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Grey Fuzzy Optimization of Total Nitrogen Load Allocation to Nonpoint Sources in Watershed

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ABSTRACT: A method for allocating allowable total nitrogen (TN) load among nonpoint sources in a watershed has been developed by adopting the two-phase grey fuzzy optimization approach with the aid of GIS (Geographic Information System). Competing goals of water quality management authorities and TN load dischargers are described with linear imprecise membership functions including interval numbers. Discharged TN loads from each cell of paddy field, upland crop field, and residential area are assumed to be transported under the conventional first-order kinetic removal. Uncertainty of river discharge and self-purification coefficients are also expressed with interval numbers. The method developed is applied to the Seimei River watershed, which is a subwatershed of the Lake Kasumigaura basin in Japan. By solving the linear programming submodels for TN load allocation to cells using the simplex method, the allowable load at each cell is procured, which could be used as a target value in the decision-making regarding effluent control policies.

Keywords: Water quality, Nonpoint source, Watershed, optimization, Fuzzy, uncertainty, GIS, Decision-making

1 INTRODUCTION

Restoration of water quality in lakes has been a serious problem in Japan. Kasumigaura Lake, which has the second largest surface water area in Japan, is one of the typical water bodies that need watershed-level water quality management due to complex mechanism of pollutant dynamics in the related water bodies and its basin and competing goals of stakeholders. Generally, mathematical programming is an effective method to describe objectives and constraints with respect to effluent control for the water quality management. Recently, for example, Karmakar and Mujumdar (2006, 2007) have presented grey fuzzy optimization models for river water quality management. Maeda et al. (2010a) has developed a robust optimization model for allocating allowable COD (Chemical Oxygen Demand) load among point and nonpoint sources in a river watershed. Li et al. (2014) has presented an inexact chance-constraint quadratic programming model for managing river water quality. Maeda et al. (2006, 2009, 2010b) have presented linear programming models for TN (Total Nitrogen) load allocation to nonpoint sources (NPSs) such as paddy field and upland crop field.

In this study, an improvement over our previous model (Maeda et al., 2009) is conducted for controlling NPS effluent using the two-phase grey fuzzy optimization framework presented by Karmakar and Mujumdar (2007). Uncertainty in TN transport and management goals are quantified with interval numbers (e.g., Huang et al., 1995), which enhances the applicability of the model solution to the real world. The presented method is numerically demonstrated by computing TN load allocation in the Seimei River basin, included in the Lake Kasumigaura basin, Japan.

2 FRAMEWORK FOR OPTIMAL ALLOCATION OF TOTAL NITROGEN LOAD

2.1 *Pollutant Sources and Management Goals*

The cell-based optimization model developed by Maeda et al. (2009) is modified by adopting the grey fuzzy optimization framework. As nonpoint sources, only paddy field, upland crop field and residential area in a watershed of a water body are chosen in this study. The watershed is horizontally divided to uniform cells. A GIS (Geographic Information System) and a DEM (Digital Elevation Model) are employed for delineating the watershed boundary, determining the flow direction for the cells, and identifying the routes of both surface and subsurface flows. Flow direction of those flows are assumed identical for simplicity, and determined using the concept of the steepest slope to one of its eight neighboring cells. Suppose that TN load issuing from a cell of nonpoint source (called 'Land Management Unit (LMU)' hereafter) is transported along the estimated route to the main river in the watershed, and finally delivered to the downstream end of the river.

Watershed-level effluent control designed in this study defines the following two competing goals:

1. Prevention of water quality deterioration at the outlet of the watershed, which is generally a goal of water quality management authorities
2. Profit maximization or cost minimization regarding water-related activities, such as farming, wastewater treatment, etc. in the dischargers' side

2.2 *Goal for Water Quality Management Authorities*

The allowable discharged load from each LMU, a decision variable in our study, is specified as an interval number due to uncertainty in self-purification function, stream discharge, precipitation, etc. The allowable load can be expressed as

$$L_{j_i}^{\pm} \equiv [L_{j_i}^{-}, L_{j_i}^{+}], \quad j = p, u, c; i = 1, \cdots, I_j \tag{1}$$

where i = cell number; j = type of nonpoint source cell (p = paddy field, u= upland crop field, c = residential area); I_j = number of type-j cells; $L_{j_i}^{\pm}$ = discharged TN load from the cell j_i (kg・day^{-1}); $L_{j_i}^{-}$, $L_{j_i}^{+}$ = lower and upper limits of discharged TN load from the cell j_i (kg・day^{-1}). It should be noted that superscript $\pm$ implies the variable is an interval number, and the subscripts – and + stand for lower and upper limits of the interval number, respectively.

