ABSTRACT: New insights into the failure mechanism piping (under-seepage) regarding the physical process as well as reliability aspects have led to a revision of the Dutch design and safety assessment rules for dikes. This paper describes how the required factor of safety for piping is derived from a top level requirements formulated in terms of an acceptable probability of flooding. The main steps herein are (a) to account for the length-effect to translate requirements on dike ring (system) level to admissible probabilities of failure on dike section (element) level and (b) the calibration of safety factors as a function of the (element) target reliability.

Keywords: code calibration, piping, under-seepage, target reliability, length-effects

Nomenclature

\(D\) [m]: thickness of sand layer

\(F_F\) [-]: force factor

\(F_G\) [-]: geometrical shape factor

\(F_R\) [-]: resistance factor

\(F_S\) [-]: scale factor

\(H\) [m]: hydraulic head difference (across structure)

\(H_c\) [m]: critical hydraulic head difference

\(L\) [m]: seepage length

\(RD\) [-]: relative density

\(RD_m\) [-]: mean relative density (small scale experiments -0.725)

\(c\) [-]: erosion coefficient

\(d_{70}\) [m]: 70-percentile value of grain size distribution of the piping-sensitive layer

\(d_{70m}\) [m]: mean value of \(d_{70}\) value in the experiments (small scale experiments -2.08e-4)

\(h\) [m+REF]: waterside water level

\(h_b\) [m+REF]: landside water level

\(\gamma_p\) [N/m³]: unit weight of particles

\(\gamma_w\) [N/m³]: unit weight of water

\(\eta\) [-]: Whites constant

\(\theta\) [DEG]: Bedding angle of sand

\(\kappa\) [m²]: intrinsic permeability

\(\alpha_R\) [-]: importance factor

\(V_R\) [-]: coefficient of variation

\(\gamma_p\) [-]: safety factor

\(h\) [m] the normative water level

\(d\) [m] Blanket layer thickness

\(h_b\) [m] decimal height
Primary flood defenses in the Netherlands undergo a 5-yearly safety assessment. Based on economic as well as societal risk acceptance criteria, the current safety standards are defined in terms of exceedance probabilities of hydraulic load conditions (see Figure 1). For failure mechanisms other than overtopping these are commonly interpreted as admissible probabilities $P_{f,adm,dr}$ of (system) failure of a dike ring (polder). That implies that the criteria to be handled for individual dike sections and failure mechanisms need to be stricter: $P_{f,adm,ds,mech} < P_{f,adm,dr}$. That is for two reasons: (a) each mechanism can cause failure and (b) failure of each individual section means system failure. We are dealing with a serial system. A third component is the length-effect. The probability of failure increases with increasing length of a dike section with statistically homogeneous properties (Vrouwenvelder, 2006).

![Normative Exceedance Frequency](image)

Figure 1. Normative exceedance probabilities of Hydraulic Load Conditions in the Netherlands

All these aspects illustrate the need to establish higher target reliabilities locally and for each failure mechanism in order to achieve a sufficiently reliable dike ring (system). In order to translate these requirements into practical terms, semi-probabilistic assessment and design rules need to be derived, using characteristic values and (partial) factors of safety instead of reliability analysis techniques.

This paper describes how a local assessment rule for piping (under-seepage) has been derived, the goal of which is to be consistent with the high level criteria in terms of probabilities of flooding. Before going into the details of the calibration code, the revised piping model is described, including a concise discussion of the model uncertainty. Subsequently, the format of the new safety assessment rules is presented and the derivation of the required factor of safety is discussed. The latter consists of two main steps: (a) derivation of the acceptable probability of (piping) failure of a dike section and (b) the calibration of the required factor of safety as function of the target reliability.

