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Risk Analysis and Observational Methods in practice: what do new codes improve?

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ABSTRACT: The construction of cuts, embankments and dam in and on different ground require the analyses of slope stability, deformations and often the use of the observational method. New codes, such as Eurocode 7 or its sister code in Switzerland SIA 267, prescribe the use of partial factors on friction angle and cohesion for slope stability analyses. This methodology has many shortcomings, as experienced in practice, in particular with steep slopes and irregular ground conditions. Examples with substantial deviation between analyses with the partial factors of safety and the global factor of safety will be presented. The use of the factor of safety on shear strength will be proposed, as had been used before in the limiting equilibrium methods, which is also called global factor of safety. The factor of safety on shear strength integrates the effects of shear strength, whether undrained shear strength, effective strength described by cohesion intercept and friction angle or curved envelopes; the effect of geometry and seepage and porewater pressures. With this approach stability analyses, deformation prediction and the observational method can be integrated this ultimately leads to safer construction.

Keywords: Slope, Safety factor, Stability, Friction, Cohesion

1 INTRODUCTION

The authors are involved in many types of stability problems in practice involving field measurements and site investigations. For the analysis the authors have used different types of analyses and have experienced the appearance of new codes with the change to the use of partial factors. Also the Observational Method has appeared in these codes to be applied in geotechnical construction. The experience has shown that the Observational Method is often poorly applied in practice. Often a poor site investigation is carried out; followed by a similar insufficient analysis and some observations and then some measurements are planned, that usually lead to the reinforcement with tie-backs. For judging the relative risk of slopes the use of probabilistic methods has proven as practical tool (Steiner et al. 1992). These findings are supported by practical examples.

1.1 *Geotechnical code with partial factors*

The new geotechnical codes such as Eurocode 7 and national (SIA 267) codes prescribe the use of separate Partial Factors of Safety on cohesion and friction angle, this reduction has led to the fact that the analyses have to be carried out with a fictitious soil material, in several cases we have noted that the obtained critical siding surface for the analyses with partial factors on cohesion and frictional strength deviate substantially from the factor of safety on shear strength. Such deviations can only be detected by carrying out analyses in parallel.

1.2 *Application of probabilistic methods*

The use of probabilistic method has proven useful in practice (Steiner et al., 1992) to judge the relative risk and the influence of the dispersion of the significant parameters, like undrained shear strength of a cohesive soil. The use of a factor of safety only would not have led to the same conclusions as with probabilistic methods.

2 BACKGROUND OF SLOPE STABILTY ANALYSES

There a many different types of analyses available, the slip circle method (Taylor, 1937), charts for estimating slopes with the consideration of cohesion, friction and pore pressure by Janbu (1954), and Bishop & Morgenstern (1960), these charts are also published in soil mechanics books (Lang et. Al 2008) . Similar charts are published in rock mechanics literature (Hoek and Bray, 1977; Wyllie and Ma, 2004). Such charts are limited to simple geometries with one type of soil or rock material. Complex ground conditions have to be simplified with respect to geometry and to geotechnical conditions. For practical cases the use of a method of slices with limit equilibrium methods has become standard practice since complicated geometric conditions and different geotechnical layers can be relatively easily analyzed (Wright,1969; Krahn, 2003; Duncan & Wright, 2005). More recently Finite element methods are used with the shear strength reduction method, SSR (Krahn,2007), which requires knowledge of deformation properties.

2.1 *Basics of Limit Equilibrium*

In the following the most important basic facts and assumptions in limit equilibrium are recalled. For a complete treatment reference is made to the literature (Duncan & Wright, 2005; Krahn, 2003 & 2004; Fredlund & Krahn, 1977). The available shear strength is defined as shown in Equation (1)

$$s = \frac{1}{F}\left[c' + (\sigma_n - u)\tan\Phi'\right] \quad (1)$$

where s = available shear strength, c'= cohesion intercept, Φ' = friction angle, σ_n = total normal stress on the base of the sliding surface, u = porewater pressure on the sliding surface, on the base of the slice.

This formulation goes back to Bishop (1955), since he had noted that the Ordinary method of slices or Fellenius' method did not fulfil equilibrium at the slices and gave substantial deviations. Krey (1936) had developed an essentially similar method as Bishop's without iteration. At that time only manual computations were feasible and the method had to be available for hand calculation. Janbu's (1957) simplified method is similar to Bishop's (1957), instead of fulfilling the moment equilibrium horizontal force equilibrium is fulfilled.

