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# A Comparative Study of Pile Design Using Eurocode 7 and RBD$^E$

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ABSTRACT: A comparative study of pile design is presented using Eurocode 7 and an expanded reliability-based design (RBD$^E$) method that is recently developed by the authors. A design example that has been used in the literature to illustrate Eurocode 7 is re-designed using RBD$^E$. The RBD$^E$ method gives designs that are consistent with the designs from Eurocode 7 or correspond to the target failure probability ($p_T$) adopted in EN 1990. The RBD$^E$ method allows design engineers to adjust the design $p_T$ easily to accommodate the needs of a particular project without additional computational efforts. In addition, design engineers have the flexibility to make assumptions and/or simplifications deemed appropriate in designs. Such flexibility is illustrated by exploring the effect of different probability distributions of soil effective friction angle on design.

*Keywords: Pile, Eurocode 7, Reliability-Based Design, Monte Carlo Simulations*

## 1 INTRODUCTION

Reliability-based design (RBD) of foundations has attracted increasing interest over the last two decades, and several RBD methodologies have emerged, such as the partial factor design method in Eurocode 7, the load and resistance factor design (LRFD) method for highway structure foundations (Barker et al. 1991, Paikowsky et al. 2004, Paikowsky et al. 2010), and the Multiple Resistance Factor Design (MRFD) method for transmission line structure foundations (Phoon et al. 2003a&b). These RBD methods aim to provide designs with appropriate degrees of reliability, which is usually expressed in probabilistic terms, such as the target probability of failure $p_T = 7.2 \times 10^{-5}$ or target reliability index $\beta_T = 3.8$ adopted in EN 1990 (European Committee for Standardization 2002). Through some calibration process, Eurocode 7 provides tabulated partial factors for actions (i.e., loads), material properties, and resistances. Design engineers select appropriate partial factors from the table and carry out design calculations using a trial-and-error approach. The calibration of partial factors in Eurocode 7 has been primarily based on deterministic methods that calibrate to the long experience of traditional design with the aid of historical and empirical methods (Orr and Breysse 2008). As the numerical values of the partial factors are obtained from deterministic methods, it is of great interest to use full probabilistic methods (e.g., Monte Carlo Simulations (MCS)) to investigate the performance of these partial factors in achieving the desired degrees of reliability. In addition, the partial factors in Eurocode 7 aim for $p_T = 7.2 \times 10^{-5}$ only, partial factors for other $p_T$ values commonly are not available. This fact limits a designer's flexibility to adjust the $p_T$ to accommodate specific needs of a particular project.

To address these limitations, an expanded reliability-based design (RBD$^E$) method was recently developed that formulates the foundation design as an expanded reliability problem (Wang et al. 2011, Wang 2011). In this paper, a comparative study of pile design using Eurocode 7 and RBD$^E$ is described. After a brief introduction of Eurocode 7 and RBD$^E$, a pile foundation design example is described that has been used to evaluate Eurocode 7 in literature (Orr 2005a). Then, the design example is re-designed using RBD$^E$ and compared with the designs from Eurocode 7. In addition, the effect of the probability distributions of soil effective friction angle on designs is explored using RBD$^E$.

## 2 PARTIAL FACTOR DESIGN METHOD IN EUROCODE 7

Eurocode 7 contains three Design Approaches (i.e., DA1 with Combination 1 (C1) or 2 (C2), DA2, and DA3), and it aims to achieve that the probability of exceeding some limit states during a specified service period of the structures is smaller than the $p_T$ valued adopted. Consider, for example, the ultimate limit state (ULS), the $p_T = 7.2 \times 10^{-5}$ is adopted in EN 1990. For the ULS design of piles under axial compression, the design equation is given as (e.g., Orr 2005b):

$$F_{c,d} \le R_{c,d} \quad (1)$$

where $F_{c,d}$ is the design action (load) and $R_{c,d}$ is the design resistance of the pile. The design vertical action, $F_{c,d}$ is given as (e.g., Orr 2005b):

$$F_{c,d} = \gamma_G G_k + \gamma_Q Q_k \quad (2)$$

where $G_k$ is the characteristic permanent load, $Q_k$ is the characteristic variable load, $\gamma_G$ and $\gamma_Q$ are the relevant partial load factors. The values of $\gamma_G$ and $\gamma_Q$ are given in Eurocode 7 and depend on the Design Approach being used. The design compressive resistance of piles is given by (Orr 2005b):

$$R_{c,d} = R_{b,d} + R_{s,d} = R_{b,k}/\gamma_b + R_{s,k}/\gamma_s \quad (3)$$

where $R_{b,d}$ and $R_{s,d}$ are the design base and shaft resistances, $R_{b,k}$ and $R_{s,k}$ are the characteristic base and shaft resistances, and $\gamma_b$ and $\gamma_s$ are the relevant partial resistance factors.

