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Reliability analysis of bearing capacity for shallow foundations based on Eurocode 7

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ABSTRACT: In Eurocode 7, the application of statistical methods and reliability analyses is encouraged though not much guidance is clearly given for the practitioner. Reliability analyses can effectively model parameter uncertainty and provide more meaningful results in terms of a reliability index and probability of failure. The current work focuses on the development of a methodology for calculating bearing resistance using reliability analysis, in terms of Eurocode 7. Random fields of soil cohesion and friction angle are generated and converted to characteristic values for checking the GEO failure criterion in ULS. The analysis is performed for a simple example of a shallow foundation, comparing the methods of Monte Carlo simulation and First Order Reliability Methods. For both methods the reliability index and probability of failure were determined. It was found that both methods can be applied for an increasing uncertainty in friction angle though FORM results for low uncertainty are closer to the deterministic solution than Monte Carlo simulations.

Keywords: Bearing Capacity, Reliability Analysis, Eurocode 7, Shallow Foundations

1 INTRODUCTION

1.1 *General*

Conventional design methods in geotechnical engineering generally consider the calculation of a reasonable global factor of safety. This global safety factor is simply a comparison of actual to required strength and indicates whether the system is safe or not, in what is termed as a deterministic approach. However, due to the large uncertainty resulting from in-situ soil variability, even in homogeneous soils, it may not always represent a realistic situation. The effect of variability in soil properties cannot be efficiently modelled in such an analysis. For these cases, the use of probability theory can be implemented in terms of a probabilistic or a reliability analysis to model ground uncertainties.

The interest here is not in calculating simply a factor of safety but to investigate the probability of failure for an engineering system. A reliability analysis presents a more meaningful approach for geotechnical design, rather than the calculation of a factor of safety, as this can be used for risk-based decision making. It also indicates the performance and reliability for a geotechnical problem. To initiate a reliability analysis, random fields of soil properties are commonly generated to derive the required statistical parameters, e.g. mean and standard deviation. A method of reliability analysis is then selected for determining the probability of failure and the reliability index. Some commonly used techniques include the Monte Carlo simulation, First Order methods and Point Estimate method. A detailed description for each of these methods is presented in Baecher and Christian (2003). The present work deals with the reliability based analysis of bearing capacity for shallow foundations, in terms of Eurocode 7 requirements.

1.2 *Background to analysis*

Reliability analysis has extensively been used in geotechnical engineering, especially in recent years. The types of analyses can be either practical reliability based computations and applications as included in Phoon (2008), or stochastic analysis using finite elements or other numerical method aiming at risk assessment, for example in Hicks (2007) and Fenton and Griffiths (2008). Applications covered may range from complex liquefaction analysis (Hicks and Onisiphorou 2005) to applications in retaining wall design Low (2005) and rock slope analysis (Onisiphorou 2010).

With regards to reliability analysis of bearing capacity, Cherubini (2000) evaluated the performance of c-φ' soils for varying correlation coefficients, while Fenton and Griffiths (2003) investigated effects of spatial variability for c-φ' soils using random finite element methods. Recently, reliability analyses published by Fenton et al (2007, 2008) state an aim of promoting the creation of reliability-based design codes for geotechnical engineering. The applications of Eurocode methodologies in combination with reliability analysis for modelling spatial variability provides a new area of research and a lot of ground still needs to be investigated (Simpson & Driscoll 1998), especially with implementation and experience gained with Eurocode 7 design. Orr & Breysse (2008) present some important issues on the EC7 partial factor approach in geotechnical design and include example reliability analyses for spread foundations. Furthermore, Hicks & Samy (2004) developed and presented a stochastic approach for determining characteristic values of soil properties for Eurocode 7, based on the confidence levels suggested by the code.