The next step for slope stability analyses came with the availability of computers and Morgenstern & Price (1963), who considered the complete equilibrium in the analysis. It is interesting to note the very limited computing power available in a major computer centre compared to today in a personal computer. Spencer (1967) developed a different formulation of the side forces inclined at a constant angle, which corresponds to a special case of Morgenstern-Price with constant function..

The problem of a sliding mass with the method of slices is highly statically indeterminate (Lambe & Whitman, 1969) and requires that assumptions on the internal stress distribution or in case of the method of slices on the lateral forces between the slices. The interslice forces involve normal and shear forces; these have also to fulfil the equilibrium conditions. The slices are assumed as rigid bodies and the static equilibrium equations have to be fulfilled (Fredlund & Krahn, 1977; Steiner, 1977).

2.2 *Fulfillment of equilibrium*

The different methods of slices fulfil the equilibrium conditions on the individual slice and the entire sliding bodies to different degrees (Table 1). The accuracy of different methods has been presented for different slopes by Whitman & Bailey, 1963; Wright, 1969; Wright et al., 1973, Krahn, 2003.

Many comparisons have been published and often Spencer's method has been recommended as the most practical to apply. One has to note that Spencer's method is a special case of the Morgenstern-Price method, namely with constant inter-slice function. Often there convergence problems of the solutions may arise, for this purpose Fredlund and Krahn, (1977) and Krahn (2004) have compared the development of the factor of safety with the inclination of the interslice forces for moment and force equilibrium and found that the differences between the methods can be attributed to the treatment of the interslice forces.

With the General Limit Equilibrium GLE (Krahn, 2004) convergence of force and moment equilibrium can be evaluated in the computer code Slope/W. Often only the simplified methods are treated in text books (Lang et al. 1990, 1996). From our experience simpler methods, such as Bishop's and Janbu's, originally developed for manual computations, may deviate in either direction from the result with complete consideration of the internal stresses for complex geometries and ground.

Table 1. Equilibrium conditions applied in method of slices

Method	Global Equilibrium			Equilibrium on slice			Inclination of inter-slice forces
	Moment	Vertical forces	Horizontal forces	Moment	Vertical forces	Horizontal forces	
Fellenius	Yes	No	No	No	No	No	None
Krey (3)	Yes	*(Yes)*	No	No	Yes	No	Horizontal
Bishop modified	Yes	*(Yes)* (1)	No	No	Yes	No	Horizontal
Janbu modified	No	*(Yes)*	*(Yes)*	No	Yes	Yes	Horizontal
Janbu general (GPS) (2)	*(Yes)*	*(Yes)*	*(Yes)*	Yes	Yes	Yes	According to line of thrust assumed
Morgenstern-Price	*(Yes)*	*(Yes)*	*(Yes)*	Yes	Yes	Yes	Variable, depends on assumed distribution
Spencer	*(Yes)*	*(Yes)*	*(Yes)*	Yes	Yes	Yes	constant

(1) Global equilibrium of vertical forces is fulfilled, because it is fulfilled for each slice.
(2) The general method of slices by Janbu implies the use of a computer.
(3) Krey's method is similar to Bishop's; the factor of safety is mostly not calculated with iteration.

2.3 *Importance of interslice forces or internal stress state on stability*

The practical important effect of the interslice forces became apparent to the senior author (Steiner, 1985) analyzing an avalanche deflection dam, similar to Figure 1.

There had been a 10 meter high dam without berms in operation for several decades, when it had to be raised to 16 m. The analysis was not straightforward, a standard analysis as retaining wall was not satisfactory and manual analysis, with Bishop's and Janbu's method then used in practice gave factors of safety around one, i.e. the dam should be unstable. The stability of the dam was analyzed with the Morgenstern-Price method, assuming a step function as illustrated on the right side of Figure 1 to simulate the internal forces closer to reality.