## 3 EXPANDED RELIABILITY-BASED DESIGN ($RBD^E$) METHOD

An expanded reliability problem herein refers to a reliability analysis of a system in which a set of system design parameters are artificially considered as uncertain with probability distributions specified by the user for design exploration purposes (Wang et al. 2011, Wang 2011). For example, consider the pile with the pile length L as the design parameter. The design process is one of finding an L value that satisfies both the ULS and SLS requirements and achieve the design target $p_T$ or $\beta_T$. In the context of $RBD^E$, the L of the pile is considered as independent discrete random variables with uniformly distributed probability mass function p(L). The pile design process is re-formulated as a process of finding failure probabilities corresponding to designs with various L values [i.e., conditional probability p(Failure|L)] and comparing them with $p_T$. Failure refers to events in which the load exceeds resistance (i.e., F > R). Feasible designs are those with p(Failure|L) $\le p_T$. Note that the uniform probability mass function p(L) does not reflect the uncertainty in L, because L represents design decisions and no uncertainty is to be associated with it. Instead, it is used to yield desired information about p(Failure|L). Using Bayes' Theorem (e.g., Ang and Tang 2007), the conditional probability p(Failure|L) is given by:

$$p(\text{Failure}|L) = \frac{p(L|\text{Failure})}{p(L)} p(\text{Failure}) \quad (4)$$

in which p(L|Failure) = conditional probability of L given failure. Since L is independent discrete uniform random variables, p(L) in Equation (4) is expressed as:

$$p(L) = \frac{1}{n_L} \quad (5)$$

in which $n_L$ = number of possible discrete values for L. Using a single run of MCS, p(Failure) and p(L|Failure) can be estimated. Details of the $RBD^E$ and MCS are given by Wang et al. (2011) and Wang (2011).

$RBD^E$ can result in a large number of feasible designs. The requirement of the economic optimization limit state (EOLS) then is adopted to finalize the design as the one with the minimum construction cost (Wang and Kulhawy 2008, Wang 2009). The construction cost of pile is estimated using published, annually-updated, unit cost data, such as Means Building Construction Cost Data (Means 2007). The construction costs for all feasible designs are calculated as the product of their unit costs and pile lengths, and the final design is determined accordingly by comparing their construction costs.

## 4 PILE FOUNDATION DESIGN EXAMPLE

Orr (2005a&b) illustrated Eurocode 7 using a bored pile design example shown in Figure 1. The bored pile with a diameter B = 0.6 m is installed in sand with a total unit weight $\gamma$ = 21 kN/m$^3$, a characteristic effective friction angle $\phi'_k$ = 35° (effective cohesion c' = 0), and SPT-N = 25. Groundwater level is at a depth of 2 m below the ground surface. The pile is designed to support an axial compression load with a characteristic permanent load $G_k$ = 1200 kN and a characteristic variable load $Q_k$ = 200 kN. The unit weight of concrete is 24 kN/m$^3$. The only design parameter is pile length L.

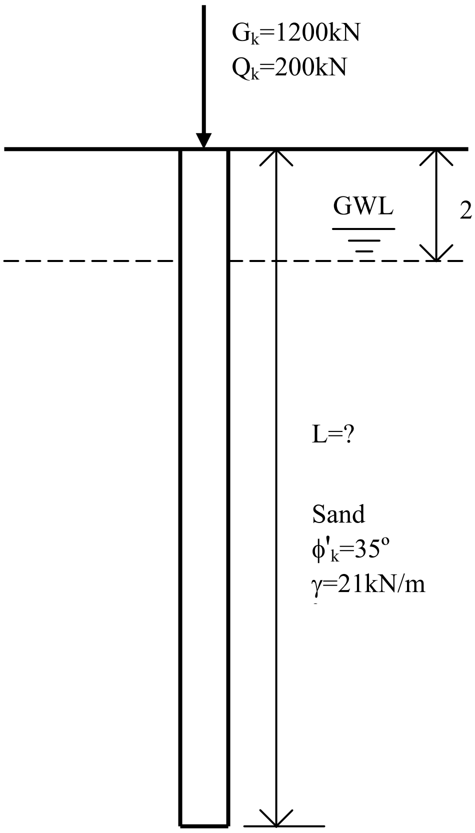


Figure 1. Pile foundation example (after Orr 2005a).