Assume that the TN loads discharged from their sources decay subject to the distance-related first-order kinetics. Using the concept of interval number, the amount of TN load that reaches the outlet can be expressed as

$$\begin{aligned} T^{+} &= \sum_{j} \sum_{i=1}^{I_j} L_{j_i}^{+} \exp\left\{-\left(\lambda^{r-} x_{j_i}^{r} + \frac{\lambda^{w-}}{Q^{+}} x_{j_i}^{w}\right)\right\} \\ &= f(L_{j_i}^{+}) \end{aligned} \tag{2}$$

$$\begin{aligned} T^{-} &= \sum_{j} \sum_{i=1}^{I_j} L_{j_i}^{-} \exp\left\{-\left(\lambda^{r+} x_{j_i}^{r} + \frac{\lambda^{w+}}{Q^{-}} x_{j_i}^{w}\right)\right\} \\ &= f(L_{j_i}^{-}) \end{aligned} \tag{3}$$

where $T^{\pm} \equiv [T^{-}, T^{+}]$ = TN load that transported to the downstream end of the main river from all the LMUs (kg・day^{-1}); $\lambda^{r\pm} \equiv \left[\lambda^{r-}, \lambda^{r+}\right]$ = watershed-wide self-purification coefficient for surface and subsurface flows (m^{-1}); $x_{j_i}^{r}$ = travel length of TN load discharged with surface and subsurface flows from the LMU j_i to the outlet at the riverbank (m); $\lambda^{w\pm} \equiv \left[\lambda^{w-}, \lambda^{w+}\right]$ = watershed-wide self-purification coefficient for river flow (m^{2}・day^{-1}); $Q^{\pm} \equiv \left[Q^{-}, Q^{+}\right]$ = discharge of river (m^{3}・day^{-1}); $x_{j_i}^{w}$ = travel length of TN load, generated from LMU j_i, from the drainage outlet at the riverbank to the downstream end of the river (m). It is noted that Eqs. (2) and (3) show T^{+} and T^{-} are functions of $L_{j_i}^{+}$ and $L_{j_i}^{-}$, respectively.

For water quality management authorities, less delivered TN load to the downstream end is preferable. Considering the competing goals of stakeholders in the watershed, the management authorities' goal can be expressed with the following linear imprecise membership function $\mu^{\pm}(T^{\pm})$

$$\mu^{\pm}(T^{\pm}) = \begin{cases} 1 & (T^{+} < T^{D-}) \\ \dfrac{T^{H\pm} - T^{\pm}}{T^{H\pm} - T^{D\pm}} & (T^{D\pm} \le T^{\pm} \le T^{H\pm}) \\ 0 & (T^{-} > T^{H+}) \end{cases} \tag{4}$$

where $T^{D\pm} \equiv [T^{D-}, T^{D+}]$ = the most desirable total transported TN load for the authorities; $T^{H\pm} \equiv [T^{H-}, T^{H+}]$ = acceptable total transported TN load for the management authorities $(T^{D\pm} \le T^{H\pm})$ (see Figure 1). The membership value of $\mu^{\pm}(T^{\pm})$ approaches 1, which is the most preferable state for the authorities, by minimizing $T^{\pm}$.

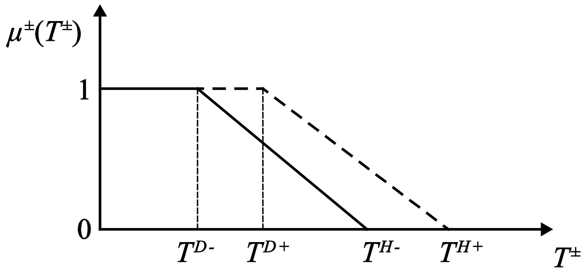

Figure 1. Linear imprecise membership function for the goal of water quality management authorities.