2 BEARING CAPACITY BASED ON EUROCODE 7

2.1 *General on Eurocode 7*

EN 1997 is based on limit state conditions defined as "the state beyond which the structure no longer fulfils the relevant design criteria". In general, Eurocode 7 considers two limit state design situations, the Ultimate Limit State (ULS), associated with collapse or failure, and the Serviceability Limit State (SLS), associated with unsatisfactory service requirements. These limit states are explained in detail in EN 1997 itself and specialised publications, e.g. by Frank et al (2004) and Orr and Farrell (1999). Furthermore, for the calculations at ULS, five different failure criteria exist, depending on possible cause of failure. These are Structural (STR), Geotechnical (GEO), Equilibrium (EQU), Uplift (UPL) and Hydraulic (HYD) and are defined in detail in EN 1990.

Eurocode 7 has considered the spatial variability and uncertainty of soil properties as significant for geotechnical design and suggests taking this uncertainty into account using statistical methods. One way of doing this is by applying the recommended partial factors to characteristic values of soil properties, which on their turn can be based on resulting statistics from extensive in-situ or laboratory data. These are considered as semi-probabilistic analyses, as referred to by Orr & Breysse (2008), as opposed to fully probabilistic or reliability analyses, though not much detail is included in the code on how to do this. It should be noted that currently a reliability analysis is mostly based on the experience and engineering judgement of the designer, rather than direct guidance from Eurocode 7.

The characteristic value of a soil property, X_k, is based on mean value but actually defined as a cautious estimate of the value affecting the occurrence of the limit state (EN1997-1). It maybe determined by

$$X_k = \mu_X (1 - kV_X) \qquad (1)$$

where μ_X = mean of property X, V_X = coefficient of variation of X (=σ_X/μ_X), k = a statistical coefficient depending on type of characteristic value, available test results and level of reliability. For defining k, the confidence level of 95% maybe too high sometimes for soils (as discussed by Orr and Breysse (2008)) leading to conservative results. Therefore, for the purpose of the current work, the characteristic value mentioned above is based on the following practical relationship by Schneider (1997).

$$X_k = \mu_X - 0.5\sigma_X \qquad (2)$$

where X = the randomly varying soil property and σ_X = standard deviation of X. Taking Equation (2) into account, random fields are generated for the characteristic values of soil properties.

2.2 *Methodology for calculating bearing capacity*

According to Clause 2.4.7.3.1, when checking the bearing resistance of a shallow foundation, the GEO criterion must be checked, which is related to failure in the ground. Therefore, the following inequality must be satisfied:

$$E_d \le R_d \qquad (3)$$

where E_d = the design value of the effect of actions, R_d = the design value of the bearing resistance of the ground. The effect of actions is represented in the analyses in the following sections solely by the design value of vertical forces V_d, which also includes the weight of foundation and overburden soil.

The bearing resistance for a shallow foundation can be calculated using the well known Terzaghi equation. A methodology based on the above equation is presented in Annex D of Eurocode 7 and is adopted here. For the present case only vertical load is considered and so the inclination factors i_c, i_q, i_γ are omitted in the equation. The equation to be used simplifies to the one given below:

$$\frac{R_k}{A} = cN_c s_c + \gamma_d D N_q s_q + \frac{1}{2} B \gamma_d N_\gamma s_\gamma \qquad (4)$$

where R_k = characteristic value of bearing resistance, c = soil cohesion, A = foundation area, N_c, N_q, N_γ = bearing capacity factors, s_c, s_q, s_γ = shape factors, q = overburden pressure at level of foundation base, B = foundation width, D = embedment depth, γ_d = design value of soil unit weight.

The bearing capacity factors, N, are given below:

$$N_c = (N_q - 1)\cot\phi' \qquad (5)$$

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \phi'/2) \qquad (6)$$

$$N_\gamma = 2(N_q - 1)\tan\phi' \qquad (7)$$

The shape factors for a rectangular footing are as follows:

$$s_c = (S_q N_q - 1)/(N_q - 1) \qquad (8)$$

$$s_\gamma = (1 - 0.3)(B/L) \qquad (9)$$

$$s_q = 1 + (B/L)\sin\phi' \qquad (10)$$

Finally, the design value of bearing resistance is given by

$$R_d = \frac{R_k}{\gamma_R} \qquad (11)$$

where γ_R = the partial factor for bearing resistance = 1.4 for Design Approach 2, as taken from Table A.5, Annex A of EN 1997-1.