Table 1. Partial factors for the pile example

| Design Approach | Permanent Load Factor, $\gamma_G$ | Variable Load Factor, $\gamma_Q$ | Base Resistance Factor, $\gamma_b$ | Shaft Resistance Factor, $\gamma_s$ | Model Factor, $\gamma_R$ | Material Factor, $\gamma_M$ |
|---|---|---|---|---|---|---|
| DA1, C1 | 1.35 | 1.5 | 1.25 | 1 | 1.5 | — |
| DA1, C2 | 1 | 1.3 | 1.6 | 1.3 | 1.5 | — |
| DA2 | 1.35 | 1.5 | 1.1 | 1.1 | 1.5 | — |
| DA3 | 1.35 | 1.5 | — | — | — | 1.25 |

Table 2. Summary of pile designs using Eurocode 7

| Design Approach | Pile Length, L (m) | Overall Factor of Safety, OFS | Probability of Failure, p(Failure) |
|---|---|---|---|
| DA1. C1 | 14.9 | 2.4 | $1.9 \times 10^{-3}$ |
| DA1. C2 | 14.6 | 2.4 | $2.6 \times 10^{-3}$ |
| DA2 | 14.0 | 2.3 | $4.4 \times 10^{-3}$ |
| DA3 | 16.7 | 2.7 | $2.4 \times 10^{-4}$ |

Using three DAs of Eurocode 7, Orr (2005b) provided a set of model solutions to this design example. The characteristic resistances are obtained as $R_{b,k} = A_b q_{bk}$ and $R_{s,k} = A_s q_{sk}$, where $q_{bk}$ and $q_{sk}$ are the characteristic base resistance and shaft friction obtained from the soil parameters and $A_b$ and $A_s$ are the areas of the pile base and pile shaft. Table 1 summarizes the partial factors used for different DAs in this example. The values of the partial factors $\gamma_b$ and $\gamma_s$ are corrected by a model factor, $\gamma_R$ = 1.5. The design resistance for DA3 is obtained by applying the partial material factor $\gamma_M$ = 1.25 to $\tan\phi'_k$ to obtain $\phi'_d$ which is used to obtain the design resistances. The base bearing resistance is given by $q_{bk} = \sigma_{v0}' N_q$, where $\sigma_{v0}'$ is the vertical effective stress at the pile base and $N_q$ is a bearing capacity factor estimated using $N_q$ versus $\phi'$ relationship proposed by Berezantzev et al. (1961). The shaft resistance is given by $q_{sk} = 0.5 (1-\sin\phi') \sigma_{v0}' \tan\phi'$.

Table 2 summarizes the L values obtained from three DAs. The L values vary from 14.0 m to 16.7 m, and the overall factors of safety, OFS (defined as $R_{c,k}/F_k$), vary from 2.3 to 2.7. This design example is re-designed in the next section using the RBD$^E$ method (Wang et al. 2011, Wang 2011).

## 5 RBD$^E$ DESIGN

The RBD$^E$ approach conceptually contains four basic steps: (1) establish deterministic calculation models, (2) model geotechnical-related uncertainties, (3) perform MCS and identify a pool of feasible designs, and (4) select the final design based on economic evaluation. To enable a consistent comparison with the design by Orr (2005b), the deterministic ULS calculation models in this section follow those adopted in the previous section. In addition, the permanent load G is treated as constant and equal to the characteristic permanent load $G_k$ (i.e. G=1200kN). The total unit weight $\gamma = 21$ kN/m$^3$ of soil is also taken as deterministic.

