Estimating the probability of piping-induced breaching of flood embankments

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ABSTRACT: A new tool for the rapid assessment of the structural performance of flood embankments during extreme hydrologic events was recently developed in the UK. This paper describes how the probability of failure due to piping through the manmade fill was included in the new methodology. Piping through the foundation (under-piping) and piping through the embankment (through-piping) are associated to different physical processes. While satisfactory mathematical models are available to study under-piping, through-piping is not currently amenable to satisfactory mathematical modelling. As a consequence it is not possible to define a performance function for reliability analyses. In this study, to overcome this obstacle, the probabilities of breaching by through-piping were estimated by elicitation of subjective judgement. In flood defence networks some characteristics of the structures are often unknown. To quantify the impact of this lack of knowledge on the performance prediction the probability of breaching was estimated in a finite, but extensive, number of scenarios, covering most cases of practical relevance.

Keywords: Erosion, Flood embankments, Piping, Reliability, River.

1 INTRODUCTION

1.1 *Reliability of flood embankments*

The probability of breaching of flood embankments (levees in American English) is an important component of flood risk modelling (Sayers *et al.* 2002). Its assessment, however, shows some problematic aspects. In particular quantifying the probability of failure due to piping through the manmade fill of the embankment's body is difficult (Figure 1). More generally the expected response to flooding events must be often assessed without knowing some key characteristics of the earth structure (epistemic uncertainty).

Through-piping is the most frequent failure mode for large embankment dams (Foster *et al.* 2000a&b, Richards & Reddy 2007). Although less data are available for fluvial embankments, also for this kind of earth structures various forms of piping seem to account for a non negligible percentage of breaches (Vorogushyn *et al.* 2009).
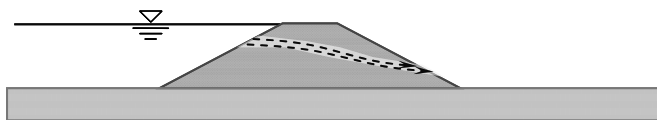

Figure 1. Schematic representation of through-piping.

1.2 *Problematic aspects in modelling through-piping*

Three main approaches are available to estimate the probability of failure of engineering systems: the statistical analysis of historical performance, the methods of reliability analysis (FORM, Monte Carlo simulation, etc.) and the elicitation of expert judgement. In the case of flood defence networks, which are extended systems with largely unrecorded and uncertain characteristics, there are not enough data on past breaches to confidently derive the probability of failure as a function of the water level. The methods of

reliability analysis require a mathematical model which separates safe states from failure states. Unfortunately, while credible models are available for other failure modes (CUR/TAW 1990), a satisfactory and complete model for trough-piping is not available to date (Fell & Fry 2007, Richards & Reddy 2007).

The availability of mathematical models for other failure modes and their successful use in the reliability analysis of flood defence networks (Vrijling 2001) has concealed this significant gap in the current methodologies. It is worth noting how the sea-dikes in the Netherlands, the first flood defences to be studied with reliability methods, are not subject to through piping due to their fill: a sand core completely surrounded by a compacted clay layer. This configuration effectively prevents an internal erosion process like the one depicted in Figure 1, even if other seepage-related adverse phenomena can still occur (Allsop *et al*. 2007).

An attempt to use mathematical models to estimate the probability of through-piping was made by USACE (1999) comparing the internal erosion model for macroscopically intact soil by Khilar *et al.* (1985) with an empirical criterion known as "Rock Island District procedure". For water levels approaching the levee crest the difference in the calculated probability of failure was of six orders of magnitude. USACE (1999) concludes that "there is no single widely accepted analytical technique or performance function in common use for predicting internal erosion".

A recent research project funded by the European Commission produced a catalogue of flood defences failure modes and associated mathematical models (Allsop *et al*. 2007); no model for the initiation of through-piping could be included. Fragility curves for through-piping produced by Bujis *et al* (2005, pp 9-11) are applied by Gouldby *et al.* (2008) to flood embankments in the UK; however the probability of failure was calculated with a criterion for the erosion of soil under impervious structures (Lane 1935), overstretching its use well beyond its range of applicability. There is no conceptual justification for the use of a model developed for a different physical process and its results should be regarded as misleading, as discussed by Redaelli & Dyer (2009) and Redaelli (2009, pp 28-30).