3 VARIABILITY OF SOIL PROPERTIES AND EXAMPLE APPLICATION

The randomly varying variables selected for the analysis are the shear strength parameters, the soil cohesion, c, and angle of internal friction, φ. It is assumed that the two random variables are uncorrelated for the purpose of the analysis, even though it is reasonable to assume a degree of cross-correlation. However, previous research work by Fenton and Griffiths (2003) has shown that the effect of correlation is not significant to the final result. Random fields are then generated from which sets of values are converted for each property. This is done by assuming a pre-described probability distribution for each property. It has been assumed that both variables follow a Normal distribution with statistical parameters as shown in Table 1 below.

Table 1. Statistical properties of random variables

Property	Point Statistics	
	μ	σ
c (kPa)	5	1.0
φ (°)	31	2-4

The assumptions for these distributions and the coefficient of variation for each parameter are based on previously published correlations and recommendations, such as Lacasse and Nadim (1996) and Phoon and Kulhawy (1999). The coefficient of variation for φ , V_φ , will be varied from 2-4 degrees in order to investigate the effects of increasing uncertainty in friction angle on system reliability. The random sets of properties, required for the Monte Carlo simulations are generated using the general equation

$$Y = F^{-1}(U)$$

where Y = random variable following a prescribed cumulative distribution *F(.)*, U = random variable uniformly distributed between 0 and 1.

The application problem considers a shallow foundation with dimensions L = 2.2m, B = 1.8m and embedment depth D = 2.0m. The soil is medium dense sand with unit weight assumed constant at γ = 19 kN/m^3. The permanent load is G_k = 650 kN and variable load is equal to Q_k = 400 kN. Using partial factors γ_G =1.35 and γ_Q = 1.5 for permanent and variable loads respectively (as taken from Tables A.3 and A.4 in Annex A of EN 1997-1), the value of V_d = 1478 kN (kept constant throught the analyses). Note that this value includes the weight of foundation and overburden soil.

4 RELIABILITY ANALYSIS USING MONTE CARLO SIMULATIONS (MCS)

4.1 *Methodology*

For the Monte Carlo simulations, 5000 realisations were performed. For each of these analyses a different characteristic value of cohesion and friction angle is used in the Equation (4) for calculating bearing capacity.

The performance function (or safety margin) for the foundation can be defined by

$$g(x_1, x_2, ..., x_i) = R_d - V_d \tag{12}$$

where x the set of random variables. The reliability index, β, following a run of the realisations for the Monte Carlo simulations can be computed by

$$\beta = \frac{\mu_g}{\sigma_g} \tag{13}$$

If a Normal distribution is assumed for the performance function $g(x_i)$, the resulting probability of failure, p_f, is given by

$$p_f = P(g < 0) = 1 - \Phi(\beta) = \Phi(-\beta) \tag{14}$$

where $\Phi(\beta)$ is the value of the cumulative standard Normal distribution.

4.2 *Results*

Figure 1 below shows the distribution of an Over-design Safety Factor (*OFS*), equal to ratio R_d/V_d deducing from Equation (3), in an analogy to the safety factor. For varying coefficients of variation for friction angle, it can be seen that this distribution results in a slightly skewed distribution, approximating a log normal distribution.