2 REVISED PIPING MODEL

2.1 The Equilibrium Model by Sellmeijer (1988)

Sellmeijer (1988) proposed a computational for piping with three main elements: groundwater flow, pipe flow through the erosion channel and limit equilibrium of soil particles in the channel. For safety assessment purposes, the following equilibrium condition was derived, which describes the critical gradient over the structure. In other words, for lower gradients ($H/L$) the erosion pipe development stops accord-
ing to the model, whereas for higher gradients the erosion may reach the upstream side and thereby en-
danger the integrity of the structure by “under-mining” it.

\[ F_R = \eta \frac{\gamma'_p}{\gamma_w} \tan \vartheta \]

\[ \frac{H}{L} = \frac{1}{c} = F_R F_S F_G \]

\[ F_S = \frac{d_{70}}{\sqrt[3]{kL}} \]

\[ F_G = \{ 0.68 - 0.1 \ln (0.25 F_S) \} \left( \frac{D}{L} \right)^{0.28} \]

The geometry and soil parameters are specified in the section Nomenclature; coefficient \( c \) is composed of three factors:
- \( F_R \): resistance factor, being the strength of the sand
- \( F_S \): scale factor, relating pore size and seepage size
- \( F_G \): geometrical shape factor

Notice that this criterion does not address the appearance of sand boils like exit gradient-based criteria do. Using this equilibrium criterion in safety assessment one implicitly allows sand boils to occur, while pipe development until the upstream side is avoided.

For more complex geometries than a simple aquifer covered by a blanket layer, both with constant thickness, the criterion (in a slightly simplified form) has been implemented in MSEEP, a numerical code for groundwater flow computations.

2.2 Revision of the Sellmeijer Model

Recently, a detailed experimental research of the piping mechanism in The Netherlands (Lopez de la Cruz et al., 2010) has provided better insights into the underlying physical phenomenon and led to a revision of the Sellmeijer model. A multivariate regression analysis enabled re-calibrating the coefficients in the model, by assessing the influence of each measured variable on the critical head simultaneously, resulting in

\[ \frac{H}{L} = \frac{1}{c} = F_R F_S F_G \]

\[ F_R = \eta \frac{\gamma'_p}{\gamma_w} \tan \vartheta \left( \frac{RD}{RD_m} \right)^{0.35} \]

\[ F_S = \frac{d_{70}}{\sqrt[3]{kL}} \left( \frac{d_{70m}}{d_{70}} \right)^{0.6} \]

\[ F_G = 0.91 \left( \frac{D}{L} \right)^{0.28} \]

Besides new coefficients the revised model contains a dependency on the relative density (\( RD \)) of the piping sensitive sand layer (i.e. aquifer, usually the upper few decimeters). However, for safety assessment purposes, the influence of \( RD \) is not taken into account. It is hard to determine in the field and of little influence despite the large uncertainty. Therefore, it was preferred to include it in the model uncertainty. Notice that for “non-standard” geometries the piping module in MSEEP is also available for the revised model and recommended for determining \( F_G \).

2.3 Model Uncertainty

The model uncertainty is accounted for by a multiplicative model with factor \( m_c \):

\[ \frac{H}{L} = m_c \frac{1}{c} = m_c F_R F_S F_G \]
The experimental results from Beek et al. (2010) have been analyzed for determining the parameters of the model factor, which is chosen to be modeled by a lognormal distribution. Its standard deviation is determined by a weighted variance-analysis, in which more weight is given to the available data from prototype scale than to the small and medium scale laboratory experiments (Lopez de la Cruz et al., 2010). The resulting standard deviation of the model factor is $\sigma_{mc} = 0.12$, the scatter of the comparison of predicted versus observed critical head difference is illustrated in Figure 2.