1.3 *Need for a new approach*

To overcome the current limitations a new methodology for the assessment of flood embankments reliability was developed. The new tool, named the Reliability Rating System, is based on a performance indicator which quantifies the probability of breaching only for a small number of water levels, above and immediately below the embankment crest. Although it does not produce a full reliability analysis, the methodology incorporates important, previously neglected geotechnical aspects and assists the final user in handling the remarkable lack of knowledge (epistemic uncertainty) typically associated with the safety assessment of flood defence networks.

One of the innovative aspects of the Reliability Rating System is the use of subjective probabilities to deal with through-piping, beside traditional reliability analyses for other failure modes like under-piping and breaching due to grass cover failure/fill erosion. This paper focuses on the determination of the probability of breaching due to through-piping. The general framework of the methodology and the other modes of failure are discussed in Redaelli (2009).

2 ESTIMATING THE PROBABILITIY OF PIPING-INDUCED BREACHING

2.1 *Lack of a satisfactory model for internal erosion*

Historically the term piping has been used in different contexts to indicate a variety of physical processes creating an often confusing terminology, particularly when disciplinary boundaries are crossed. For flood embankments the most relevant forms of piping are internal erosion – initiated along cracks or zones of concentrated leakage – and backward erosion – initiated at the exit point of seepage in a homogeneous granular material. Criteria to check safety against backward erosion in the foundation of water retaining structures can be found in the literature (Lane 1935, Weijers & Sellmeijer 1993).

Al least one theoretical formulation (Zaslavsky & Kassif 1965) and a mathematical model (Kilar *et al.* 1985) are available for piping through fine grained soils; however they refer to macroscopically homogeneous materials and do not consider erosion along cracks or zones of concentrated seepage. Several authors have studied the removal of grains due to water flowing in an opening (Worman & Olafsdottir 1992, Mohamed 2002, Bonelli *et al.* 2006). These models can correctly reproduce the process of pipe-growth and breach evolution. However the ability to capture accurately the presence or genesis of heterogeneities and anomalies which initiate the erosion still eludes the efforts of researchers. No mathematical model is

currently capable of describing the complete chain of events leading to breaching by through-piping (Richards & Reddy 2007, p 398) and even if some parts of the process are relatively well understood the associated quantitative tools are still being refined (Brown & Bridle 2008). For this reason, in embankment dam engineering, the probability of breaching by through-piping is not calculated with the methods of reliability analysis. Instead the statistical analysis of historical performance or the elicitation of subjective probabilities, often supported by the event tree approach, are regularly used (Fell *et al.* 2000, Fell & Fry 2007). Not enough data are currently available on the past performance of flood embankments to support a robust statistical analysis. The only remaining option is the elicitation of subjective judgement.

2.2 *Quantifying subjective judgement*

Subjective probabilities, in combination with the event tree technique, are extensively used by owners of large portfolios of dams (Beacher & Christian 2003). Published examples and guidance are available in the North American (Ladon-Jones *et al.* 1996), European (Johansen *et al.* 1997) and Australian literature (Fell *et al.* 2004).

In the event tree approach the failure is decomposed in chains of simpler events. These component events are organised in a graphical representation - the tree - which starts with an initiating event and then branches repeatedly, identifying some sequences leading to failure and some others leading to a safe state. The probability of some, or all, the component events along failure chains can be assessed, in absence of other solutions, by expert judgement. In order to produce credible results the judgement elicitation process must be rigorously structured and follow a precise procedure (Vick 1999 & 2002, Beacher & Christian 2003, USACE 2006). Techniques like the association with verbal descriptive statement (Lichtenstein & Newman 1967) and the "action approach to elicitation" (betting, reference lottery, wheel of fortune, etc.) are available to facilitate the assessment.

