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Performance Based Design and Eurocode

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ABSTRACT: Along with the introduction of the Eurocodes, an extensive and continuing discussion process regarding the implemented safety concept has been triggered in the engineering community. Typically, these discussions soon tend to focus on technical aspects of the suggested design *procedure* – and, unfortunately, obscure the view of underlying essentials of the Eurocode safety *concept*.

This paper advocates a holistic perception of the structural design process. By shortly reflecting the design process, it aims to identify common characteristics of 'good' structural design: A common postulate is always the definition of, or the agreement upon, specific structural requirements that conform to the intended function and life-time of the structure – *performance criteria.*

Then, what are suitable engineering tools, tools that support the development of a qualified design? At this point, the paper attempts to contribute to some demystification of reliability based design. Not least by practical geotechnical case studies, reliability based design is motivated as an engineering tool which adds value to engineering decision making and finally helps to contribute to a 'good' – performance based – structural design.

Finally, what is to say about the normative background? Are the Eurocode concept and the notion of performance based design extremes? To shed some light on these relevant, practically motivated questions, the EN 1990 is critically reviewed and discussed.

Keywords: Structural Design Process, Eurocode Design Concept, Reliability Analysis, Performance Based Design

1 STRUCTURAL DESIGN

1.1 *'Good' design*

What is the essence of 'good' structural design? To attempt a cautious answer to this question, an exemplary design case is considered.

The 'tree swing project' nicely illustrates possible pitfalls over the project planning phase by means of a rather simple structure, a tree swing (Figure 1). The accentuated representation reveals the problematic nature of non-integral thinking: The client orders a tree swing. The structural engineer then succeeds to come up with a design that provides for sufficient structural resistance. Good structural design? In this fictitious case, the assessment seems clear; in fact however, the question cannot be answered satisfactorily, without considering the envisioned service of the structure.

Scenario A) (unlikely): The tree swing is meant as an artwork, no specific use is assigned. In this case, the design might meet the requirements.

Scenario B) (more likely): The tree swing is meant as a gift for the client's little son. With this background the assessment differs. The design indeed ensures the required safety standards; however, it fails to meet the intended function of the structure. During the design process, a necessary constraint, sufficient structural resistance, has become an end in itself and finally the primary goal of the design.

The disappointment of scenario B) could have been avoided, if client and planners had agreed upon specific *performance criteria* a priori – structural requirements that conform to the intended function of the structure. Moreover, if client and planners had commonly tried to identify the desired performance, there might have been opportunity to realize, that the solution that meets the client's needs best, is a tire swing – rather than a board swing.

a) What the client had in mind ` b) How it was designed c) What the client really needed

Figure 1. Variants of a tree swing [http://www.projectcartoon.com/].

1.2 *Evolution of structural design concepts*

For a long time, the structural design process was almost entirely based on empirical knowledge which had primarily been gained by trial and error. As a consequence, structures were repetitive and increases in scale were incremental. Along with the evolution of the theory of structures, material science and computational possibilities, the first design concepts were established. In the beginning of this process, simple instructions and guidelines documented the state of knowledge. Successively, with origin in the early 20th century, a comprehensive system of technical directives and codes was developed. The underlying safety concept was and is continuously adapted to the attained practical and experimental experience, the increased theoretical knowledge and – not least – the computational possibilities.

Currently adopted design concepts can be classified into deterministic, semi-probabilistic and full-probabilistic approaches. Differences can readily be contrasted, when considering the design goal 'structural safety' (ultimate limit state). It is important to realize, that the common background of all approaches is the control of the vast amount of inherent uncertainties. These uncertainties, to name a few, are induced by variation of material properties, by the limited predictability of loading or by the construction process itself. Hence, the objective can be identified as limiting the probability of failure to an acceptable level.

The deterministic approach adopts the simplest safety definition

$$R/\gamma \geq S \tag{1}$$

where R denotes the resistance, S is the effect of the action and γ is an empirically determined global safety factor that is meant to incorporate all uncertainties (Figure 2a).

