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Use of the Subgrid Technique in Operational Models

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ABSTRACT: The basic idea of subgrid models is the use of available high-resolution bathymetric data at subgrid level in computations that are performed on relatively coarse grids allowing large time steps. For that purpose, an algorithm that correctly represents the precise mass balance in regions where wetting and drying occur was derived in [10] and [11]. Computational grid-cells are permitted to be wet, partially wet or dry and no drying threshold is needed. Based on the subgrid technique, practical applications involving various scenarios were implemented including an operational forecast model for water level, salinity and temperature of the Elbe Estuary in Germany. The grid generation procedure allows a detailed boundary fitting at subgrid level. The computational grid is made of flow aligned quadrilaterals including few triangles where necessary. User-defined grid subdivision at subgrid level allows a correct representation of the volume up to measurement accuracy. Bottom friction requires a particular treatment. Based on the conveyance approach, an appropriate empirical correction was worked out. The aforementioned features make the subgrid technique very efficient, robust and accurate. Comparison of predicted water levels with the comparatively highly resolved classical unstructured grid model shows very good agreement. The speedup in computational performance due to the use of the subgrid technique is about a factor of 20. A typical daily forecast can be carried out in less than 10 minutes on standard PC-like hardware. The subgrid technique is therefore a promising framework to perform accurate temporal and spatial large scale simulations of coastal and estuarine flow and transport processes at low computational cost.

Keywords: Subgrid modeling, Wetting and drying, Semi-implicit schemes, Operational forecast models

1 INTRODUCTION

The simulation of coastal flows involving wetting and drying of large tidal areas implies two major difficulties: (i) the need for stable, accurate and efficient algorithms; (ii) the use of model grids that are able to reproduce the bathymetric details and boundaries up to an acceptable level. A general review of popular wetting and drying algorithms can be found in [1, 19]. Over the last two decades a family of numerical models for solving 2D and 3D shallow water equations based on the efficient and robust semi-implicit finite difference method has been proposed by Casulli and his co-authors [5, 6, 7, 8]. The models range from the 2D structured linear *TRIM* (Tidal Residual and Intertidal Mudflat) model up to the 3D unstructured piece-wise linear *UnTRIM* model. The need to capture bathymetric details in complex regions, with a better boundary fit and to reduce the grid resolution in large and open regions, like tidal flats, motivated the transition from structured to unstructured orthogonal grids. What is new, on the other hand, is that modern remote sensing technologies can deliver very detailed land surface height data that should be considered for more accurate simulations. In that case, and even if some compromise is made with regard to grid resolution of an unstructured grid, simulations still will require large grids which can be computationally very demanding.

Recently, new techniques have been proposed for flood simulation such us the raster-based models working with high-resolution topographic data [2, 13, 20], or models that make use of subgrid-scale details to a limited extent [3, 21]. The subgrid technique, first published in [10], is based on the idea of making use of the available detailed subgrid bathymetric information while performing computations on relatively coarse grids permitting large time steps. Consequently, accuracy and efficiency are drastically enhanced. Compared to the classical linear method, where the underlying bathymetry is solely discretized by the computational grid, the resulting system is mildly nonlinear requiring only few extra Newton iterations to be solved. The algorithm guarantees rigorous mass conservation and non-negative water depths for any time step size. The generalization of the semi-implicit algorithm for three dimensional flows [10] to intrinsically account for detailed bathymetric subgrid data was demonstrated in [11]. A special treatment, proposed in the present work, is applied to bottom friction for friction dominated flows. Based on the subgrid technique, flood simulations with subgrid digital elevation models were performed [17]. The aforementioned features motivated the decision to use the *UnTRIM*[2] subgrid system for the simulation of the Elbe Estuary system.

The subgrid technique is accompanied by new challenges associated with issues of grid (subgrid) generation. For that, new techniques have been developed. The present paper aims to give some insights in the subgrid technique and to shed some light on issues like grid generation and numerical efficiency. One implementation of the technique at the Federal Waterways Engineering and Research Institute (*BAW*) in an operational forecast model for the Elbe Estuary in Germany will be discussed. We refer to [15] for an extensive presentation of the present work.