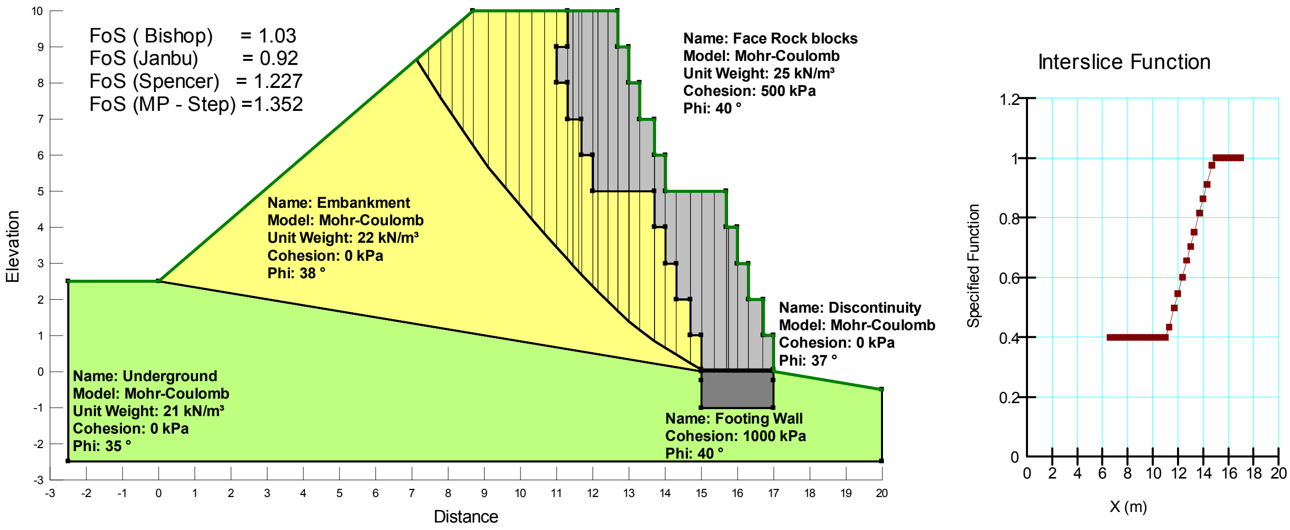

Figure 1. Analysis of a snow avalanche deflection dam with different method of slices: Section and results on left side; right side: interslice step function describing the inclination of the thrust line in Morgenstern Price method.

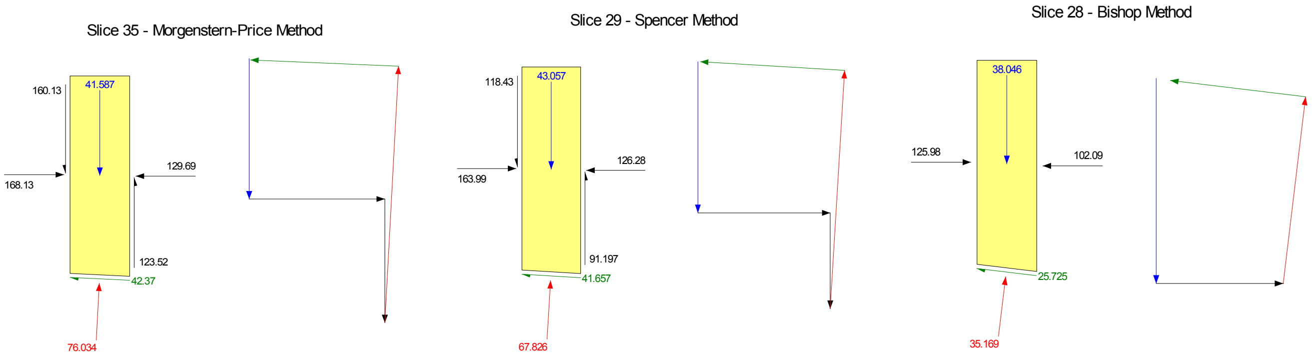

Figure 2. Comparison of force equilibrium in zone of transition of sliding surface from embankment into the block wall for Morgenstern-Price method with step function (left), Spencer's method (centre) and Bishop's method (right).

The value of the step function was assumed 1, as long as the slices cut through the block wall and 0.4 when the slices are completely located in the embankment fill and are linearly decreasing in between, with this the effect of the vertical component of earth pressure was simulated. The sliding surface must pass through the discontinuities between the horizontal rock blocks. The analysis with the Morgenstern-Price method yields a factor of safety FoS = 1.35, with Bishop's method FoS = 1.03 and with Janbu FoS= 0.92. With Spencer's method, equivalent to Morgenstern and Price with a constant function, gives a factor of safety FoS = 1.227. The forces acting o a slice at the transition of the sliding surface into the block wall are presented in Figure 2. The force polygons are very revealing: with Bishops method (right side of Figure 2) the normal force on the base of the slice is les than half of the force for the case with the step function and slightly more than half of Spencer' method. The force polygons show that more vertical and forces are considered in these analyses.