A more refined version is represented by partial safety factors:

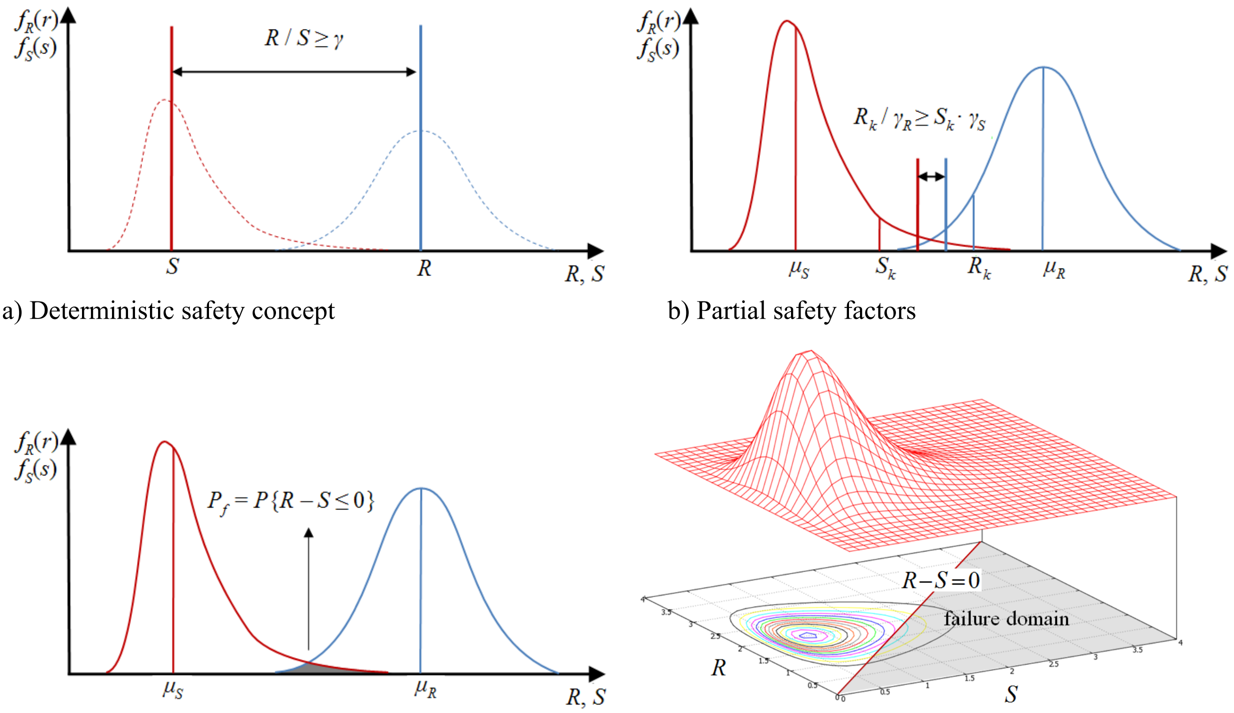
$$R_k/\gamma_R \geq S_k \cdot \gamma_S \tag{2}$$

In this case, uncertainties on action and resistance side are accounted for, separately. They are reflected by the load factor γ_S and the resistance factor γ_R (Figure 2b). Computation remains purely deterministic but since the partial safety factors are calibrated to match a predefined probability of failure level P_d (=acceptable probability of failure), the approach is classified as semi-probabilistic [Marek et al. (1998), Honjo et al. (2009)].

The full probabilistic approach finally abandons the definition of (partial) safety factors; it simply imposes the constraint on the probability of failure P_f directly (Figures 2c-d):

$$P_f = P\{R - S \leq 0\} < P_d \tag{3}$$

This most general approach ultimately leaves the domain of deterministic calculation; some background in reliability theory is required. In return, it is consistent and general-purpose. In particular, it does not rely on the differentiation between action and resistance – an important aspect, when considering, e.g., soil-structure interaction scenarios. As will be illustrated further, the concept can be employed for serviceability assessment in a straight-forward manner; in fact, *any criterion* that is considered relevant for successful *performance* of the structure can be adopted. Last but not least, this direct approach inherently respects the design philosophy that *a designer should keep track of the most likely behavior of the structure throughout the design calculations as much as possible* [cf. Honjo et al. (2009)] – which is of particular relevance in nonlinear settings.



a) Deterministic safety concept
b) Partial safety factors
c) Probabilistic safety concept
d) Probabilistic safety concept – alternative illustration

Figure 2. Illustration of safety concepts.