2 GRID GENERATION

Performing simulations using the subgrid technique requires building models with subgrid resolution. *Smile Consult GmbH Hannover* developed the Janet preprocessor for the generation of unstructured orthogonal grids with subgrid scale bathymetry that comply with the subgrid algorithm. The objective was to investigate different grid generation strategies with subgrid technology for varying computational grid sizes and apply them to typical BAW applications. In this section, some important outlines of grid generation are presented.

Available topographic data are typically stored in a relational database management system. The database interface of the grid generator (preprocessor) can process different kinds of data sets. All data sets are described with metadata. The different interpolation strategies use a database enabled spatial search engine. For every computational edge a set of sub-edges, each with one length and one depth (depth-classes of the sub-edges), is created. Similarly, for every computational polygon a set of sub-polygons, each with one area and one depth (depth-classes of sub-polygons), is created. The number of depth-classes may vary for each polygon or edge.

For the subgrid generation the following strategies were developed:

- Subdivision (SD) strategy with a uniform subdivision of all edges and polygons in sub-edges and sub-polygons as shown in Figure 1. The resolution of the subdivision is user-defined. The database driven interpolation delivers the corresponding depth at subgrid level.
- Terracing Topography (TT) strategy with non-uniform depth-level aligned sub-edges and sub-polygons in the form of bathymetric "terraces" as shown in Figure 2. The depth of the isobaths is user-defined. The generation of isobaths is performed on a temporarily interpolated raster (database interpolation).

The generated subgrid is subject to some constraints for optimization and validation purposes:

- Computational cells with no or tiny sub-cell area ($<$ 1%) are filtered and removed from the grid.
- Flow separation between computational cells that are physically not connected should be insured everywhere.
- Edges with no sub-edges are given one subedge with 0.0 m length (default).
- Inflow edges must have at least one sub-edge with sufficiently large lengths.
- Terrain data should strictly overlap inflow sides.
- For both subgrid generation strategies, the boundary fit is performed at subgrid level.

Another important aspect during grid generation is the possible 1D discretization of tributaries with solely one computational polygon over the cross section and subgrid bathymetry as illustrated in Figure 3. Most of the aforementioned constraints are naturally taken into account in an unstructured grid including flow alignment for less numerical diffusion.

The previous techniques were successfully used to generate different structured and unstructured grid series for the Elbe Estuary with varying computational grid sizes but identical subgrid bathymetry. Grid variants of the Elbe Estuary were designed for systematic investigations with the new technique. In section 4, a detailed description of one of the grids used in a real world application is given.

3 METHODOLOGY

In this section we will recall the basics of the semiimplicit solution algorithm and the associated numerical approximations used in *UnTRIM²* which refers to the new subgrid based computational core [10]. The solution algorithm remains very similar to the one used in the classic linear or piecewise linear *UnTRIM* [8]. The solvers involved in the present work are the piecewise linear *UnTRIM* (designated by classical) and the nonlinear subgrid *UnTRIM²* (designated by subgrid).