This example illustrates that the internal forces in a slope may play an important role in slope stability and must be considered. It is also evident that there is no unique solution for a safety factor as the internal stresses are influenced by the stress history of the ground. The deviation between the different assumptions of complete methods appears not as large, as between the simplified methods and complete methods.

2.4 *Estimation of the internal stresses*

The direct estimation of the internal stresses in the slope is empirical and requires some experience. In order to facilitate the estimate of the initial internal stresses Krahn (2007) proposed to run a Finite Element analysis of the slope first and then to introduce the obtained stresses in slope stability analyses and to run limit equilibrium analyses. This method is called the strength summation method (SSM) and has the advantage that there are less convergence problems as the factor of safety is directly determined.

2.5 *Application of the Finite Element method: Shear strength reduction method (SSR)*

With the finite element method deformations and the stress state in the ground can be modelled, depending on the accuracy of the constitutive models. The stability of slopes can be estimated by applying the Shear Strength Reduction method (SSR). With this method along slip surfaces the factor of safety is determined and the shear strength reduced (cohesion and friction angle) by the same factor. This procedure is carried out until a slip surface form; where the factor of safety with the reduced shear strength reaches one. The factor with which the shear strength is reduced is the factor of safety on shear strength. Problems may arise with convergence since the method approaches an instable condition in the analysis.

3 CASE STUDY OF A SLOPE WITH SIMPLE GEOMETRY

Considered is a slope of 5 m height and an inclination of $\beta = 40°$ without influence of water table (seepage) or external loads. The soil parameters are unit weight $\gamma = 20$ kN/m^3; friction angle: $\varphi_k' = 30°$ ($\varphi_d' = 25.7°$); cohesion intercept: $c_k' = 5$ kN/m^2 ($c_d' = 3.33$ kN/m^2). Stability calculations with the method of slices and limit equilibrium with the Morgenstern-Price method with constant inter-slice function for characteristic and design shear strength values. As a variant the strength summation method SSM, where the limit equilibrium method is combined with a stress distribution based on finite element stress-strain analysis has been used. The third method is shear strength reduction (SSR) method with finite element with characteristic soil parameters.

The results are shown in Figure 3 for the limit equilibrium analysis with the Morgenstern-Price method and the constant function, actually equivalent to Spencer’s method, on the left side the results obtained with characteristic values and the factor of safety on shear strength are shown, on the left side the sliding surface with design values, i.e. characteristic values reduced by partial factors. The results for the strength summation method are presented in Figure 4 and in Figure 5 the results for the analysis with the shear strength reduction and the finite element method. The obtained factors of safety on shear strength for calculations with characteristic shear strength and level of utilization μ are compiled in Table 2

Although for all cases the estimated FoS or μ are nearly equal, the detailed shape of the sliding surface deviates from one method of calculation to the other. From a general inspection one might conclude that the analysis with characteristic values and the determination of the factor of safety on shear strength does not deviate substantially from analyses carried out with design values, i.e characteristic values reduced by partial factors. Apparently for not to steep slopes with simple geometries or “text-book” slopes only small deviations between the different approaches are found.

