7)$$

where A_b is the gross pile base area,

$$A_b = \pi \left(\frac{d_0}{2} \right)^2 \quad (8)$$

and d_o is the outside diameter of the pipe pile base. For open-ended piles, $q_{b,10\%}$ from Eq. (7) is defined as

$$q_{b,10\%} = \left(\frac{q_{b,10\%}}{q_c} \right) q_c \quad (9)$$

where

$$\frac{q_{b,10\%}}{q_c} = -0.00443IFR(\%) + 0.557 \quad (10)$$

IFR is the incremental filling ratio, a measure of the state of plugging of the pile at any point during driving (Paik et al. 2003). Lee et al. (2003) provide guidance on estimating values of IFR when field measurements are not available. The $q_{b,10\%}/q_c$ relationship (Eq. 10) was developed by Lee et al. (2003) based on calibration chamber test results of $q_{b,10\%}$. The uncertainty of this relationship was assessed similarly to the method discussed for shaft capacity.

Table 1. Summary of design of driven open-ended (OE) pipe piles in sand using results from CPT or SPT. Resistance Factors (RF) are given for use with ASCE-7 and AASHTO load factors. FS indicates an approximate value of safety factor corresponding to the resistance factors given.

Design Method	RF using ASCE-7 LF s	RF using AASHTO LF s	Representative FS
Direct Design – CPT			
OE pipe shaft (Eq. 6)	0.41	0.45	3.7
OE pipe base (Eq. 9)	0.54	0.59	2.8
Property-Based Design – CPT			
OE pipe shaft (Eq. 11)	0.47	0.50	3.7
OE pipe base (Eq. 14)	0.42	0.45	3.5
Property-Based Design – SPT			
OE pipe shaft (Eq. 11)	0.42	0.45	4.0
OE pipe base (Eq. 14)	0.40	0.42	3.7

2.3 Property-Based Estimation of Shaft Capacity

For open-ended piles q_{sL} from Eq. (5) is computed from

$$q_{sL} = \frac{K_s}{K_0} \tan\left(\frac{\delta_c}{\phi_c} \phi_c\right) (K_0 \sigma'_v) \quad (11)$$

where K_s/K_0 is the ratio of the coefficient of lateral earth pressure after pile installation to the coefficient of lateral earth pressure at rest K_0 , δ_c/ϕ_c is the ratio of the interface friction angle δ_c to the critical-state friction angle ϕ_c , and $(K_0 \sigma'_v)$ represents the initial horizontal effective stress at the depth of the pile segment considered. Based on data presented by Paik and Salgado (2003), the ratio K_s/K_0 is computed as

$$\frac{K_s}{K_0} = \beta(7.2 - 4.8PLR) \quad (12)$$

where

$$\beta = 0.00018D_R^2(\%) - 0.0089D_R(\%) + 0.329 \quad (13)$$

with $20\% < D_R < 90\%$. PLR is the plug length ratio ($0 \leq PLR \leq 1$), defined as the ratio of plug length to pile penetration length, and $D_R(\%)$ is the relative density, expressed as a percent.

The prediction of the unit limit shaft resistance, q_{sL} , represented by Eq. (12), contains uncertainties from three sources: 1) the increase in lateral earth pressure due to pile driving and pile loading expressed by K_s/K_0 , 2) the coefficient of interface friction $\tan\delta_c$, and 3) the pre-driving lateral earth pressure ($\sigma'_h = K_0 \sigma'_v$).

The combined uncertainty in the ratio K_s/K_0 , defined by Eq. (12), is due to the variable uncertainties of D_R and PLR and to the model uncertainty present in the results of Paik and Salgado (2003). Foye et al. (2006a) examined the uncertainty of relative density determined from the CPT. The uncertainty in the

K_s/K_0 model relationship was deduced directly from the scatter in the results of Paik and Salgado (2003) since D_R , σ'_h , and degree of plugging were closely controlled or measured.

The uncertainty of PLR predictions was assessed by considering the recommendations by Lee et al. (2003) and the data presented by Paik and Salgado (2003). Given that the sand around the pile can be described as loose, medium-dense, or dense, the designer will be able to estimate PLR to within ± 0.15 of the actual value when aided by prototype pile driving results. The least biased probability density function to describe the uncertainty in this estimate is a uniform distribution with bounds ± 0.15 of the expected value. Without the benefit of such results, PLR may be completely unknown. The most extreme expression of this uncertainty is a uniform distribution with bounds $PLR = 0$ and $PLR = 1$. The consequences of this difference in knowledge of PLR are explored below.