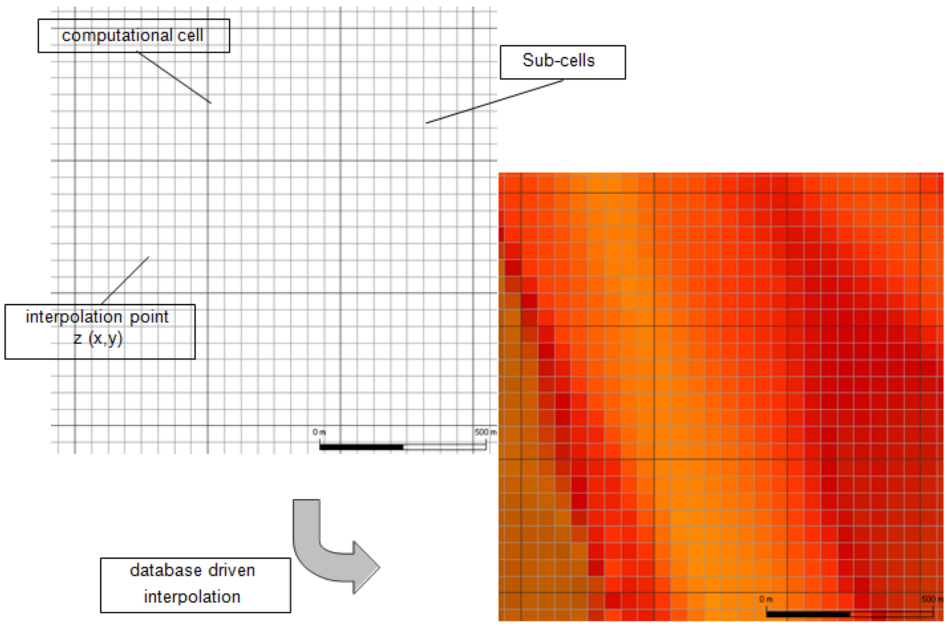

Figure 1. Grid generation using the subdivision (SD) strategy. Left: Computational grid and subgrid interpolation. Right: Corresponding subgrid bathymetry.

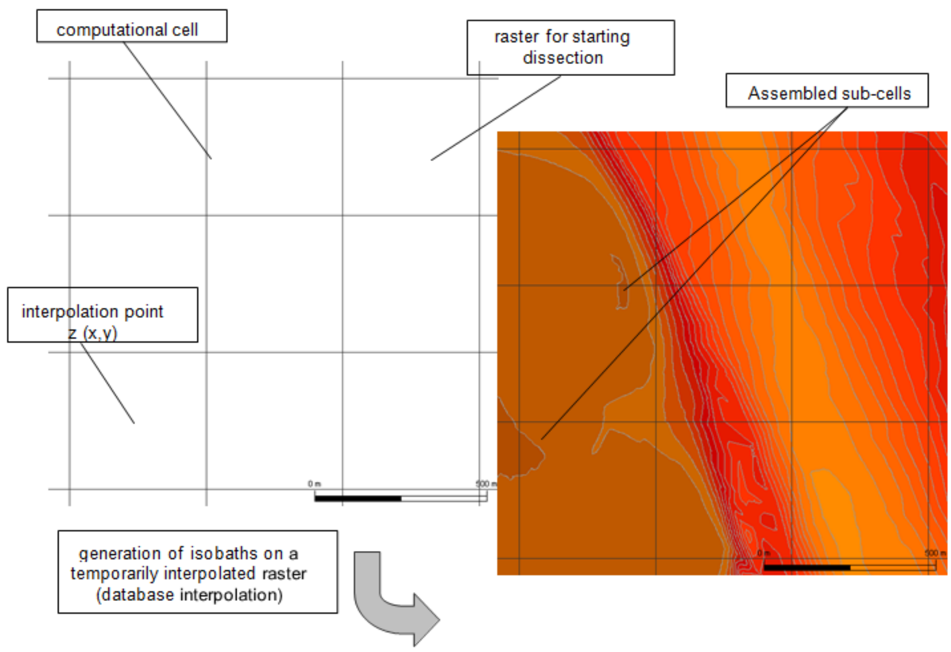

Figure 2. Grid generation using the Terracing Topography (TT) strategy. Left: Computational grid and auxiliary grid interpolation. Right: Corresponding subgrid bathymetry.