2 RELIABILITY ASSESSMENT

In many cases, the definition of the probability of failure given in Eq. (3) is not sufficient, since it may not be possible to reduce the design condition to a simple R versus S relation. In a more general context, there may be many different parameters that are expected to present an uncertain behavior. Typical examples are dimensions, loads, material properties as well as any other variable that is employed in structural analysis and design procedures.

The corresponding generalized reliability concept features two basic ingredients.

In a first step, the uncertain parameters are modeled by a vector of basic random variables $\mathbf{X} = [X_1, X_2, \ldots, X_n]^T$ with a joint probability density function (PDF) $f_{\mathbf{X}}(\mathbf{x})$. The joint PDF is usually approximated using marginal distributions and correlations [e.g. see Der Kiureghian and Liu (1986)], which are estimated based on observed data and engineering judgment. The joint PDF for the two variable-case $\mathbf{X} = [R, S]^T$ is shown in Figure 2d.

The second component is concerned with the intended function of the structure, i.e. the design goals. Criteria that are considered as relevant for a successful performance of the structure are expressed by means of a corresponding *performance function* $g(.)$, defined in terms of the vector $\mathbf{X}$. The performance function $g(\mathbf{X})$ is defined by convention, such that failure of successful performance occurs when $g(\mathbf{X}) \leq 0$. Note that a more general definition of the performance function can include several performance criteria, leading to the parallel- or series-system reliability problem. The probability of failure is computed by:

$$P_f = P\{g(\mathbf{X}) \leq 0\} = \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\,\mathbf{x} \qquad (4)$$

The notion of *performance criteria* overcomes the constrictive character of Eq. (3) and leads to a general definition of the probability of failure. The strength of this approach lies in the fact that the performance criteria can be tailored individually to the specific requirements of the structure to be designed. However, it is important to note the trivial result that, for the special case where $\mathbf{X} = [R, S]^T$ and $g(\mathbf{X}) = R - S$, Eq. (3) is recovered (Figure 2d).

A corresponding measure of safety may be chosen as any decreasing function of P_f. The standard reliability measure is the generalized reliability index [Ditlevsen and Madsen (1996)], defined as:

$$\beta = -\Phi^{-1}\left(P_f\right) \qquad (5)$$

where $\Phi^{-1}(.)$ is the inverse of the standard normal cumulative distribution function.

The difficulty in the evaluation of the integral in Eq. (4) lies in the fact that the performance function is usually not known in an explicit form, but depends on the outcome of a structural analysis calculation for any given realization of the random variables. Since the computational time for the underlying structural analysis may be considerable, methods that aim at limiting the required number of structural computations have been developed. A series of reliability methods have been coupled to the SOFiSTiK structural finite element analysis and design program [Papaioannou et al. (2009)], in an attempt to achieve a balance between accuracy and efficiency for a variety of possible performance criteria. These include first order reliability methods as well as a number of simulation techniques. In addition, the methods may be combined with a response surface approximation of the performance function to further enhance the efficiency of the reliability assessment. In the following section, the SOFiSTiK reliability module is employed for the reliability-based design of a geotechnical application.

3 CASE STUDY

The considered design case study is a deep excavation with a vertical retaining wall and a horizontal supporting strut (Figure 3). The soil consists of two layers of clay. A conservative estimation of the soil strength parameters is given in intervals, as shown in Table 1. The design goal is the determination of the wall thickness against a serviceability requirement.

The soil is modeled in 2D with elasto-plastic plane strain finite elements and the soil's shear strength is described by the Mohr-Coulomb criterion. For the wall, beam finite elements are adopted, while the interface between wall and soil is resolved by special elements featuring a reduced shear capacity compared to the soil base material. A horizontal spring is used to model the supporting strut. The computational model is depicted in Figure 4a.

A displacement-based deterministic design for the wall thickness is first performed considering the lowest values of the strength parameters given in Table 1. For serviceability reasons, the design is targeted at a restriction of the maximum inward wall displacement at the bottom of the trench to a threshold value of $u_{\max}$ = 3.5cm. Apart from accounting for the lower bound soil strength only, no further safety margins are incorporated in this design. The design process yielded a wall thickness of 1.0m. Figure 4b shows the deformed configuration for the deterministic design of the wall thickness at the final excavation step.