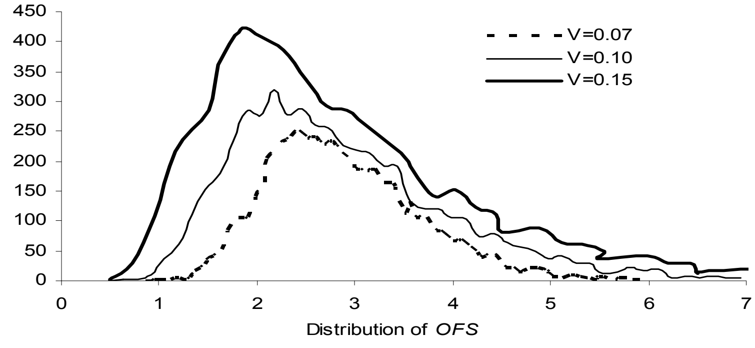

Figure 1. Distribution of Over-design Safety Factor (*OFS*) for varying V of friction angle

The mean *OFS* visibly decreases as uncertainty becomes larger in the characteristic value of friction angle (increased *V*). It should be noted that the deterministic solution gives a value of 3.1 well above the reliability based solution. The same skewed distribution has been observed for the safety margin $g(x_i)$, therefore it was decided to calculate the probability of failure, p_f, using the lognormal probability distribution instead of Equation (14), in order to avoid highly conservative results.

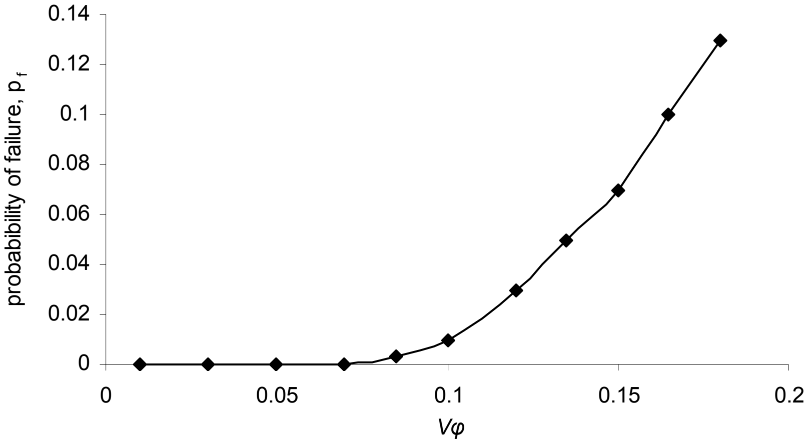

Figure 2. Probability of failure p_f for varying *V* of friction angle (Monte Carlo)

The results are summarised in Figure 2 below for varying *V* of friction angle. For cases of low variation such as V_φ = 0.05-0.08 (and less uncertainty), probability of failure is relatively low (less than 0.0002). As variation increases with $V_\varphi > 0.1$, a sudden increase is evidenced in probability of failure. This is reflected in low values of reliability index, β ranging from 1.0 to 3.0.

5 ANALYSIS USING FIRST ORDER RELIABILITY METHOD (FORM)

5.1 *Method of analysis*

The present analysis is based on the approach suggested by Hasofer and Lind (1974) for finding the minimum distance d (represented by the reliability index β) between the mean value and the failure surface for $g(x) = 0$, where *g* the safety margin as defined by Equation (12). For the present work, the analysis using FORM is performed using solver in spreadsheets, based on the matrix formulation by Low & Tang (2007). The starting trial values are equal to the characteristic values of the soil properties

This equation used for the reliability index is

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \qquad (15)$$

where β = reliability index, x_i = random variables, μ_i = mean values of *ith* variable, σ_i = standard deviation of *ith* variable, *R* = correlation matrix, *F* = failure domain, i.e. $g(x)$=0. As discussed in Section 3 above, cohesion and friction angle are assumed uncorrelated. The point statistics used are tabulated in Table 1 of Section 3 and the probability of failure is calculated from Equation (14).

5.2 *Results*

Figure 3 shows summarised results for the FORM analysis. The variation of the reliability index with regards to an increase of uncertainty in friction angle is plotted. As can be seen the reliability index ranges from 3-5 for low variation while steadily droping to lower values of $\beta < 2$ for larger *V*. Figure 4 presents the values of probability of failure for various coefficients of variation for friction angle and is comparable to Figure 2 for the Monte Carlo simulations. The FORM method gives a lower probability of failure than Monte Carlo, especially as uncertainty becomes significant.