![Figure 2. Observed vs. predicted (revised piping model) critical piping gradients](image)

3 SAFETY ASSESSMENT RULE

The revised piping model can be used to assess the resistance against piping in terms of the critical head difference:

$$H_c = m_i F_R F_S F_G L$$

(4)

For safety assessments this value can be compared to the head difference the structure experiences. For the Dutch safety assessment rules it was decided to handle the same reduction term as in the current guidelines:

$$H = h - h_b - 0.3d$$

(5)

Consequently, the format of the new safety assessment rule is chosen such that the ratio of the critical head difference $H_c$ (resistance) and the head difference including reduction term (load) using characteristic values (5% respectively 95% quantiles) is required to be larger than the safety factor $\gamma_p$:

$$\frac{R_k}{S_k} = \frac{H_{c,k}}{H_k} = \frac{m_{pk} F_{R,k} F_{S,k} F_{G,k} L_k}{h_k - h_{b,k} - 0.3d_k} > \gamma_p$$

(6)

Notice that the characteristic factors $F_{i,k}$ are determined by using 5%/95%-quantiles for their input parameters. Furthermore, the characteristic (or, in fact, design value) for the water level $h_k$ is taken to be the normative water level (MHW) as defined in the Hydraulic Boundary Conditions (Rijkswaterstaat, 2007). The procedure to determine the characteristic values is beyond the scope of this study; reference is made to Eurocode 0.

4 TARGET RELIABILITY AND FACTOR OF SAFETY

The main goal of the calibration is to determine (partial) safety factors that, if consequently used in design or safety assessment, lead to a structure that is at least as safe as the predetermined target reliability.
Commonly, in codes and standards (e.g., Eurocode) the target reliability is chosen from safety classes that reflect the severity of the consequences - the more severe the consequences the higher the target reliability. For the Dutch flood defense system that basic concept is the same, except that the target reliability is defined by the exceedance probabilities\(^1\) (Figure 1) as probabilities of flooding. In other words, these are admissible probabilities of (system) failure. The probability that any of the elements of a dike ring (i.e., dike or other flood defenses) fails is defined as: \(P_{\text{f,adm,dr}} \text{[1/yr]}\). That means that this probability cannot be used to define one target reliability for a particular structure in the system directly.

The first step is a pragmatic one: The probability of system failure is distributed over the failure mechanisms in the system that play a significant role. For piping the admissible probability of failure is 10\% of the total: \(P_{\text{f,adm,dr,p}} = 0.1 \ P_{\text{f,adm,dr}}\). This distribution over failure mechanisms can be treated as economic optimization problem, however, these considerations are beyond the scope of this paper.

The second step is to translate dike ring requirements into dike section requirements. The latter beings the level at which designs and safety assessments are carried out. The key element in this step is the length-effect (Vrouwenvelder, 2006). The probabilities of failure of the flood defenses that form a dike ring are partially correlated. Usually, there is a large (spatial) correlation between the loads on different sections, where also the resistance properties are highly independent. That implies that the probability of failure somewhere in the dike ring is larger than the probability of failure of one (or the weakest) element (i.e., dike section): \(P_{\text{f,adm,dr,p}} > P_{\text{f,adm,ds,p}}\). This is accounted for by incorporating the length effect. The details are further discussed in section 5.

Having determined the target reliability, the actual calibration code is applied. A convenient starting point is to pick standardized values for importance factors such as given in the Eurocode. For example, for a dominant load parameter: \(\alpha_R = 0.8\) with the standard formulae for partial resistance factors. However, from the FLORIS project Rijkswaterstaat (2005) it is known that for piping the importance factors can vary significantly and even exceed \(\alpha_R = 0.8\). Therefore, an appropriate value for \(\gamma_p\) for varying conditions is examined directly by the analysis described in Figure 4. Further details are discussed in section 6.

### 5 LENGTH EFFECTS

Applying zero-level crossing theory for the input parameters to the piping model, a relationship between the admissible probability of failure for a dike section and the admissible probability of failure on dike ring together is established. This includes the influence of the dike sections lengths \(L_{dr,s}\) that are sensitive to piping (i.e., potentially contribute to the probability of failure). A detailed description of this analysis is beyond the scope of this paper but more details can be found in Lopez de la Cruz (2010). The result is represented in Figure 5.