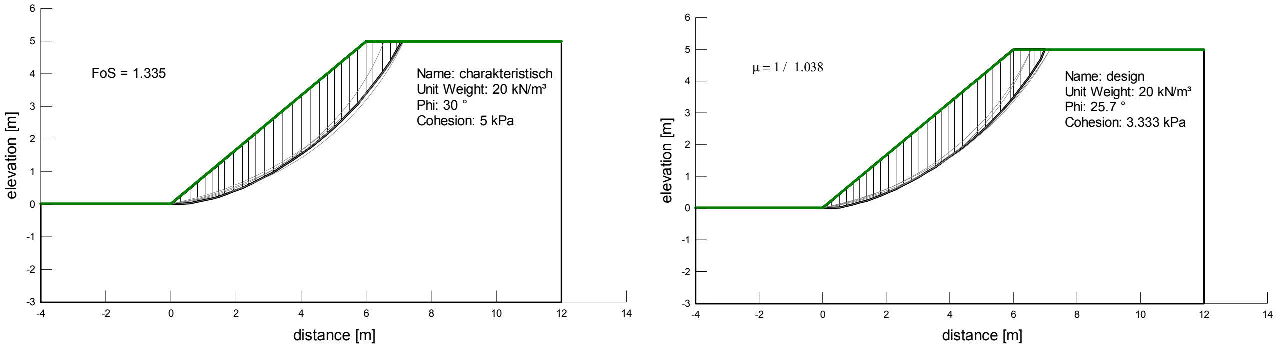

Figure 3. Sliding surfaces determined with limit equilibrium method (Morgenstern-Price with constant function) and factor of safety und shear strength and characteristic values (left) and with design values (partial factors).

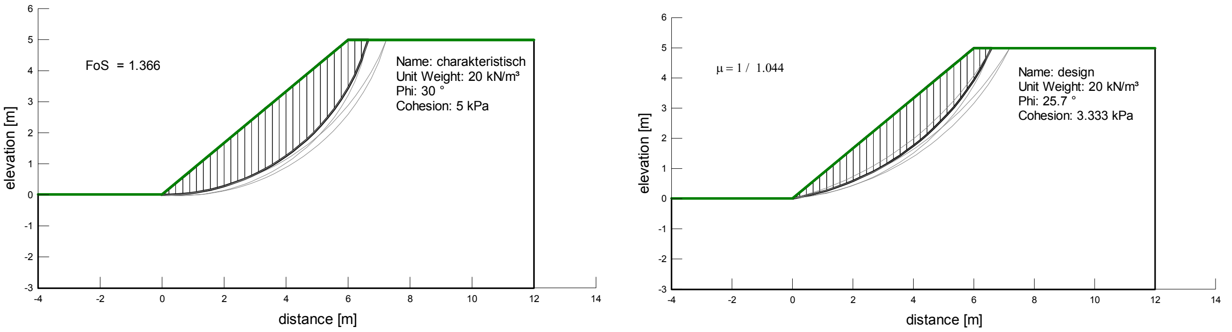

Figure 4. Comparison of sliding surfaces with strength summation method (SSM) and factor of safety on characteristic values (left side) and with design values and partial factors (right side)

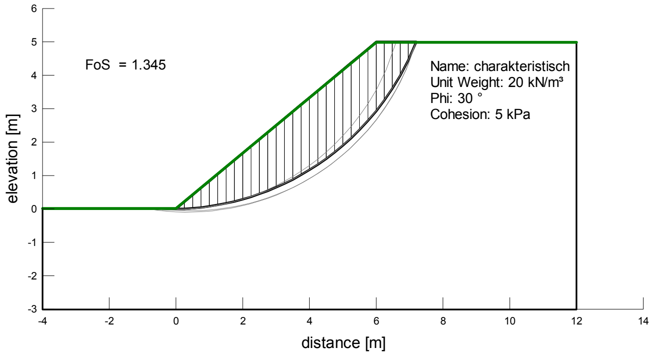

Figure 5. Critical slip surface determined with Finite Element method and shear strength reduction method (SSR) and characteristic values

Table 2. Results of stability calculation with different methods for the case study of a simple slope

Method	Parameter	Limit equilibrium (LE),	Strength summation (SSM)	Shear Strength reduction method (SSR)
Factor of safety on shear strength	Factor of safety on shear strength: FoS	1.335 Morgenstern-Price: constant 1.348 Bishop; 1.286 Janbu	1.366	1.345
Partial factors on strength	Level of utilization μ	0.964 Morgenstern-Price: constant 0.952 Bishop; 1.0163 Janbu	0.958	Not possible to determine

4 APPLICATION OF STABILITY ANALYSES IN PRACTICE

From their experience with real structures involving steeper slopes and heterogeneous foundations the authors have found that there are substantial differences between sliding surfaces determined with the factor of safety on shear strength and with design strength and level of utilization.

4.1 *Geogrid reinforced structure*

The 25 m high structure, reinforced with geo-grids shows substantially deviating critical sliding surfaces (Figure 6 left). The slip surface (Figure 1 right) obtained with design values does not appear plausible.