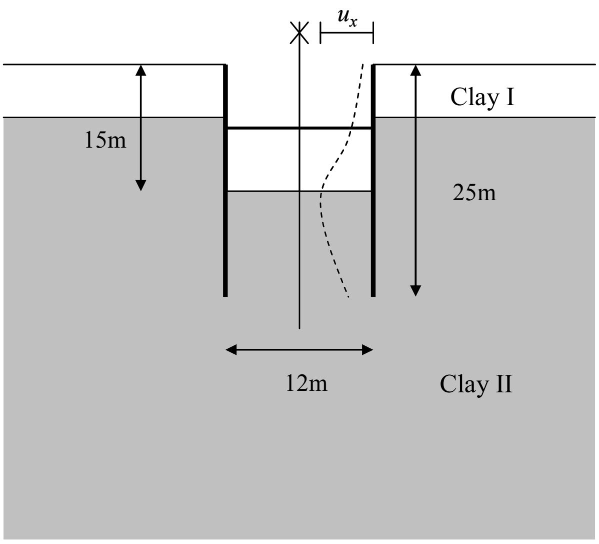
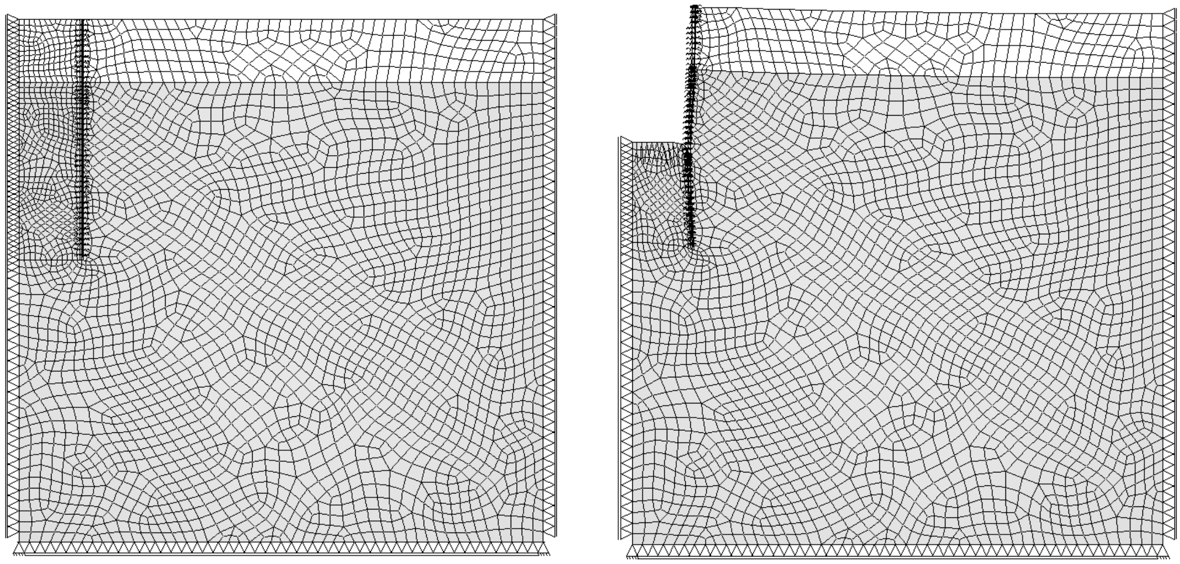


Figure 3. Deep excavation with retaining wall.

Table 1. Estimated strength parameters of the soil materials

Parameter	Clay I	Clay II
Friction angle φ [º]	12-22	18-28
Cohesion c [kPa]	5-10	15-25



a) Finite element mesh b) Deformations at the final excavation step magnified by a factor of 20

Figure 4. Numerical model.