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\(^1\) The normative exceedance probabilities are not exactly the admissible probabilities of flooding, but in the context of code calibration for failure mechanisms other than overtopping are interpreted as such.
The target reliability $\beta_{req} = \Phi^{-1}(P_{f,adm,ds,p})$ increases with the (piping-sensitive) cumulative length of dike sections $L_{dr,s}$ forming the dike ring. For practical purposes, the following formula is proposed which fits the relations in Figure 5 very well:

$$P_{f,adm,dr,s} = \frac{0.1 P_{f,adm,dr}}{1 + \alpha / l_{eq} \cdot L_{dr,s}}$$

where

- $\alpha$: Calibration factor
- $l_{eq}$: Correlation length of the limit estate function for piping

For characteristic Dutch conditions a value of $\alpha / l_{eq} = 0.0028$ is proposed.

6 UNCERTAINTIES AND CALIBRATION DATA SETS

In order to check the suitability of $\gamma_p$ as described in Figure 4, a set of conditions to analyze (parameter sets) are needed as well as their probability distributions per parameter. For the latter, it is recur to the probabilistic modeling in the FLORIS project. The distribution types and variation coefficients (or standard deviations) per parameter are given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Type</th>
<th>Mean</th>
<th>Spread (V=CoV, $\sigma$=Std)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Log-</td>
<td>nominal</td>
<td>V=5.0</td>
</tr>
<tr>
<td></td>
<td>normal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Log-</td>
<td>nominal</td>
<td>V=1e-6</td>
</tr>
<tr>
<td></td>
<td>normal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Log-</td>
<td>nominal</td>
<td>V=0.24</td>
</tr>
<tr>
<td></td>
<td>normal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d70</td>
<td>Log-</td>
<td>nominal</td>
<td>V=6e-5</td>
</tr>
<tr>
<td></td>
<td>normal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eta</td>
<td>Normal</td>
<td>0.25</td>
<td>V=0</td>
</tr>
<tr>
<td>theta</td>
<td>Normal</td>
<td>37</td>
<td>$\sigma$=0</td>
</tr>
<tr>
<td>m_p</td>
<td>Log-</td>
<td>0.12</td>
<td>$\sigma$=0.12</td>
</tr>
<tr>
<td>h_b</td>
<td>normal</td>
<td>nominal</td>
<td>$\sigma$=0.10</td>
</tr>
<tr>
<td>h</td>
<td>Gumbel</td>
<td>nominal</td>
<td>$\sigma$=0.39</td>
</tr>
</tbody>
</table>
The assessment rule should hold for the range of conditions that is expected to be encountered in practice. To establish these conditions, for each parameter with nominal mean value in the table a low, medium and high value (for typical Dutch conditions) have been chosen. The resistance parameters are summarized in Table 2.

Table 2. Nominal Values of Piping Resistance Parameters for Calibration Sets

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unfavourable</th>
<th>Average</th>
<th>Favourable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_D$ [m]</td>
<td>50</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>$\mu_k$ [m/s]</td>
<td>1E-04</td>
<td>1E-05</td>
<td>1E-06</td>
</tr>
<tr>
<td>$\mu_{D70}$ [m]</td>
<td>1,2E-04</td>
<td>2,0E-04</td>
<td>4,0E-04</td>
</tr>
<tr>
<td>$\mu_d$ [m]</td>
<td>0,1</td>
<td>2,5</td>
<td>6,0</td>
</tr>
</tbody>
</table>

For the load parameters (influencing the head difference $H$) three parameter sets are chosen to represent different hydraulic load regimes together with the according acceptable probability of flooding (see Table 3).