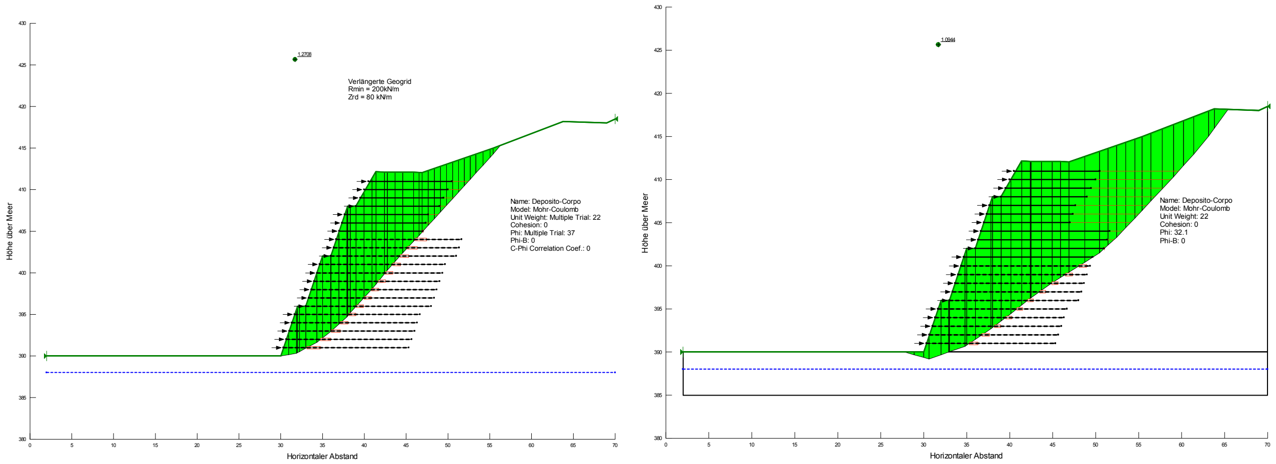

Figure 6. Comparison of Geo-grid reinforced wall in cohesionless soil (c’=o; Φ’= 37°) on left side designed with factor of safety on shear strength (former global factor of safety) F = 1.27 compared to design with partial factor of safety on right side (γ_Φ= 1.2) and Degree of Utilization $\mu = 0.91$.

4.2 *Fill deposit on soft clay deposit*

The geogrid structure is founded on a 15 to 20 m thick glaciolacustrine deposits (pink layer) overlain by about 10 m of sand and gravel.

The design with characteristic values and factor of safety on shear strength (Figure 7) give a critical slip surface through the reinforced slope whereas with the design values a much deeper reaching critical sliding surface (Figure 8) is obtained.

The sliding surface obtained by the determination of the minimal factor of safety on shear strength is more plausible. In this particular case the soft clay layer was modeled by undrained strength, which was reduced by the partial factor γ_c =1.5. Since the stress distribution plays an important role and is determined with consideration of the activated shear strength, this led to this unlikely sliding mass. The resulting critical sliding surface is the result of fictitious material properties obtained by reducing the real properties by a partial factor of safety. In the above particular case the observational method is used and from the analyses the true sliding surfaces should be known, as one would expect the largest displacement to occur close to this zone. For the design also finite element analyses were carried out with characteristic values, since for the evaluation of the stability the deformation occurring during construction have to estimated and then compared to measured displacements.