In a second step, a probabilistic design is performed, accounting for randomness in the strength parameters of the soil. Two cases are considered. In the first investigation (Case I), the conservative Uniform distribution is employed, assuming the same probability density for any value of the strength parameters in the given intervals (Table 2). For the second case (Case II), the Beta distribution is used, taking as mean values the middle points in the intervals and allowing for a 10% coefficient of variation (COV) inside the intervals (Table 3). The Beta distribution includes lower and upper bounds, but distributes the probability content according to the second order properties (mean and variance). This means that we assume additional information than the given estimation in Table 1, which reduces the uncertainty in the model. Therefore, Case II should lead to a less conservative estimation of the reliability. However, it should be noted that the Beta distribution is a much more realistic assumption for the description of physical quantities compared to the Uniform distribution. For both cases, the typical value of –0.3 is chosen for the correlation coefficient between the strength parameters of the same materials.

A displacement-based performance function is adopted, restricting the horizontal wall displacement at the bottom of the trench:

$$g(\mathbf{X}) = u_{\max} - u_x(\mathbf{X}) \quad (6)$$

For this, $u_{\max}$ = 3.5cm is chosen again, which represents the maximum displacement of the deterministic design.

Table 2. Random variables of the soil strength properties (Case I)

Parameter	Distribution	Min	Max
Friction angle (Clay I) φ_{I} [º]	Uniform	12	22
Cohesion (Clay I) c_{I} [kPa]	Uniform	5	10
Friction angle (Clay II) φ_{II} [º]	Uniform	18	28
Cohesion (Clay II) c_{II} [kPa]	Uniform	15	25

Table 3. Random variables of the soil strength properties (Case II)

Parameter	Distribution	Mean	COV	Min	Max
Friction angle (Clay I) φ_{I} [º]	Beta	17	0.1	12	22
Cohesion (Clay I) c_{I} [kPa]	Beta	7.5	0.1	5	10
Friction angle (Clay II) φ_{II} [º]	Beta	23	0.1	18	28
Cohesion (Clay II) c_{II} [kPa]	Beta	20	0.1	15	25

A series of reliability analyses were carried out for different values of the wall thickness applying the first order reliability method (FORM) combined with a quadratic response surface. The FORM is an approximation method, which computes the probability of failure by performing a first order approximation of the failure domain at the most probable failure point (MPFP) of an independent standard normal space derived from an isoprobabilistic transformation of the basic random variables. The MPFP is computed by solving an equality-constrained quadratic optimization problem [Der Kiureghian (2005)]. The quadratic response surface is computed applying a central composite design at the basic random variable space [Faravelli (1989)]. The reliability indices computed for the different values of the wall thickness are shown in Figure 5.

For the assumed scenario of a serviceability-driven design, the EN 1990 directive quantifies the target serviceability failure probability to a value of 2×10^{-3} (Table 4), which corresponds to a reliability index of 2.9 [CEN (2002)]. Based on this condition, the probabilistic design is computed by interpolating between the combinations of wall thicknesses and calculated reliability indices (Figure 5). The design results in a wall thickness of 0.92m for the conservative probabilistic scenario (Case I), which considerably reduces the thickness compared to the deterministic design (1m). Moreover, in the case where more data is available to support the assumption of a more realistic probability distribution (Case II), the wall thickness design may further decrease to 0.88m.

With this result, a possible benefit of using a fully probabilistic design approach is imposingly highlighted. The findings illustrate the potential of a direct account of parameter uncertainties to realize a significantly more economical design; note that this can be achieved without eroding the targeted safety level.

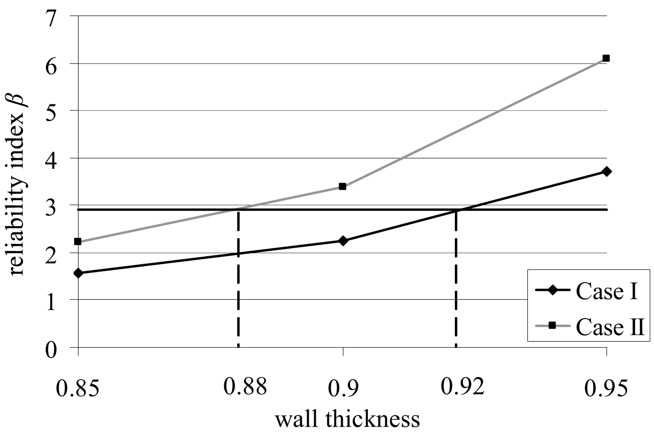


Figure 5. Reliability index vs. wall thickness.