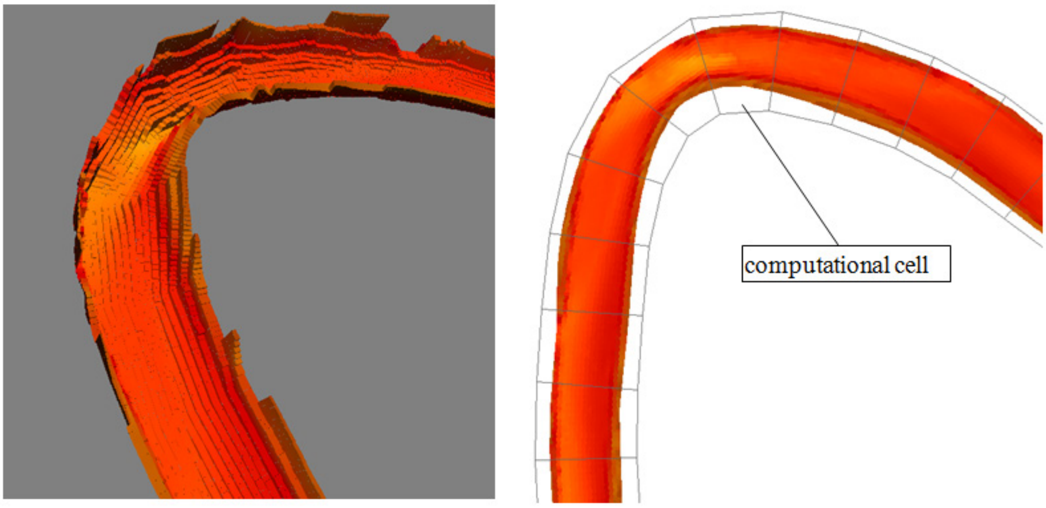

Figure 3. Subgrid bathymetry and 1D discretization of a tributary river of the Elbe Estuary using the SD strategy. Left: 3D view of the subgrid bathymetry. Right: Computational grid with solely one computational cell over the cross section.

Numerical approximation *UnTRIM*[2] solves the three-dimensional, time dependent, nonlinear differential equations related to hydrostatic and non-hydrostatic free-surface flow problems on an unstructured orthogonal grid to cover problems with complicated geometry. Numerically, *UnTRIM*[2] is based on a semi-implicit finitedifference scheme that allows unconditional stability.

Terms affecting stability like bottom friction, wind stress and vertical mixing are treated implicitly. An Eulerian-Lagrangian explicit finite difference operator is used to account for the discretization of advection and horizontal dispersion. For stability, the implicitness factor θ should be taken in the range $0.5 \leq \theta \leq 1$. We refer to [7] and [8] for a detailed description of the algorithm. Scalar transport processes are treated by an explicit mass conserving finite volume scheme with sub-cycling if necessary to ensure stability. The scheme allows higher accuracy through the use of flux limiters [9]. In the case of baroclinic flow, the transport equations are coupled with the momentum equations through density gradients. The baroclinic forcing terms are solved explicitly in the momentum equations. The equations of transport are solved lagged one time step.

In this case the numerical scheme is subject to a weak *CFL* (Courant-Friedrichs-Lewy) stability condition. It is also subject to a weak stability condition due to the explicit treatment of horizontal diffusion in the momentum equations.

Bottom friction A highly resolved bathymetry at subgrid level with the assumption of constant velocity along a computational edge require a modified treatment of processes involved at the lowest layer of the model particularly bottom friction . It is well known from cross-section integrated models that the combination of the cross-sectional mean velocity with a constant bottom friction parameter tends to systematically overestimate the energy dissipation in shallow regions leading to higher flow resistance. This problem can be mitigated by the use of the conveyance approach [16] which allows the computation of a uniform and consistent crosssectional energy dissipation. Unfortunately, the conveyance approach only works for 2D vertically averaged models. Some preliminary meaningful experiences conducted on simple geometries like a U-shaped channel showed that there is a systematic overestimation of bottom friction particularly when the resolution of the computational grid becomes low. Based on the conveyance approach, an appropriate empirical correction was worked out [15].

4 EXAMPLES OF SUBGRID MODELS

Three major estuaries are located at the German North Sea coast: Elbe, Weser and Ems. In this study we have focused on the Elbe Estuary. In addition to a classical grid, a series of computational grids using subgrid scale bathymetry were generated:

- UG400SD24: based on the SD strategy with a computational unstructured grid having 400 m average resolution, 24 subdivisions along the edges and 24×24 subdivisions inside polygons.
- UG200SD12: based on the SD strategy with a computational unstructured grid having 200 m average resolution, 12 subdivisions along the edges and 12×12 subdivisions inside polygons.

- UG200SD06: based on the SD strategy with a computational unstructured grid having 100 m average resolution, 6 subdivisions along the edges and 6×6 subdivisions inside polygons.
- UG050SD03: based on the SD strategy with a computational unstructured grid having 50 m average resolution, 3 subdivisions along the edges and 3×3 subdivisions inside polygons.