### 5.1 *Uncertainty Modeling*

Uncertainties in design loads and material properties in Eurocode 7 are reflected through their respective characteristic values. The uncertain variables in this design example include the variable load, effective friction angle of soil, and length of pile, as shown in Table 3. The characteristic value for a design load in EN 1990 is defined as the load magnitude that corresponds to 5% or 2% probability of exceedance (i.e., an upper 95% or 98% fractile of its probability distribution) (European Committee for Standardization 2002). The variable load Q is considered as a lognormal random variable with a coefficient of variation $COV_Q = 0.5$, and its characteristic value $Q_k$ is taken as the upper 95% fractile of the probability distribution. Then, the mean value (i.e., $Q_m$) of variable load is calculated as:

$$Q_m = \frac{Q_k}{\left(1+1.645COV_Q\right)} \qquad (6)$$

Using $Q_k = 200$ kN (Orr 2005a&b) and $COV_Q = 0.5$, $Q_m$ is estimated as 110 kN.

The effective friction angle $\phi'$ of soil is considered as a lognormal random variable with a coefficient of variation $COV_{\phi'} = 0.1$. The mean value (i.e., $\phi'_m$) of effective friction angle of soil is calculated as (Schneider 1997):

$$\phi'_m = \frac{\phi'_k}{\left(1-0.5COV_{\phi'}\right)} \qquad (7)$$

Using $\phi'_k = 35^o$ (Orr 2005a&b) and $COV_{\phi'} = 0.1$, $\phi'_m$ is estimated as $36.84^o$.

In addition, the pile design parameter L is treated as independent discrete uniform random variable. The possible L values vary from 12 m to 21 m with an increment of 0.3 m.

Table 3. Uncertain modeling in RBD$^E$ design

| Variables | Variable Load, Q | | Effective friction angle of soil, $\phi'$ | | | | Pile Length, L | | |
|---|---|---|---|---|---|---|---|---|---|
| | Mean | COV* | Min* | Max* | Mean | COV* | Min* | Max* | Interval |
| Values | 110 kN | 0.5 | 24$^o$ | 40$^o$ | 36.84$^o$ | 0.1 | 12 m | 21 m | 0.3 m |
| Distribution Type | Lognormal Distribution | | Lognormal Distribution | | | | Discrete Uniform Distribution | | |

* COV = Coefficient of Variation, Min = Minimum, Max = Maximum

### 5.2 *Monte Carlo Simulation (MCS)*

MCS is performed using the software package Matlab (Mathworks 2010), which is equipped with random number generators for various probability distributions, such as “lognrnd” for lognormal variables, “normrnd” for normal variables, and “rand” for uniform variables. Random samples of the lognormally distributed variable load and effective friction angle of soil are generated by “lognrnd” with their respective means and standard deviations. Because the $N_q$ versus ϕ' relationship proposed by Berezantzev et al. (1961) is only applicable for the ϕ' values between 24° and 40°, the ϕ' values are taken as 24° and 40°, respectively, when the ϕ' values generated from the random number generator are smaller than 24° or larger than 40°. Random samples of uniformly distributed L are generated by "rand". For each set of random samples, the loads and resistances of the pile are calculated. The pile is considered "failed" when the sum of the permanent and variable loads exceeds the bearing resistance. A single run of MCS with a sample size of 9,000,000 is performed for $RBD^E$, and the p(L|Failure), p(Failure), and conditional failure probability p(Failure|L) are estimated from MCS using Equation (4) accordingly.