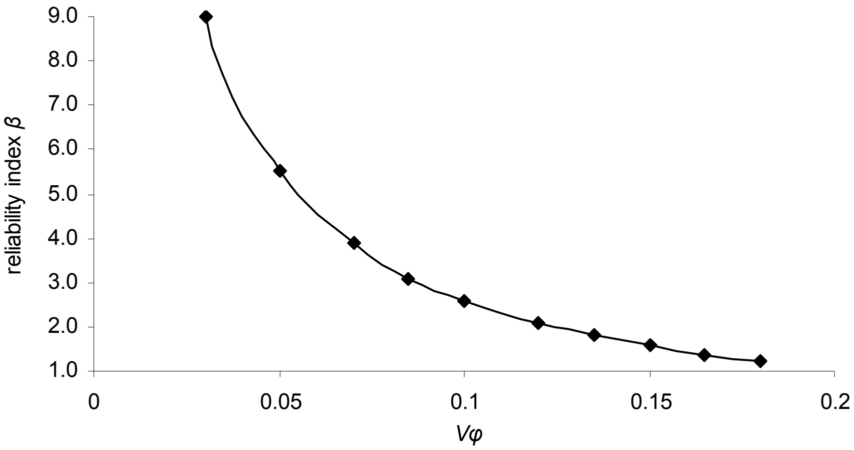

Figure 3. Reliability index β for varying V of friction angle

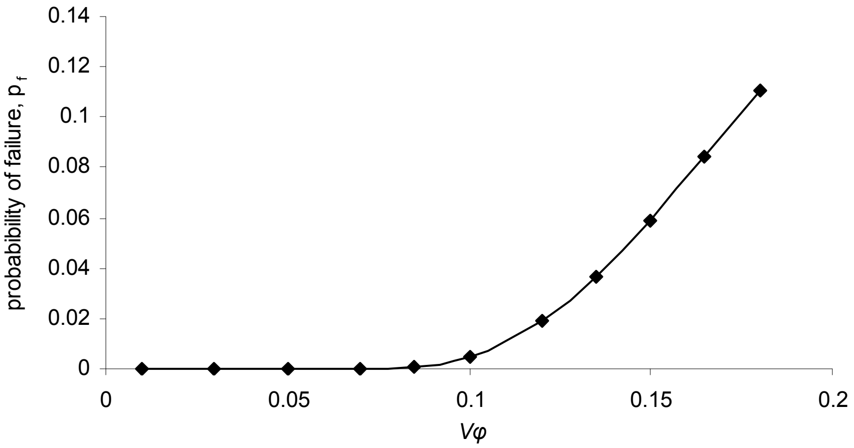

Figure 4. Probability of failure p_f for varying V of friction angle (FORM)

6 CONCLUSIONS

Conventional deterministic analyses may not always capture the soil variability and the uncertainty in soil properties. In these cases, a reliability analyses maybe more appropriate as statistical properties can be used to determine the probability of failure and reliability index. The importance of reliability analyses has been recognised by the recently implemented Eurocode 7 and is encouraged to use whenever suitable. However, experience needs to be gained among researchers and practitioners in various applications in geotechnical engineering.

The current research work presents a methodology for calculating the bearing resistance of the ground using reliability analysis. A recommended deterministic solution by Eurocode7 has been implemented while characteristic values were input as random variables for cohesion and friction angle. The application considered a shallow foundation with vertical loads and the system was analysed using two well known reliability techniques, the Monte Carlo simulations and the First Order Reliability Method (FORM).

The effect of varying the friction angle was investigated for the bearing capacity of a shallow foundation. It was found that the reliability index for Monte Carlo simulations was lower than the FORM solution, when variability is small. When uncertainty increases, the reliability indices for the two methods increase for both methods and are closer together. This is possibly due to an improved modelling of uncertainty using the Monte Carlo simulations. Overall, the two methods approach the deterministic solution as uncertainty increases, though FORM result gives a closer approximation as Monte Carlo is also dependent on the number of realisations.

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