2.3 *Goals for Dischargers*

Goals of TN dischargers at LMUs of paddy field, upland crop field, and residential area can also be expressed with the imprecise membership functions $\mu^{\pm}(L_{j_i}^{\pm})$, defined as

$$\mu^{\pm}(L_{j_i}^{\pm}) = \begin{cases} 0 & (L_{j_i}^{+} < L_{j_i}^{L-}) \\ \dfrac{L_{j_i}^{\pm} - L_{j_i}^{L\pm}}{L_{j_i}^{D\pm} - L_{j_i}^{L\pm}} & (L_{j_i}^{L\pm} \le L_{j_i}^{\pm} \le L_{j_i}^{D\pm}) \\ 1 & (L_{j_i}^{-} > L_{j_i}^{D+}) \end{cases} \tag{5}$$

where $L_{j_i}^{L\pm} \equiv [L_{j_i}^{L-}, L_{j_i}^{L+}]$ = acceptable discharged TN load for discharger at LMU j_i; $L_{j_i}^{D\pm} \equiv [L_{j_i}^{D-}, L_{j_i}^{D+}]$ = the most desirable discharged TN load for discharger at LMU j_i ($L_{j_i}^{D\pm} \ge L_{j_i}^{L\pm}$) (see Figure 2).

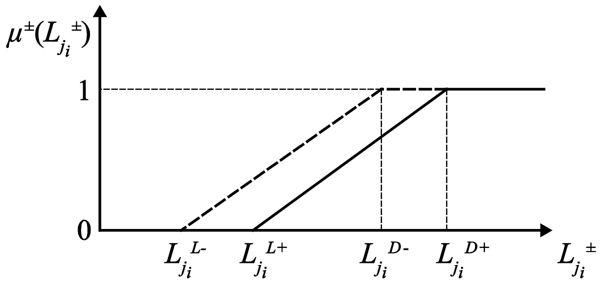

Figure 2. Linear imprecise membership function for the goal of discharger at LMU j_i.

3 GREY FUZZY OPTIMIZATION MODEL

A grey fuzzy optimization model that harmonizes goals mentioned above under constraints can be expressed as the following mathematical programming problem

$$\text{Maximize} \quad \xi^{\pm} \tag{6}$$

subject to
1. Achievement level of goal of water quality management authorities

$$\frac{T^{H\pm} - T^{\pm}}{T^{H\pm} - T^{D\pm}} \geq \xi^{\pm} \tag{7}$$

2. Achievement level of goals of TN dischargers

$$\frac{L_{j_i}^{\pm} - L_{j_i}^{L\pm}}{L_{j_i}^{D\pm} - L_{j_i}^{L\pm}} \geq \xi^{\pm}, \quad \forall i, j \tag{8}$$

3. Lower and upper limits on transported TN load to the watershed outlet

$$T^{D\pm} \leq T^{\pm} \leq T^{H\pm} \tag{9}$$

4. Lower and upper limits on discharged TN load at each LMU

$$L_{j_i}^{L\pm} \leq L_{j_i}^{\pm} \leq L_{j_i}^{D\pm}, \quad \forall i, j \tag{10}$$

5. Lower and upper limits on watershed-wide satisfaction level

$$0 \leq \xi^{\pm} \leq 1 \tag{11}$$

$$\text{when} \quad T^{\pm} = f(L_{j_i}^{\pm}) \tag{12}$$

where $\xi^{\pm} \equiv [\xi^{-}, \xi^{+}]$ = watershed-wide satisfaction level equivalent to overall goal fulfillment level of all the stakeholders considered in the watershed. Note that variables in the optimization model [Eqs. (6) - (12)] are $L_{j_i}^{\pm}$, $\xi^{\pm}$, and $T^{\pm}$. By solving this optimization model, the allowable TN loads $L_{j_i}^{\pm}$ for all LMUs are procured.

Values of $T^{D\pm}$, $T^{H\pm}$, $L_{j_i}^{L\pm}$, and $L_{j_i}^{D\pm}$ that compose the membership functions can be determined using actual data on such as fertilization amount, water quality standard, precipitation, etc. The flow length in Eqs. (2) and (3) are estimated in the GIS and the DEM.