Table 3. Parameter Sets of Hydraulic Load Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coast</th>
<th>River</th>
<th>Estuary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(h&gt;MHW)$</td>
<td>10000</td>
<td>1250</td>
<td>4000</td>
</tr>
<tr>
<td>$h_k = MHW$ [m+NAP]</td>
<td>3,3</td>
<td>6,3</td>
<td>3,1</td>
</tr>
<tr>
<td>$\Delta h_{10}$ (decimate height) [m]</td>
<td>0,75</td>
<td>0,7</td>
<td>0,35</td>
</tr>
<tr>
<td>$\mu_{h_b}$ [m+NAP]</td>
<td>-2,5</td>
<td>4,2</td>
<td>-1,0</td>
</tr>
</tbody>
</table>

The first three parameters in the table above can be used to define a the Gumble distribution for the water level.

The “decimate height” is the water level difference that increases the exceedance probability of the water level by a factor 10 with respect to the normative water level with known exceedance probability:

$$P(h > MHW) = 10 \cdot P(h - \Delta h_{10}) \quad (9)$$

Combining each load parameter set with each of the resistance variables, we obtain 243 ($=3^5$) calibration parameter sets.

7 CALIBRATION RESULTS

The calibration analysis has been carried out for each of the 243 parameter sets (previous section) and for six different values of the partial resistance factor $\gamma_p$: 1.0, 1.2, 1.4, 1.6, 1.8. This section presents a summary of the calibration results.

Each of the points in the figure 6 shows which value of the reliability index (horizontal axis) is found for the underlying parameter set and the $\gamma_p$ (vertical axis).

The results present some clusters due to the hydraulic load regimes and the selected blanket layer thickness, which acts in this case as a load reduction term. In order to illustrate this, the data points are plotted with different colors and shapes (see legend).

From Figure 6, it can be seen that the load conditions together with the blanket layer thickness determine the performance of the safety factor.
The black line presents the proposed linear relationship between the required reliability and the partial resistance factor:

\[ \gamma_p = 0.6 \beta_{req} - 1.5 \quad (1.2 < \gamma_p < 1.8) \quad (10) \]

The line is chosen on the left of the scatter points, approximately through the 95%-quantiles per safety factor level. That means that for 95% of the (calibration) cases the reliability achieved by applying the safety factor is higher than the target reliability, while 5% of the (rather extreme) cases would result in a slightly unconservative design, which was supposed to be acceptable. The lower limit of \( \gamma_p \) was a political decision maintaining safety factor of 1.2 that is currently in use as a lower limit.

A point in the upper left region of the scatter implies a region with under-performance. However, all of these points are triangles, meaning that they represent river load conditions. In the Netherlands, those areas have safety requirements that do not exceed \( \beta_{req} = 4.7 \) even for the longest dike rings. Therefore, these points are irrelevant.

The green in the figure 6, indicates the range of \( \gamma_p \)-values expected to be applied using the proposed assessment rule in the Netherlands, based on the relevant range of required reliability index including the length-effect. The resulting range is \( 1.2 < \gamma_p < 1.6 \) with the remark that values of 1.4 are expected to be exceeded only in rare cases such as long dike rings with low acceptable probability of flooding.

The red dashed and dotted lines indicate the resulting safety factor values after the standard level-I equations for lognormal distributed resistance is applied for different combinations of importance factor (\( \alpha_R \)) and coefficient of variation (\( \nu_R \)).

8 DISCUSSION

A target reliability-dependent safety factor for piping is derived for design and safety assessment in the Netherlands. The incorporation of the presented results in the design guidelines is still pending. The target reliability is derived from specific flood protection norms in the Netherlands instead of consequence classes such as used in the Eurocode. In the derivation, it is accounted for system reliability aspects such as accounting for several mechanisms as well as for the length-effect.

Taking into account the scatter of the calibration points around the proposed relation between the safety factor and the target reliability shows a typical aspect of semi-probabilistic design and safety assessments. For a significant range of the chosen conditions, the approach leads to over-design. This can
be avoided by more differentiation. For example, by establishing different safety factors for different load regimes or by reliability-based design (reliability analysis in safety assessment).

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