In most cases the opinions hold by experts are based on intuition, qualitative knowledge, personal experience and other ways of simplified reasoning. The mental processes behind the integration of information of various types into subjective probabilities are studied by cognitive psychology with the aim of achieving coherence and calibration. Subjective probabilities are coherent if they conform to probability theory; they are well calibrated if they reflect frequencies that are (or would be) observed in the real world. The informal methods used by people to estimate subjective probabilities are called heuristics. Several studies show that, in some circumstances, heuristics can lead to systematic errors called cognitive biases (Edwards & Tversky 1967, Kaneman *et al.* 1982, Taversky & Kahneman 1983). In the elicitation process presented here care was taken to minimise the anchoring bias, the representativeness bias, the conjunction fallacy, the availability bias. Overconfidence and misperception of extreme probabilities (Fischhoff *et al.* 1977, Vick 1997) were also considered. The interested reader can find a more detailed discussion of heuristics and biases, and their implication in geotechnical engineering practice in Beacher & Christian (2003).

2.3 *The elicitation process*

2.3.1 *Context*
In the Reliability Rating System (Redaelli 2009) the probabilities of breaching due to various failure modes are estimated for a small number of water levels. In particular the probability of breaching due to through-piping is evaluated only with the water level at the crest of the embankment. This paper describes results used in the Reliability Rating System; therefore all the probabilities of breaching mentioned in the following text do refer to this specific loading condition.

The Reliability Rating System is conceived for comparison among different embankments in a flood defence network, without any attempt to quantify the probability of network failure. Correlations in space are not included in the methodology and, similarly to other regional-scale tools (Gouldby *et al.* 2008), individual defences are identified splitting the network in sections not longer than 600m.

2.3.2 *Structured procedure*
In order to provide the end user with a tool for the quick quantification of reliability it was important to construct a methodology able to cover the wide range of scenarios possibly encountered in practice. This was done:

a) identifying the basic characteristics affecting the performance of embankments in flooding conditions;

b) establishing a realistic range of variation for each of these characteristics;

c) dividing the range in a tractable number of quantitative or qualitative classes;

d) estimating the probability of breaching for each relevant combination of classes.

As tables reporting the estimated probabilities of failures were complied, the final users have simply to locate the characteristics of an existing embankment in the appropriate classes to obtain an estimate of the probability of breaching by each failure mode and an overall index of expected performance.

The assessing panel which estimated the probability of failure by through-piping was composed by three persons: the author, then a PhD student at the University of Strathclyde (Glasgow), M. Dyer, then appointed to the Chair in Construction Innovation at Trinity College (Dublin), and S. Utili, at the time postdoctoral researcher at the University of Strathclyde (later Lecturer at the University of Oxford). All participants have a background in civil engineering with particular emphasis on geotechnics and experience in the geotechnical aspects of flood defences safety deriving form both professional practice and academic research.

The elicitation process was performed in a period of six months between March and September 2008 and was structured according to state-of-the-art recommendations (Vick 1999 & 2000, Beacher & Christian 2003, USACE 2006). With the aim of achieving coherence and good calibration the procedure was organised in five phases:

1. Motivating phase: developing an effecting working relationship among the assessors and clarifying the aims of the process.
2. Training phase: highlighting the biases potentially affecting the assessment in order to avoid, or at least mitigate, them.
3. Structuring phase: in which the problem was analysed and decomposed to an appropriate level of detail; this phase was introduced by an extensive literature review.
4. Assessing phase: consisting in the separate quantification of probabilities by each individual and by the subsequent discussion within the panel, finally leading to consensus on the final results.
5. Documenting phase: recording the process and the conclusions for verification and credibility.

2.3.3 *Initial formulation*

In principle the structural failure of a flood embankment could, and perhaps should, be studied with the event tree method. However the panel found that, while in embankment dam engineering, their use is supported by a reasonably detailed knowledge of the earth structure, for flood embankment a comparable level of knowledge is unimaginable. This makes the use of event tree still possible but more challenging and time consuming than affordable in the study presented here. For this reason, after an initial attempt, the assessors abandoned the event tree approach in favour of a different format.