Since the optimization model [Eqs. (6) - (12)] embodies the interval numbers, it must be converted to linear programming models that includes no interval numbers. Adopting the two-phase grey fuzzy optimization approach developed by Karmakar and Mujumdar (2007) to our problem results in producing eight submodels (linear programming models) that can directly be solved by the simplex method. Submodels 1 to 4 are produced when the goals of water quality management authorities expressed in Eq. (4) is considered first in the solution procedure, and the optimal solution of them is specified as that in 'Case 1'. On the other hand, Submodels 5 to 8 are created when the goals of TN load dischargers represented in Eq. (5) are considered first, whose solution is named that in 'Case 2'.

4 APPLICATION

The optimization model developed is applied to the Seimei River watershed, which is a sub-watershed of the Lake Kasumigaura basin in Japan. Using the DEM (Figure 3) at a resolution of 50m (width) ×60m

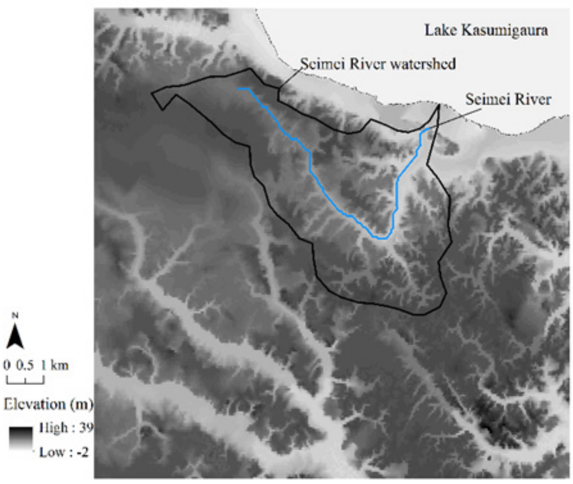

Figure 3. Surface elevation in and around the Seimei River watershed.

(height) and ArcGIS 10.2, the flow direction of each cell is estimated by the steepest gradient method with surface elevation. In the Seimei River watershed (25.6km^2), 4,496 cells at a resolution of 50m×60m are specified as LMUs using ArcGIS 10.2. The numbers of LMUs of paddy field, upland crop field, and residential area are 1201, 1753, and 1542, respectively (see Figure 4).

Values of parameters such as self-purification coefficients, discharge of river flow, and permissible and desirable levels related to goals for the authorities and dischargers, etc. are determined with observed data in 2013 and the literature (Skop and Sørensen, 1998) (see Table 1). Flow length, i.e., length of route where TN load issued from a LMU is transported, is estimated using the obtained flow direction data in the GIS.

The obtained optimal solutions in Cases 1 and 2 are listed in Table 2. The grey degree (Karmakar and Mujumdar, 2007) of decision variable, $Gd(L_{j_i}^{\pm})$, is defined as

$$Gd(L_{j_i}^{\pm}) = \frac{L_{j_i}^{+} - L_{j_i}^{-}}{\frac{1}{2}\left(L_{j_i}^{-} + L_{j_i}^{+}\right)}. \tag{13}$$

Table 1. Parameters used in the application

Parameter	Value	
λ^{r-}, λ^{r+} (m^{-1})	0.000850,	0.000850
λ^{w-}, λ^{w+} (m^2・day^{-1})	0.480,	0.480
Q^{-}, Q^{+} (m^3・day^{-1})	4168.8,	16675.2
$L_{p_i}^{D-}$, $L_{p_i}^{D+}$ (kg・day^{-1})	3.38,	4.13
$L_{p_i}^{L-}$, $L_{p_i}^{L+}$ (kg・day^{-1})	0.00,	0.452
$L_{u_i}^{D-}$, $L_{u_i}^{D+}$ (kg・day^{-1})	8.80,	10.8
$L_{u_i}^{L-}$, $L_{u_i}^{L+}$ (kg・day^{-1})	0.00,	1.18
$L_{c_i}^{D-}$, $L_{c_i}^{D+}$ (kg・day^{-1})	1.03,	1.26
$L_{c_i}^{L-}$, $L_{c_i}^{L+}$ (kg・day^{-1})	0.00,	0.138
T^{D-}, T^{D+} (kg・day^{-1})	0.00,	72.8
T^{H-}, T^{H+} (kg・day^{-1})	3720,	4090

Table 2. Optimal solutions expressed as interval numbers and grey degrees of decision variables in Cases 1 and 2. Case 1: goal of water quality management authorities is considered first; Case 2: goals of TN load dischargers are considered first.