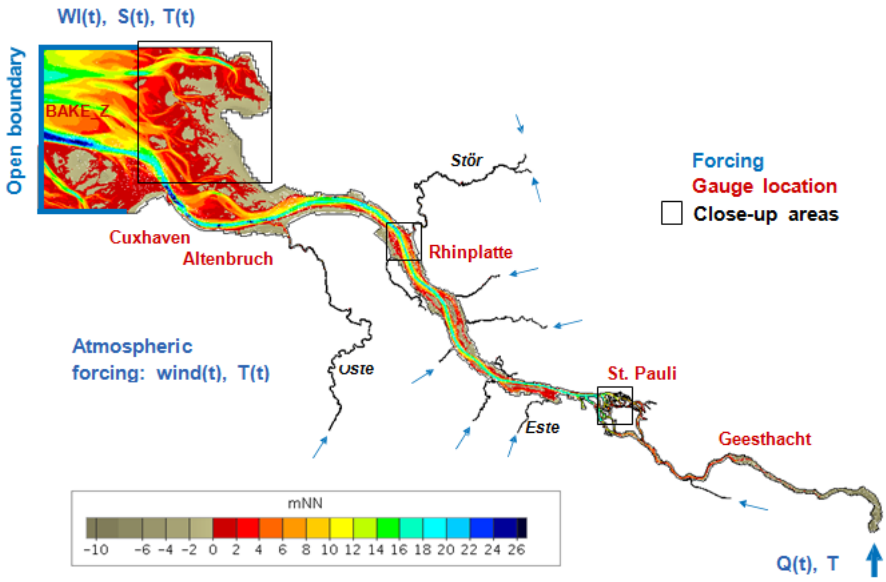

Figure 4. Bathymetry of the Elbe Estuary with the UG400SD24 model (400 m average computational grid resolution, 24 sub-edges per edge and 24 × 24 sub-polygons per polygon). Locations of tidal gauges, discharges and close-up areas are depicted as well.

These grids were generated at the starting phase during the implementation of the *UnTRIM*^{2} subgrid technique at *BAW*. The aim was to settle the kind of computational grids that should be considered (structured or unstructured) as well as to figure out the extent to which the resolution of the computational grids can be coarsened. The results were compared to the classical unstructured model which stands for the reference. An extensive work was done in this direction including the introduction of the bottom friction correction.

The outcome of this preliminary study was that the sensitivity on computational resolution was mild to moderate and that even the coarsest computational grid (UG400SD24) was still able to deliver satisfactory results as long as bottom friction correction with a suitable calibration factor is switched on. The trade-off between numerical efficiency and accuracy clearly put the coarsest grid forward for potential full scale simulations like in the case of the Elbe Estuary model. Thus, we will restrict ourselves to the coarsest UG400SD24 model depicted in Figure 4. For comparison, a typical classical unstructured grid is used. Characteristics of both models are presented in section 5. Figure 5 highlights a section of the Elbe Estuary at the junction with the tributary river *Stör*. It gives an impression of how differently bathymetric details are represented in a classical grid and a grid using the subgrid technique. Moreover, the tributary river is taken into account in a one dimensional way. It was neglected in the classical grid due numerical constraints. On the second close-up area shown in Figure 6, one can clearly see that the bathymetry of the tidal flats can be resolved even better in a coarse computational grid model using subgrid scale bathymetry (UG400SD24) compared to a highly resolved classical grid model. With subgrid technique, the simulation of flooding and drying in these areas can be performed more realistically. For any water level, an excellent approximation of the water volume as well as the active cross section for the flow is guaranteed.

Narrow harbor channels and basins do also represent a challenge for traditional grid generation. Geometries of these regions are often complex requiring a particular attention during meshing. The elaboration of such grids remains always a fussy business mainly due to the need of smooth transitions between channels of different sizes. In some hydrodynamical systems, this problem is bypassed by the replacement of the detailed harbor branching with simple geometries containing the relevant volume of water. With subgrid technique, harbor channels and basins are realistically and accurately represented at subgrid level including boundary fit as shown in Figure 7. The harbor area can be covered using a coarse computational grid as long as one is not interested in local details of flow, e.g. flow separation near harbor basin entrances.

5 APPLICATION

In this section one application of the subgrid technique is presented. The model covers the Elbe Estuary and focuses on operational water level, salinity and temperature forecast. The model was run first in hindcast simulations for calibration and validation purposes.

5.1 *The Elbe Estuary operational model*

The Elbe Estuary is a very important waterway. Ship traffic requests precise and preferably long term water level forecasts. Particular events like storm surges, periods of low or high fresh water discharge have great economic significance and require therefore detailed temporal and spatial forecasts for water level, current velocity, salinity and eventually temperature in the case of ice sheet formation.