This alternative approach builds on the formal structure of a historical performance method, known as University of New South Wales method (Foster *et al.* 2000a, b), which was modified to adapt it to the use of subjective probabilities. The adopted formula is:

$$P(B_{tp}) = \prod_i w_i \times P_{ref} \qquad (1)$$

where $P(B_{tp})$ = probability of breaching by through-piping, w_i = weights, i = characteristic affecting the performance, P_{ref} = probability of breaching by through-piping of a reference embankments with fixed characteristics.

Equation (1) reduces the task of the panel to the estimate of the probability of failure for a reference embankment and a system of weights to account for the different characteristics of individual structure. Hence the assessors went through the five phases described in the previous Section; in particular, during the structuring phase:

- the relevant characteristics influencing the probability of through-piping were identified;
- for each of these characteristics the level of information available in practice was discussed and an appropriate subdivision in classes chosen accordingly.

These activities have been guided by the literature on the condition assessment and performance estimation of flood embankments, with particular attention to the British reality (e.g. Environment Agency 2006, Morris *et al.* 2007). In the assessing phase:

- a reference embankment was chosen and a weight of 1.0 assigned the each one of its characteristics;
- the weights for each class of the various characteristics were established;
- the probability of failure for the reference embankment was estimated.

The last two steps had to be repeated iteratively: first to bring internal coherence to the system of weights devised by each individual; then to harmonise the different systems and achieve consensus through discussion. These activities were guided by the literature on the performance of embankment dams (e.g. Foster *et al.* 2000a&b, Fell *et al.* 2004, FEMA 2005a&b) combined with a significant dose of engineering judgement to adapt the indications to the case of flood embankments.

Achieving a well calibrated estimate of the probability of breaching for the reference embankment is problematic, partly because failure is the result of a complex process, partly because the probability is likely to be, in most cases, very low, thus falling in the field affected by the overconfidence bias. However the guidance offered by USACE (1999) in the form of verbal descriptors of the performance associated with probabilities of failure was used for this purpose.

Table 1. Weights estimated by the panel for the characteristics affecting the resistance to piping; the conditions of the reference embankment (optimal resistance) are in *italics*.

CHARACTERISTIC	CLASSES & WEIGHTS					
Animal burrowing w_{burr}	Yes, likely to completely cross the earthfill 200		Yes, unlikely to completely cross the earthfill 50		*No* 1	
Seepage w_{seep}	Muddy leakage 50		Clear leakage 30	Pooling water 10	*None* 1	
Differential settlement w_{sett}	Inducing recognisable cracking 30		Visible, no recognisable cracking 10		*Not visible* 1	
Compaction w_{com}	No compaction 25		Some compaction 10		*Good compaction** 1	
Culvetrt w_{cul}	Many poor details** 25	Few poor details 10		Optimal condition 15	*None* 1	
Fill type w_{soil}	Silt LL<50*** 21	Clean Sand 15	Clayey Sand Silty Sand 6	Silt LL>50 5	Clay LL<50 4	*Clay LL>50* 1
Plant roots w_{root}	Trees 15		Bushes 5		*Grass only* 1	
Fill origin w_{geo}	Alluvial 8	Aeolian, Colluvial 5	Residual, Lacustrine, Marine, Volcanic 4		*Glacial* 1	

* According to BS 6031:2009 or equivalent modern standard; ** list and discussion of relevant details can be found in Redaelli (2009); ***LL = liquid limit.

2.3.4 *Refined formulation*

During the structuring phase the panel identified eight main characteristics which affect the piping resistance; these characteristics are visible in the first column of Table 1. A convenient subdivision in classes for each of the characteristic was also identified. It was also decided to take as reference an embankment with optimal characteristics for piping-resistance and a length of 500m.

After some attempts the assessors found impossible, using Equation (1), to reflect a sufficient worsening of the performance when only one or few negative factors were present and simultaneously satisfy the basic condition $P(B_{tp}) \leq 1$ for combinations of several negative factors. The problem was overcome adopting a more complex formula, in which coefficients of influence are introduced:

$$P(B_{tp}) = \prod_j \max(w_j \times r_j, 1.0) \times P_{ref} \qquad (2)$$

where j = position of the weight in the list of characteristics, arranged form the most to the least influential.