Solution	$Gd(L_{j_i}^{\pm})$
Case 1:	
$L_{p_i}^{\pm} = [1.3698, 1.3698]$ (kg・day^{-1}), $i = 1, \cdots, 1201$	0
$L_{u_i}^{\pm} = [3.5647, 3.5647]$ (kg・day^{-1}), $i = 1, \cdots, 1753$	0
$L_{c_i}^{\pm} = [0.4173, 0.4173]$ (kg・day^{-1}), $i = 1, \cdots, 1542$	0
$T^{\pm} = [2384, 3403]$ (kg・day^{-1})	
$\xi^{\pm} = [0.0775, 0.4678]$	
Case 2:	
$L_{p_i}^{\pm} = [1.0920, 1.0965]$ (kg・day^{-1}), $i = 1, \cdots, 1201$	4.11×10^{-3}
$L_{u_i}^{\pm} = [2.8536, 2.8536]$ (kg・day^{-1}), $i = 1, \cdots, 1753$	0
$L_{c_i}^{\pm} = [0.33326, 0.33405]$ (kg・day^{-1}), $i = 1, \cdots, 1479, 1481, \cdots 1542$	2.37×10^{-3}
$= [0.33326, 0.33451]$ (kg・day^{-1}), $i = 1480$	3.74×10^{-3}
$T^{\pm} = [1906, 2724]$ (kg・day^{-1})	
$\xi^{\pm} = [0.1550, 0.3745]$	

$Gd(L^{\pm}_{j_i})$ can be used to measure the magnitude of uncertainty in the variable. Since all the decision variables in Case 1 have zero grey degree as shown in Table 2, the solution in Case 1 is judged more favorable to use as a management alternative in decision-making.

The substantial deviation of upper and lower bounds of the transported load $T^{\pm}$ in Case 1 caused only by the assumed uncertainty of river discharge $Q^{\pm}$, as can be seen in Eqs. (2) and (3) and Table 1. In the future, influence of uncertain variation of self-purification on allowable TN loads needs to be examined.

The optimal allocation of allowable discharged TN load in Case 1 is shown in Figure 5. In the distribution of the optimal value in Case 1, land use type results in the main influencing factor. However, two kinds of optimal interval values in the residential area LMUs, shown in Table 2, explain the flow length also affects the allocation in Case 2, i.e., the greater the flow length related to a LMU is, the greater allowable TN load becomes

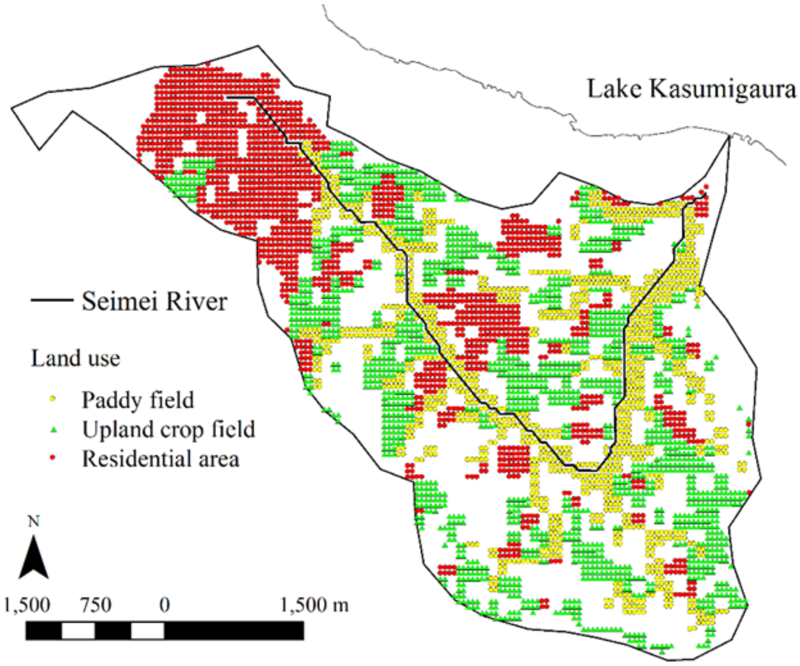

Figure 4. Distribution of land management units (LMUs) of considered nonpoint source cells, i.e., paddy field, upland crop field, and residential area in the Seimei River watershed.