Table 4. Target reliability index β and corresponding probability of failure P_f values (EN 1990)

Limit state	Target reliability index β		Target probability of failure P_f	
	1 year	50 years	1 year	50 years
ULS	4.7	3.8	1×10^{-6}	7.2×10^{-5}
SLS	2.9	1.5	2×10^{-3}	6.7×10^{-2}

4 PERFORMANCE BASED DESIGN AND EUROCODE

In the previous section, an application of reliability-based design based on a tailored performance criterion was presented; reliability levels were adopted in line with the Eurocode design directive. The applied design concept, termed here performance-based design, is defined as follows: *A structure shall be designed in such a way that it will with appropriate degrees of reliability and in an economical way attain the required performance*. In this definition, we attempt a generalization of different aspects included in the Eurocode design concept. The generalization is based on the central notion of *required performance.* This notion may include any subjective requirement that is considered critical for the successful performance of a specific structure. The following question arises: How does this definition compare to the Eurocode?

The Eurocode design concept is based on two fundamental requirements for the reliability of structures; structural safety and serviceability [CEN (2002)]. Appropriate design criteria and diversified reliability levels are reflected by the provision of corresponding limit states, the ultimate and serviceability limit states, respectively. The ultimate limit state includes failure conditions due to a number of reasons, such as loss of equilibrium, excessive deformations, loss of stability and fatigue. The serviceability limit state is concerned with the functioning of the structure under normal use as well as the comfort of people and the appearance of construction works. In addition, it is noted that *usually the serviceability requirements are agreed for each individual project* [cf. CEN (2002)]. These two categories of criteria require different minimum reliability levels (see Table 4). The reliability of the design should be verified against all relevant limit states, which will depend on the specific characteristics of each individual structure. We assert that a careful choice of these relevant criteria will lead to a successful *performance* of the structure. In fact, the above devised definition of performance based design is only a slight variation of the 'basic requirements', stated in EN 1990 [cf. CEN (2002)].

The Eurocode allows for two different approaches for the verification of structural reliability; the partial safety factor or semi-probabilistic method and the full probabilistic method (see Section 1.2). The application of the partial safety factor method is based on choosing the correct safety factor for each design parameter and targeted limit state. The provided partial safety factors are meant to be calibrated for each different limit state to match the corresponding required reliability level. On the other hand, the full probabilistic approach requires the direct evaluation of the structural reliability and probability of failure, respectively. Based on the general definition of the probability of failure (see Section 2), it seems that this approach, while being in perfect alignment with EN 1990, is appropriate for application to the generalized concept of performance-based design. As demonstrated in Section 3, this combined approach employs a reliability-based design technique.

Evidently, the notion of performance-based design and its direct link to probabilistic methods seem to be in good agreement with the Eurocode design concept. Nevertheless, the lack of a general directive for the application of full probabilistic procedures leads to apparent difficulties for the designer. These include the fact that he has to do an extensive literature review to acquire available data and models. Moreover, this will lead to subjective designs, depending on different amounts and types of information collected by different designers. A solution to these problems is suggested by the Joint Committee on Structural Safety (JCSS) that published a Probabilistic Model Code (PMC) [see Vrouwenvelder (2002)]. The suggestions given in the PMC may be applied in conjunction with the performance-based design concept leading to objective reliability-based designs.

5 CONCLUSION

This paper attempts to identify a general performance-based design concept, tailored to the needs of individual structures, that contributes to a 'good' structural design. Moreover, we argue that a consistent realization of this design concept requires the consideration of probabilistic approaches and ultimately leads to a reliability-based design. In light of a geotechnical design application, we demonstrate how this approach can add value to engineering decision making, compared to standard design approaches, and that its application can potentially lead to more economical designs. Nota bene, this approach conforms well to the basic design concept of the Eurocode directive.

The significant progress regarding advanced algorithms and increased computational power have made full probabilistic procedures feasible for practical engineering applications. Nevertheless, it must be admitted that to date there is an obvious lack of guidance regarding the application of these procedures. However, there are signs of an increasing awareness of the potential of performance based design in conjunction with probabilistic reliability assessment. At the same time, promising developments – like the draft of the above mentioned Probabilistic Model Code – may well contribute to establish a best practice regarding the concept's application to practical engineering tasks.

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