To calculate the probability of failure a list of the eight key characteristics is made, ordering them from the most influential (highest weight) to the least influential (smallest weight). Then each weight is multiplied by a coefficient of influence reduction, which has unit value for the first characteristic, then progressively lower values for the following characteristics. Given that the reference embankment is the one with optimal resistance the individual weights w_i, also after multiplication with the coefficients of influence reduction r_i, cannot be less than 1. The panel has found convenient expressing the coefficients of influence reduction with the formula:

$$r_i = 1/a^{j-1} \qquad\qquad\qquad\qquad (3)$$

where a is a constant, which becomes part of the judgement elicitation process.

2.4 Outcome of the elicitation process

The weights estimated by the panel of assessors at the end of the elicitation process are reported in Table 1. The panel also proposed coefficients of influence reduction based on

$$a = 3 \qquad \text{which leads to} \qquad r_1 = 1, \quad r_2 = 1/3, \quad r_3 = 1/9, \quad \cdots$$

The probability of breaching for the reference embankment, with water at crest level, was estimated to be in the range of:

$$P_{ref} = 9.0 \times 10^{-5}$$

The probability of failure of the embankment with the highest proneness to piping, calculated using the ordered list of characteristics given in Table 2, is:

$$P\left(B_{tp}\right) = 9.0 \times 10^{-5} \times 1.1 \times 10^{4} = 0.99$$

Table 2. Weights and coefficients of influence reduction of the embankment with the most unfavourable characteristics for piping resistance; only the first three characteristics affect the probability of failure.

j	Characteristic	Class	w_i	r_i	Max[$w_i \times r_i$, 1]
1	Animal burrowing	Yes, likely to cross fill	200	1	200
2	Seepage observed	Muddy leakage	50	1/3	16.7
3	Differential settlement	Yes, inducing cracking	30	1/9	3.33
4	Compaction	Not compacted	25	1/27	1.0
5	Culvert	Many poor details	25	1/81	1.0
…	…	…	…	…	…

The interval covered by the presented approach is shown in Figure 2 against the performance descriptors by USACE (1999). The reference embankment is estimated to have a "good" performance also in the very severe loading condition assumed, while the "worst case" embankment, with 99% chance of failure, is almost certainly breaching.

3 DISCUSSION

3.1 Handling epistemic uncertainty

In many practical cases some of the characteristics affecting the piping-resistance are uncertain. The values provided here can quantify of the impact of this lack of knowledge on the performance prediction. For example Figure 3 shows how the interval of possible probabilities of breaching for an embankment with optimal characteristics, but unknown compaction and fill type/origin, becomes significantly narrower once the information on the fill is gathered.

3.2 Use and limitations

The subjective probabilities elicitation was conducted following a rigorously structured procedure according to state-of the-art recommendations. Nevertheless the small size of the panel and the relatively uniform background of the participants suggest that better results could be obtained by a larger panel which included a wider spectrum of competencies and covered more completely the different roles that individuals should play in the elicitation process (Baecher & Christian 2003, p 510).