In the Elbe, the weir near *Geesthacht* (see Figure 4) represents the artificial limit between the river and the estuary. The river feeds the estuary with time varying fresh water runoff. The seaside boundary at the North Sea is influenced by tidal dynamics, waves, external surges and storm surges. During the flood tide, salt water with higher density intrudes into the estuary. By means of hydrodynamic models, the variation in water level, current, salinity and temperature can be simulated with reasonable accuracy.

In order to accurately represent the progression of the tidal wave, an appropriate grid resolution on top of a prevailing topography is required. This is the scope of the operational model for the Elbe Estuary we present in this section.

The numerical model for water level, salinity and temperature of the Elbe Estuary at *BAW* was first developed using the classical highly resolved unstructured piece-wise linear *UnTRIM*. Later, the classical model was replaced by *UnTRIM*^{2} [10] based on subgrid technique. The forcing is identical for both models. The daily meteorological forcing data (wind and temperature) are delivered by the German Weather Service *(DWD*) using the *COSMO-EU* model [12] which covers almost the whole Europe including the Baltic Sea, the Mediterranean Sea and the Black Sea with 665×657 grid points in a 0.0625° (~ 7 km) resolution. The meteorological model delivers ground air temperature and zonal and meridional wind velocities at 10 m height. The coarse forecast data of the meteorological forcing are interpolated from the meteorological grid (7km × 7km) onto the *UnTRIM*^{2} computational grid. The coarse resolution of the meteorological grid leads to an underestimation of wind velocities in grid cells that are partially wet like for instance in narrow parts of the estuary. Therefore, the so called *WAsP* (Wind Atlas Analysis and Application Program) factors [14] were introduced by *DWD* to enhance the wind fields over the estuary.

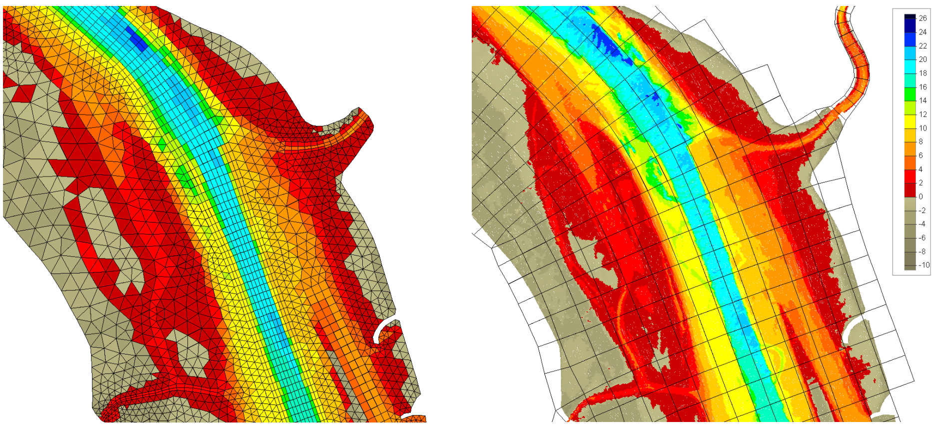

Figure 5. Bathymetry at the junction of the Elbe Estuary with the *Stör* tributary using classical unstructured grid (left) and subgrid technique (right)

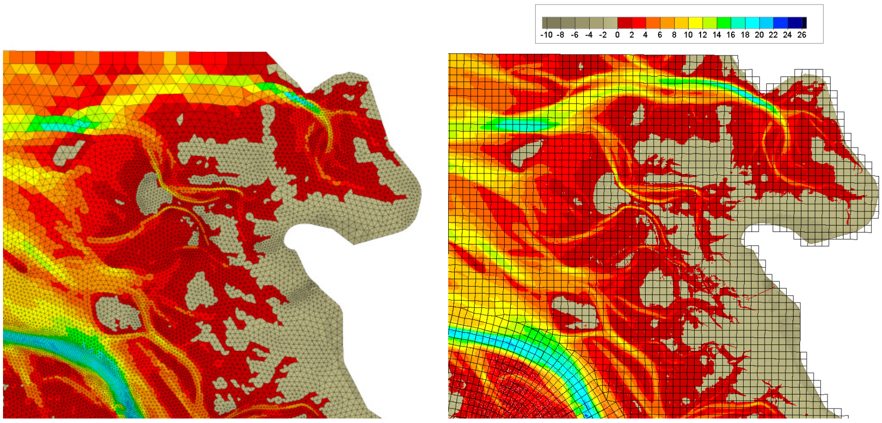

Figure 6. Bathymetry of tidal flats with classical unstructured grid (left) and subgrid technique (right)