The uncertainty of δ_c was assessed by considering ϕ_c and the ratio δ_c/ϕ_c . Uncertainty in critical state friction angle ϕ_c was obtained from results reported by Bolton (1986). Uncertainty in the ratio δ_c/ϕ_c was assessed by considering the results of high-quality, direct-interface shear tests by Lehane et al. (1993), Jardine and Chow (1998), and Rao et al. (1998).

The uncertainty of σ'_h cannot be systematically assessed. Although σ'_v can be easily computed in practice, K_0 is not so easily assessed. Correlations have been found between K_0 and ϕ and between K_0 and void ratio e or D_R and overconsolidation ratio, which is often unknown. However, the determination of ϕ and D_R from *in situ* tests is dependent on K_0 . Hence, in typical design practice, an assumption of K_0 is required.

2.4 Property-Based Estimation of Base Capacity

For open-ended piles $q_{b,10\%}$ was found by Paik and Salgado (2003) to be related to the relative density, $D_R(\%)$, and the effective lateral earth pressure, σ'_h , as

$$q_{b,10\%} = \alpha \left(326 - 295 \frac{IFR(\%)}{100} \right) \sigma'_h \quad (14)$$

where

$$\alpha = 0.0112D_R(\%) - 0.0141 \quad (15)$$

with $20\% < D_R < 90\%$. The prediction of unit base resistance q_b for open-ended pipe piles, represented by Eq. (13) contains uncertainties from two sources: 1) uncertainty in the $q_{b,10\%}/\sigma'_h$ relationship and 2) uncertainty in the initial lateral earth pressure σ'_h . As seen in Eq. (14), $q_{b,10\%}/\sigma'_h$ also depends on D_R . The uncertainty of these relationships was assessed using the same methodology discussed for pile shaft resistance.

3 RESISTANCE FACTORS AS INDICATORS OF UNCERTAINTY

Table 1 summarizes the resistance factors determined for each of the presented pile design methods when the reliability index $\beta = 3.0$. Resistance factors are presented for use in conjunction with both the ASCE-7 (ASCE 2000) load factors and the AASHTO (AASHTO 1998) load factors. Lower resistance factors are expected for design methods with greater uncertainty. Direct design methods are expected to have less uncertainty than property-based methods. This expectation is met by the RF results for open-ended pipe pile base resistance. Also, *in situ* test methods that introduce greater uncertainty are expected to produce lower RF values. This expectation is confirmed by the relative values of RF obtained for CPT- and SPT-based methods.

3.1 Effect of IFR and PLR Estimate Confidence on Resistance Factor

Table 2 illustrates the effect of different degrees of confidence in IFR estimates on the reliability of the direct design of open-ended pipe pile base resistance (Eq. 9). Since parallel analysis of the effect of estimates of PLR on shaft resistance has similar results, this discussion is limited to IFR and base resistance. Two cases are considered; 1) when IFR is unknown, it can assume any value between 0 and 100% with equal likelihood (uniform distribution) and 2) when IFR is estimated from prototype pile test results and is modeled as uniformly distributed within $\pm 15\%$ of the estimated value. For both cases, Table 2 includes

the calculated results for the RF value required to achieve a reliability index $\beta = 3.0$, the value of β for $RF = 0.54$ as obtained in Table 1, and the probability of failure P_f if $RF = 0.54$.

According to Table 2, if a pile is designed using $RF = 0.54$ without measuring IFR for a test pile (the completely unknown case), $P_f = 7.44\%$. If a test pile is used and IFR measured, the same $RF = 0.54$ value will lead to $P_f = 0.131\%$, a considerable difference in risk to the project. It is clearly unacceptable to have a 7.44% probability of failure. Also, designing piles twice as conservatively as may be necessary is a waste, as is indicated by the ratio of RF values for $\beta = 3.0$. Therefore, this example shows how the difference in RF values are an indirect indicator of relative risk. Also, the difference in RF values highlights the value of measuring IFR in this specific case and of obtaining information about other variables in other types of project at costs that are comparatively small.

The following sequence is suggested by these results: 1) design the piles following a conservative estimate of IFR (higher values), 2) measure IFR during the installation of test piles at the site, and 3) revise the design as needed to reflect the site conditions. This technique allows information to be introduced and used to improve the estimated reliability or economy of the project.

Table 2. Resistance factors RF for $\beta = 3.0$, reliability indices β for $RF = 0.54$, and probabilities of failure P_f for ULS design checks of OE pipe pile base resistance using the Paik et al. (2003) method (Eq 9).