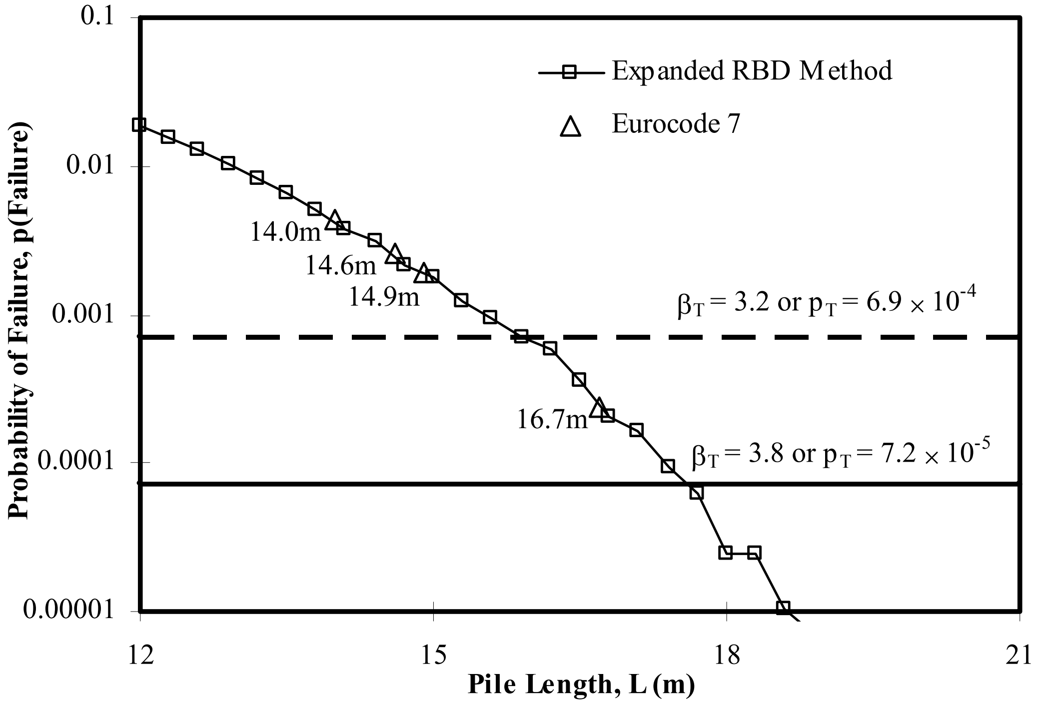


Figure 2. Conditional probability of failure from Monte Carlo Simulation.

## 5.3 *Results*

Figure 2 shows the conditional failure probability p(Failure|L) obtained from a single run of MCS. Note that p(Failure|L) is a variation of failure probability as a function of L. Failure probability p(Failure) decreases as L increases. Figure 2 also includes the $p_T = 7.2 \times 10^{-5}$ adopted in EN 1990, and feasible designs are those that fall below the $p_T$, as shown in the figure. In this example, the feasible designs are the piles with L ≥ 17.7 m. The economic requirement then is adopted to determine the final design (Wang and Kulhawy 2008, Wang 2009). Since the construction cost is the product of pile depths and unit costs, the economic design for a given value of B is the one with the minimum L value. Therefore, among the pool of feasible designs obtained from $RBD^E$, the final design is a pile with L = 17.7 m.

# 6 RESULT COMPARISONS

Figure 2 also includes the failure probability p(Failure) for the design pile length obtained from Eurocode 7 (see Table 2). Each p(Failure) is obtained by performing a run of MCS with a sample size of 1,000,000, and the exact values of p(Failure) are summarized in Table 2.

These p(Failure) values vary from $4.4 \times 10^{-3}$ to $2.4 \times 10^{-4}$, and they follow the p(Failure) versus L curve obtained from the $RBD^E$ method. They are all however significantly larger than the $p_T = 7.2 \times 10^{-5}$ adopted in EN 1990. This implies that using the partial factors recommended in Eurocode 7 does not guarantee automatic fulfillment of its target reliability. It is interesting to note that, however, these p(Failure) values are consistent with the empirical foundation failure rates of about $10^{-2}$ to $10^{-3}$ (Baecher 1987).

In addition, it is worth noting that, in $RBD^E$, feasible designs for different $p_T$ values are inferred directly from Figure 2 without additional computational efforts, which allows designers to adjust easily the design $p_T$ to accommodate the needs of a particular project. To illustrate such flexibility, Figure 2 also includes the $p_T = 6.9 \times 10^{-4}$ (i.e., $\beta_T = 3.2$) that have been adopted in the reliability – based designs of foundations for transmission line structures in North America (Phoon et al. 2003a&b). The corresponding design is a pile with L = 16.2 m, which falls among the range of pile length obtained from Eurocode 7.

## 7 EFFECT OF PROBABILITY DISTRIBUTIONS OF SOIL EFFECTIVE FRICTION ANGLE

The $RBD^E$ method allows designers to make assumptions and/or simplifications deemed appropriate in designs. This section illustrates this flexibility by using different probability distributions of soil effective friction angle $\phi'$ in the design and exploring its effect. The $\phi'$ is considered as a normal random variable with the same mean value (i.e., $\phi'_m = 36.84^o$) and coefficient of variation (i.e., $COV_{\phi'} = 0.1$). Random samples of the normally distributed effective friction angle of soil are generated by the Matlab function "normrnd" with its mean and standard deviation. The $\phi'$ values are taken as 24° and 40°, respectively, when the $\phi'$ values generated from the random number generator are smaller than 24° or larger than 40°. A single run of MCS with a sample size of 9,000,000 is performed for $RBD^E$, and the results are shown in Figure 3.