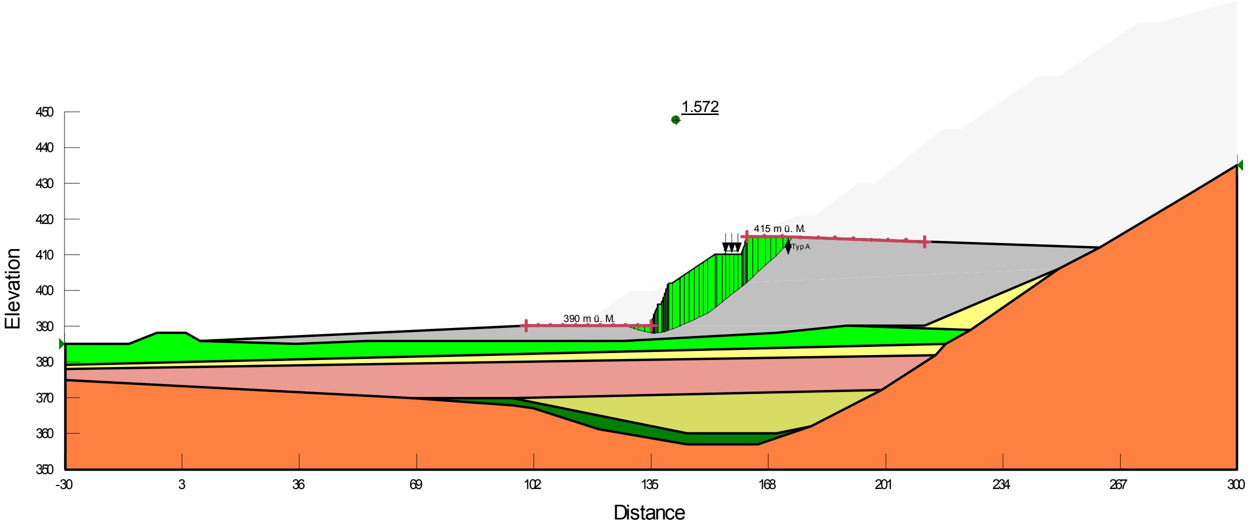

Figure 7. Stability Analysis of geogrid supported embankment on foundation with soft clay layer. FoS =1.572

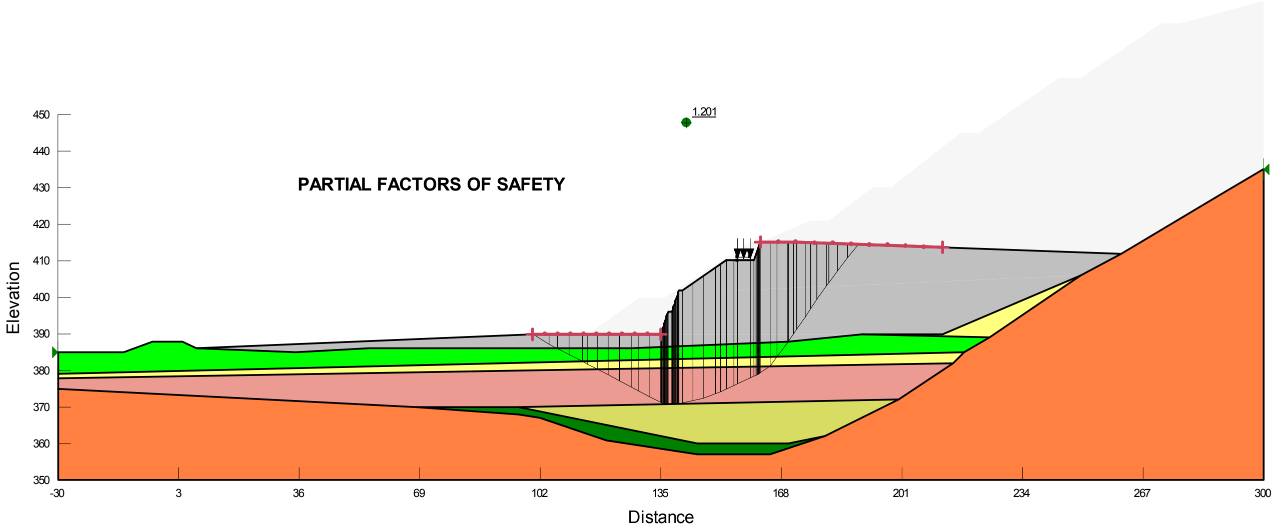

Figure 8. Critical slip surface determined with strength parameters reduced by partial factors for same geometry and geology as shown in Figure 3. Partial factor of on friction γ_Φ= 1.2 and cohesion γ_c =1.5. Degree of Utilization = 0.84.

4.3 *General shortcomings of the method with partial factors of safety*

For a method to be valid in practice it must work in all cases. One cannot proof with examples that the method is generally valid, however, one can proof with examples that the method has fundamental shortcomings. As shown earlier the stress state inside the sliding mass is important, as the initial state of stress depends on the geologic history of the ground. With partial factors of safety one assumes that strength parameters are independent and geometry (height) of the slope does not play a role. With partial factors an artificial material is created with little relation to reality and as consequence the analyses are performed with a fictitious equilibrium in the sliding mass.