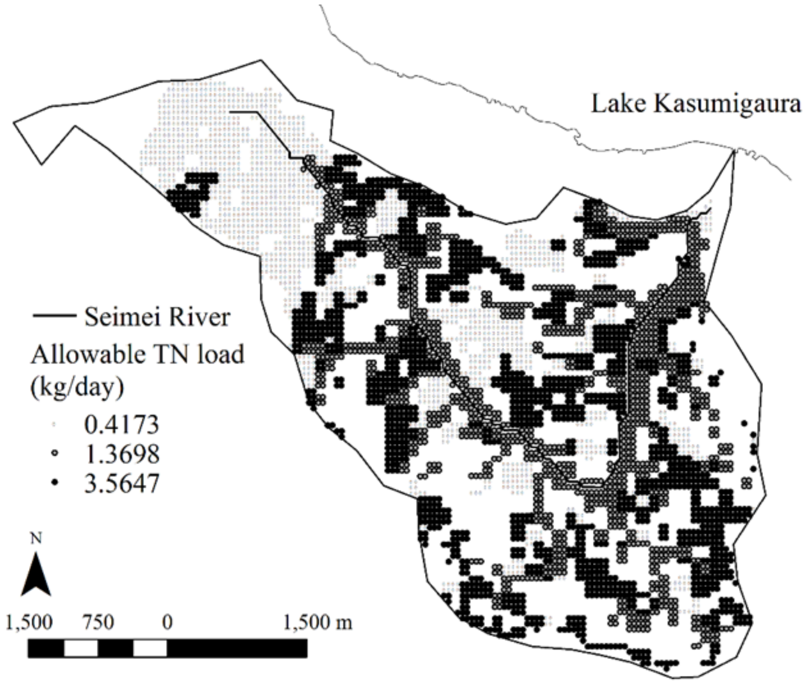

Figure 5. Distribution of computed optimal allowable discharged TN load.

5 CONCLUSIONS

A GIS-aided optimization method is presented for managing TN load from nonpoint sources in a watershed. The method includes estimation of flow routes by hydrological analysis with the DEM and the overlay of maps of land use and flow direction in the GIS. The two-phase grey fuzzy optimization framework is employed to formulate optimal allocation problem of the allowable discharged TN load among cells (LMUs) of paddy field, upland crop field, and residential area. The method developed is applied to the Seimei River watershed, which is a part of the Lake Kasumigaura basin, Japan. It is demonstrated that optimal solution or allowable TN load for each LMU is numerically procured and the presented method could contribute to more rational effluent control at the watershed level.

NOTATION

$L^{\pm}_{j_i}$	discharged TN load from the cell j_i
I_j	number of type-j cells
$T^{\pm}$	total transported TN load at the downstream end
$\lambda^{r\pm}$	watershed-wide self-purification coefficient for surface and subsurface flows
$\lambda^{w\pm}$	watershed-wide self-purification coefficient for river flow
$Q^{\pm}$	discharge of river
$x^{r}_{j_i}$	travel length of TN load discharged with surface and subsurface flows from the LMU j_i to the outlet at the riverbank
$x^{w}_{j_i}$	travel length of TN load, generated from LMU j_i, from the drainage outlet at the riverbank to the downstream end of the river
$T^{D\pm}$	the most desirable transported TN load for the authorities
$T^{H\pm}$	acceptable transported TN load for the authorities
$\mu^{\pm}(T^{\pm})$	linear imprecise membership function on total transported TN load
$L^{L\pm}_{j_i}$	acceptable discharged TN load for discharger at LMU j_i
$L^{D\pm}_{j_i}$	the most desirable discharged TN load for discharger at LMU j_i
$\mu^{\pm}(L^{\pm})$	linear imprecise membership function on discharged TN load
$\xi^{\pm}$	watershed-wide satisfaction level
Gd	grey degree

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