Figure 3 compares the results for normal distribution with those for lognormal distribution. The p(failure) values both decreases significantly as the pile length increases. The p(failure) versus L relationship for the normal distribution moves towards the upper right corner of the plot, indicating a significant increase of failure probability for the same L value or a significant increase of design L value for the same $p_T$ value. For the $p_T = 7.2 \times 10^{-5}$ adopted in EN 1990, the feasible designs are the pile with $L \geq 18.9$ m, and therefore, the final design is the pile with L = 18.9. For $p_T = 6.9 \times 10^{-4}$ (i.e. $\beta = 3.2$), the final design pile length is 17.4 m. These results indicate that probability distribution types have significant effect on the design.

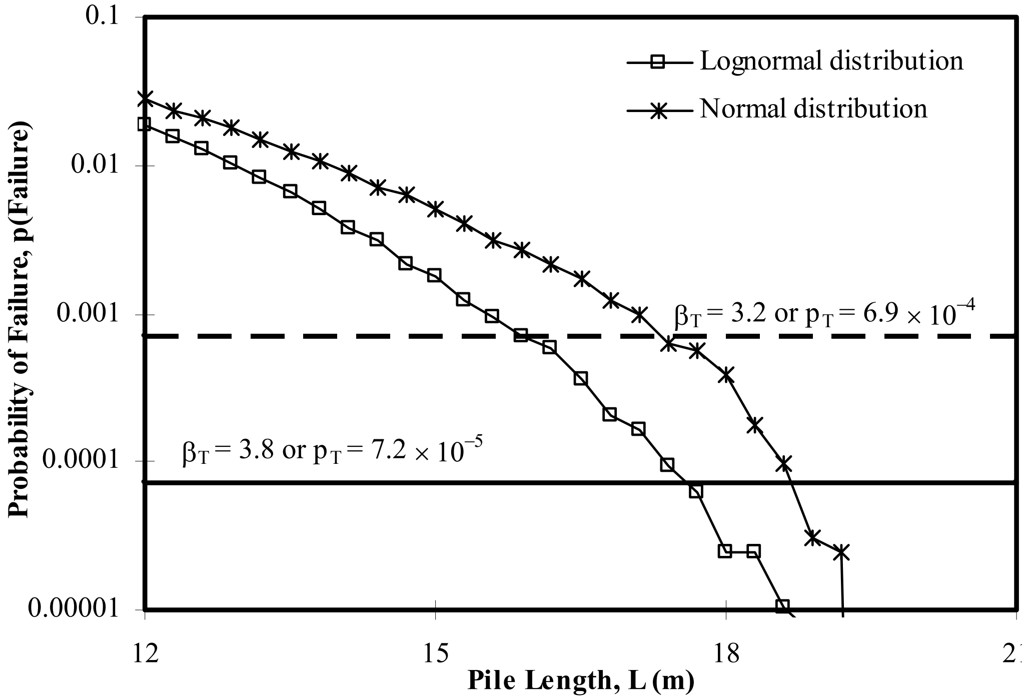


Figure 3. Effect of probability distributions of soil effective friction angle.

## 8 SUMMARY AND CONCLUSIONS

A comparative study was presented for pile design using Eurocode 7 and $RBD^E$. An example that was used to illustrate Eurocode 7 was re-designed using $RBD^E$. The $RBD^E$ method gives designs that are consistent with the designs from Eurocode 7 or correspond to the target failure probability $p_T$ adopted in EN 1990. It is also found that, using the partial factors recommended in Eurocode 7 does not guarantee automatic fulfillment of its target reliability, although the resulting failure probabilities are consistent with the empirical rates of foundation failure. In addition, it is worth noting that, in $RBD^E$, feasible designs for different $p_T$ values are inferred directly without additional computational efforts, which allows designers to adjust easily the design $p_T$ to accommodate the needs of a particular project. The $RBD^E$ method also gives designers the flexibility to make assumptions and/or simplifications deemed appropriate in designs. This flexibility was illustrated by using different probability distributions for soil effective friction angle in the design and exploring its effect. It was found that probability distribution types have significant effect on the design.

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