IFR determination	RF for $\beta = 3.0$	β if $RF = 0.54$	P_f if $RF = 0.54$
completely unknown	0.26	1.44	7.44%
estimated from prototype	0.54	3.00	0.131%

3.2 Effect of Design Method Uncertainty on Resistance Factor

The ratio $q_{b,10\%}/q_c$ (Eq. 9) for closed-ended pipe piles can be assessed by considering the results of high-quality instrumented pile load test results by Vesic (1970), BCP Committee (1971), Gregersen et al. (1973), Beringen et al. (1979), Briaud et al. (1989), Altae et al. (1992, 1993), and Paik et al. (2003) (Figure 1). Based on these results, a viable relationship for $q_{b,10\%}/q_c$ appears to be

$$\frac{q_{b,10\%}}{q_c} = 1.02 - 0.0051D_R (\%) \quad (16)$$

where $D_R(\%)$ is the relative density, expressed as a percent. Figure 3 shows that the distribution of residual $q_{b,10\%}/q_c$ values (trend subtracted from data) with respect to Eq. (16) resembles a normal distribution with $COV = 0.17$. Since this field test database includes uncertainty due to inherent soil variability, CPT testing, and the relationship between measured values of q_c and $q_{b,10\%}$, this distribution was used directly to determine resistance factors for use with Eq. (9) and Eq. (16), without further consideration of additional probabilistic models for soil and CPT testing. Hence, these data most closely resemble the scenario presented in the lower left quadrant of Figure 1.

These results – both in terms of the suggested design value and the resulting resistance factors – compare favorably to the Paik and Salgado (2003) calibration chamber results, as expressed in Eq. (10). Regarding the suggested design value, an $IFR = 0$ indicates the plug length is not growing even as the pile penetration length is increasing, which suggests the pile is responding as a closed-ended pile. In the close-ended or fully plugged condition, Eq. (10) becomes

$$\frac{q_{b,10\%}}{q_c} = 0.557 \quad (17)$$

Since the fully plugged condition was approached in the Paik and Salgado (2003) study only for open-ended piles driven in dense sands (80-100%), Eq. (17) is in agreement with Eq. (16). The distribution of residual $q_{b,10\%}/q_c$ values by Paik and Salgado (2003) with respect to Eq. (10) resembles a normal distribution with $COV = 0.10$. Aggregation of this uncertainty with the uncertainty due to estimates of q_c in the calibration chamber results in a distribution of $q_{b,10\%}/q_c$ values best fit by a beta distribution with bounds $0.54f(IFR)$ and $5.92f(IFR)$, and distribution parameters $\alpha = 3.5$ and $\beta = 40.2$.

Reliability analyses based on the load test database and on the Paik et al. (2003) results are used to develop resistance factors for use with Eqs. (16) and (17). These resistance factors are compared to examine the differences in the two sources of data and the effect of these differences on the results of the reliability analyses. First, the case of the probability distributions actually obtained during this study is presented. In this case, the value of RF obtained for use with Eq. (17) in conjunction with ASCE (2000)

load factors is $RF = 0.54$. This RF value is identical to the value obtained for use with Eq. (10) because the sources of the design method uncertainty, calibration chamber uncertainty, and CPT measurement uncertainty are also identical.

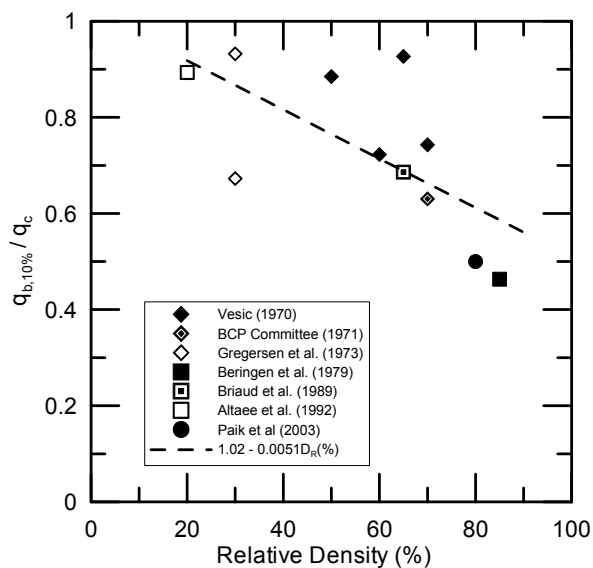


Figure 2. Plot of $q_{b,10\%}/q_c$ from database of high-quality instrumented pile load tests.