5 PROBABILISTIC METHODS

The following practical example deals with a small slope that had to be excavated in a built-up area. The soil is a dense gravel and with an estimated friction angle (Φ'_k= 38°; Triangular distribution: 36; 38; 40°) and some difficult to estimate cohesion (c'_k = 15 kPa; Triangular distribution: 1, 15, 20 kPa). The corresponding design values are: Φ'_d= 32.8°; c'_d = 10 kPa. The analysis with Spencer's method (Figure 9) yields a factor of safety on shear strength FoS =1.535. With partial factors a level of utilization μ= 0.86 was obtained. One would judge with both methods that the excavation would be safe. The consideration of the distribution of the Factor of safety on shear strength (Figure 9right side) indicate a probability close to 2% that FoS < 1, i.e. there is a substantial risk that a failure could occur. For this reason we proposed to the owner to use a vertical wall supported with soil nail that eliminates this risk.

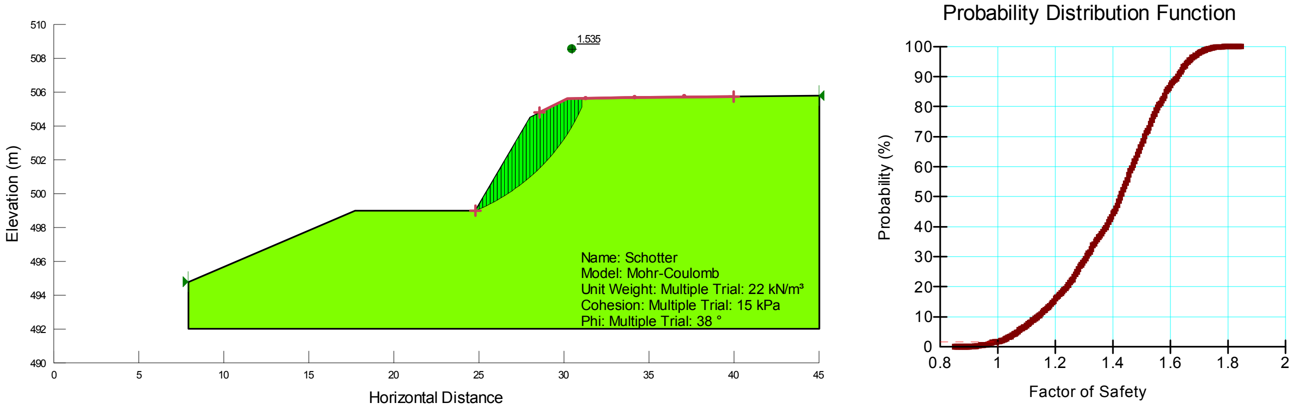

Figure 9. Analyses of slope in gravel with factor of safety on shear strength (FoS = 1.535), partial factors of safety (level of utilization μ= 0.86) and probabilistic method with $p_f \approx 2\%$

For this case an estimate of the strength parameters was used. Probabilistic methods allow one to judge the effects of measured dispersion of soil properties on safety (Steiner et al. 1992) and the risk and consequences of slope failures (Christian et al. 1994; Baecher & Christian, 2003). Probability and reliability approaches allow taking into consideration the spatial variability of the ground, although this may not be an easy task (El-Ramly et al., 2006). Slope stability programs (Krahn, 2004) can simulate the variation on a single slip surface; this may provide a better understanding of the effect of parameters involved. For evaluating the overall probability of failure the evaluation of many slip surfaces may be necessary (Cho, 2010). Silva et al. (2008) have proposed a framework for subjective assessment of slope stability.

6 CONCLUSIONS

Based on practical experience we conclude that the application of partial factors of safety as described in Eurocode 7 (EN 1997-1) and national codes (SIA 267) for slope stability do not provide a reliable tool for judging the safety of slopes. Our experience is from steeper slopes and embankment with heterogeneous conditions that are present in mountainous regions, like the Alps. In more complex cases slip surfaces obtained substantially deviate from the slip surfaces obtained with the factor of safety on shear strength. These slip surfaces appear not plausible.

The use of the Factor of safety on shear strength allows considering the effect of geometry, different materials and pore pressures in the ground (seepage) in stability analyses with a stress state in the ground corresponding to the real state. The effect of individual parameters or modification can more easily compared

With this approach also deformation computations necessary for comparing with measurements of displacements can be integrated. Predictions of displacements made with numerical methods can be compared to field measurements (observational method) and, if necessary, appropriate actions taken.

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