The presented approach was dictated by the need to develop the elicitation process in a very parsimonious way, which made good use of limited time and resources. This work demonstrates how the appropriate use of subjective judgement can lead to sensible, if not extremely well calibrated, predictions. Hopefully more complete studies on this subject will be performed in the near future within the civil engineering community.
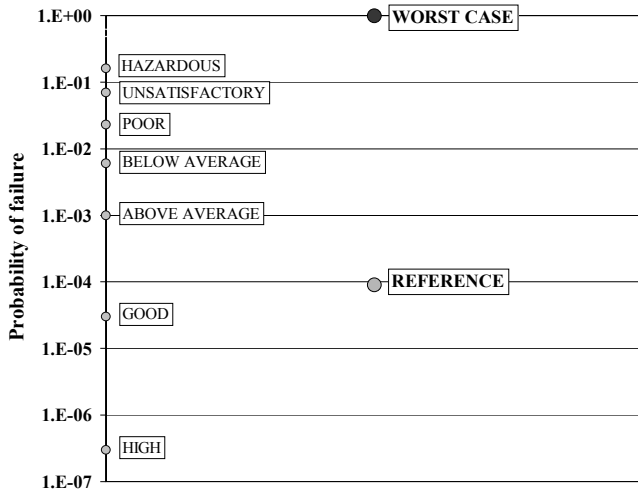
Figure 2. Expected performance, with water at crest level, for the reference embankment (optimal resistance to piping) and for the "worst possible" embankment; values are compared with verbal descriptors from USACE (1999).
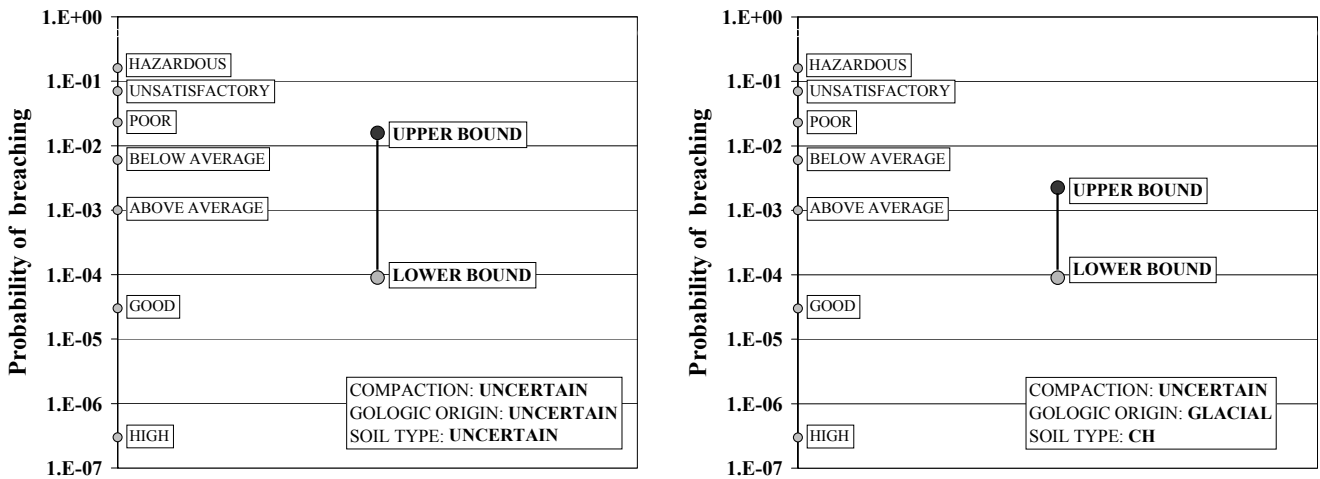

Figure 3. Bounds on the probability of breaching for an embankment with uncertainty about compaction, type of the soil in the earthfill and geologic origin of the earthfill (left). The interval narrows once the information on the fill becomes available (right); CH = high plasticity clay (liquid limit above 50), according to the Unified Soil Classification System.

3.3 Future research: the need for through-piping fragility curves

The probabilities of breaching presented here are all estimated for a specific loading condition. These vales have been derived to offer some guidance regarding the safety against through piping during extreme floods. They are also employed, with a large set of tabulated results on other failure modes, in a simplified reliability assessment methodology proposed by Redaelli (2009), which provides an index of overall performance.

To conduct a complete flood risk analysis, however, the knowledge of the probability of breaching in one loading condition is insufficient: fragility curves, defining the conditional probability of failure given the load for each relevant water level, are needed (Figure 4). While several works on fragility curves for under-piping are available and the underlying theory and assumption are widely accepted, to the author's knowledge no satisfactory fragility curves for piping through the fill of fluvial embankments has been developed to date. This is largely due to the absence of a credible mathematical model ant the consequential impossibility to adopt the traditional methods of structural reliability.