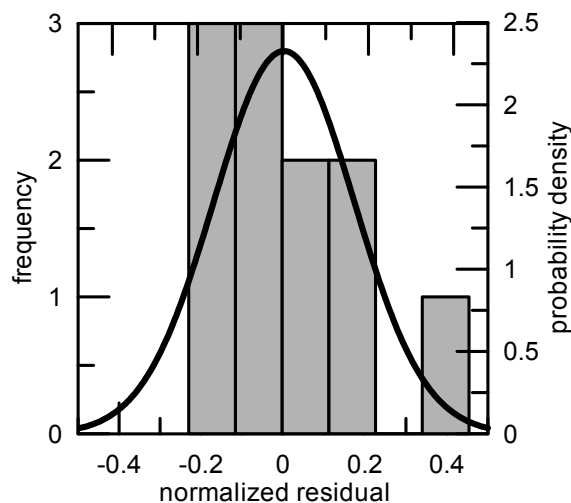


Figure 3. Histogram of data scatter about $q_{b,10\%}/q_c$ relationship for closed-ended pipe piles in sand based on field results (Figure 2). Line indicates a normal distribution with $COV = 0.17$.

The value of RF obtained for use with Eq. (16), calculated from the pile load test database, is also $RF = 0.54$. While this coincidence lends some credence to the analysis methods discussed in this paper, further investigation of the relationship between $q_{b,10\%}/q_c$, COV and RF is useful to understand the significance of the uncertainty aggregation technique.

Figure 4 plots the values of RF obtained for different input values of COV defining the normal distribution representing the uncertainty in design relationship $q_{b,10\%}/q_c$. Figure 4 shows that RF is significantly less sensitive to COV for Eq. (17) (Paik and Salgado 2003) than for Eq. (16) (pile test database). The reason is that other sources of uncertainty (e.g., q_c measurement) are separately evaluated in the

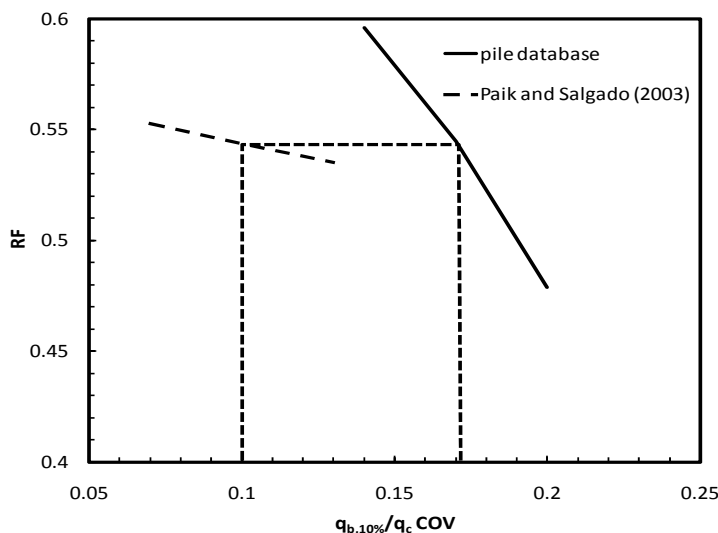


Figure 4. Plot of calculated resistance factors, varying the COV representing the uncertainty of the $q_{b,10\%}/q_c$ model relationship obtained from both the database of high-quality instrumented pile load tests and the Paik and Salgado (2003) calibration chamber data. Short dashed lines indicate the RF values obtained for the actual distributions found in the study.

analysis of Eq. (17) – as in the upper-left and lower-left quadrants of Figure 1 – that cannot be separately evaluated in the analysis of Eq. (16). Accordingly, the separate analysis of the various sources of uncertainty may allow more confident assessments of RF values since this approach is less sensitive to errors in the assessment of individual probability distributions. Conversely, small errors in the assessment of load test databases, or revisions of these databases, may result in significant changes in recommended RF values.

4 CONCLUSIONS

In this paper, we computed resistance factors appropriate for use in the direct and property-based design of driven pipe piles in sands. Resistance factors are a useful quantitative tool to express the relative uncertainty of load capacity of different pile types and pile design methods. Property-based design methods tend to have higher uncertainty (lower RF) but apply to general cases. Direct design methods tend to have lower uncertainty (higher RF) but apply only to cases resembling the specific piles and soils of the source direct design database.

By systematically disassembling the design equations and considering the uncertainty associated with each design variable or relationship separately, we quantified the various sources of uncertainty for each method. Using this technique, we identified where the sources of uncertainty are different between design methods. An example of RF calculations was given to illustrate the significance of this uncertainty aggregation technique. The example shows that separately accounting for each source of uncertainty can reduce the sensitivity of RF values to individual model or measurement distributions. Furthermore, as these distributions are researched and updated, this technique allows a modular approach to updating the relevant models and calculating new values of RF .

Finally, we presented an example showing the difference in resistance factors obtained when IFR is measured or unknown. This example illustrated the ability of resistance factors to convey the design value of this additional information. Conversely, the impact on the reliability or risk of the design by omitting this information can be similarly assessed.

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