Considering that through-piping plays an important role in the safety of flood embankment the urgency of developing a satisfactory set of fragility curves for this failure mode should be perceived by all researchers active in this field. It is here suggested that an elicitation process, similar to the one presented here but involving a larger and more complete panel of experts, should be arranged to work at the definition of through-piping fragility curves. Working on a longer time scale and mobilising larger resources a more advanced approach could be adopted, possibly making use of event trees like the one shown in Figure 5.
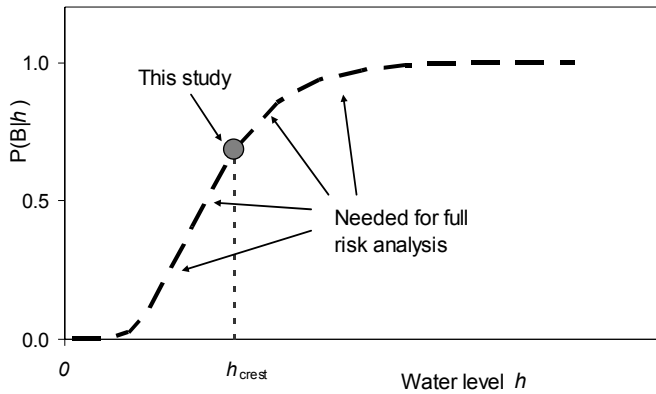
Figure 4. This study provides estimates of the probability of failure for one loading condition (water at the crest level); for a complete flood risk analysis fragility curves are needed.
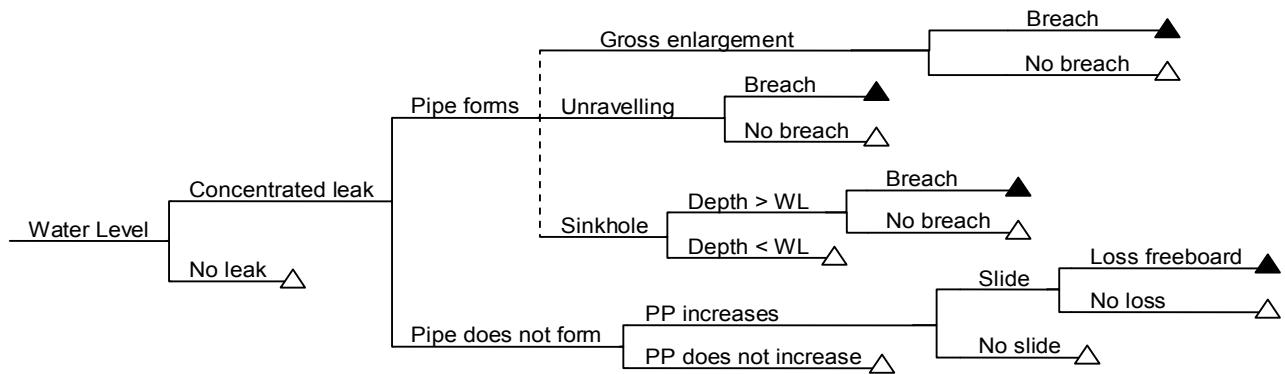

Figure 5. Event tree for breaching by piping through the earthfill of flood embankments proposed by Redaelli (2009). Dotted branches connect events that are not mutually exclusive; WL = water level, PP = pore pressure.

4 CONCLUSION

Through-piping is an important failure mode for flood embankments which cannot be studied with the methods of reliability analysis due to the lack of a satisfactory and complete mathematical model. This paper presents probabilities of piping-induced breaching which were estimated via a rigorously structured process of judgement elicitation. These probabilities of failure are linked to eight key characteristics of flood embankments and were assessed for a wide range of scenarios, covering most cases of practical relevance.

The presented results enable a first assessment of piping resistance of existing embankment and offer a way of quantifying the impact of epistemic uncertainty on the performance prediction. These results are part of a wider body of work on the development of an innovative methodology for the simplified reliability assessment of flood embankments, which combines the traditional methods of reliability analysis and the elicitation of subjective probabilities to incorporate various failure modes in a quantitative measure of the overall expected performance (Redaelli 2009).

The probabilities of breaching reported here were estimated for a single loading condition, which is water level at the crest of the embankment. To support full flood risk analyses it is necessary to produce fragility curves that provide the conditional probability of failure given each relevant value of the water level. It is urgent that the flood risk community addresses the current absence of adequate fragility curves for through-piping.

5 ACKNOWLEDGMENTS

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