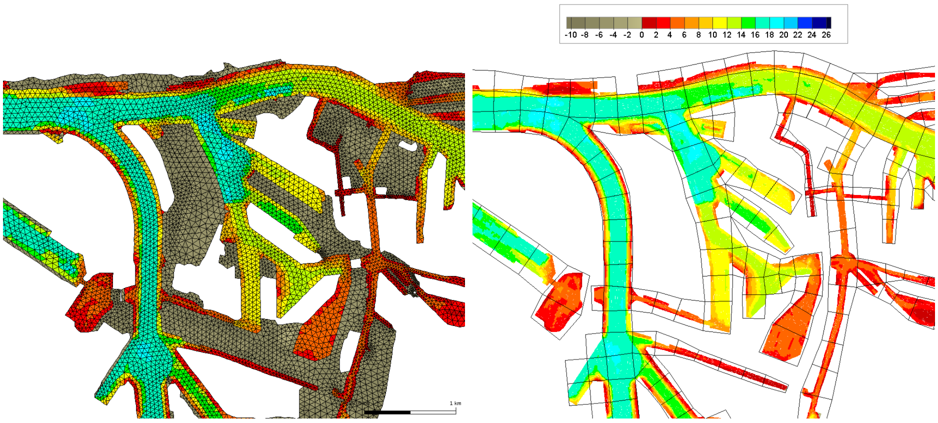

Figure 7. Bathymetry of Hamburg harbor using classical unstrucrured grid (left) and subgrid technique (right)

Along with a fine *WAsP* grid (250m × 250m) and 12 wind direction parameters (each every 30°), *WAsP* correction factors are computed and multiplied with the interpolated meteorological data. Open boundary data are delivered by the German Maritime and Hydrographic Agency *BSH* using the coastal model *HIROMB-BOOS-MODEL* (*HBM*) [4] which is the successor of the *BSHcmod* model. The operational service at BSH is based on a 3D baroclinic circulation-model run on a North Sea/Baltic Sea grid with a horizontal resolution of 3 nautical miles and a fully two-way nested 0.5 nautical mile grid of the German Bight/Western Baltic. The forecasted data of water level, temperature and salinity are processed to fit with the *UnTRIM*^{2} open boundary. For salinity and temperature at open boundary vertically averaged values are used. At the inflow boundary, measured fresh water discharge from the Federal Institute of Hydrology (*BfG*) at the gauge *Neu Darchau* is used. Water temperatures at inflow are daily measurements at gauge *Cumlosen* delivered by the *River Basin Community Elbe* (*FGG Elbe*).

A typical forecast simulation is performed for 24 hours. Comparison is performed between the coarse computational grid model with subgrid bathymetry (UG400SD24) and the classical model.

5.1.1 *Results*

Obviously, the quality of the forecasts delivered by the estuary operational model depends strongly on the accuracy of the forcing at boundaries, in particular water levels predicted by the *HBM* model at the North Sea boundary. A measurement gauge (*Bake Z*) located right on the open boundary (see Figure 4) allows to assess the quality of such predictions. Discrepancies at open boundaries will automatically be transferred along the estuary. Therefore, the model was first calibrated and validated for different scenarios using measured water level at the seaside boundary. The main calibration parameters that were investigated are bottom friction and turbulence (see section 3).

Calibration and validation

Apart from mean conditions, the year 2006 involved various particular events including a storm surge and very high/very low discharges and was hence chosen for the calibration and validation processes. Figure 8 shows results for a period of 11 days out of a 14 days (22.10.2006-04.11.2006) model run. The results highlight water level predictions before, during and after the occurrence of a storm surge. They were obtained by a 3D simulation including salinity and temperature. The quality of the results as well as the numerical performance of classical and subgrid models are discussed here.

The water level at the St. Pauli gauge (see Figure 4) is an important benchmark for model results. In the upper part of Figure 8, both, tidal signal and tidally averaged (over 12 hours and 25 minutes) water levels are in good agreement with measurement for classical and subgrid models. If we look at differences between model results and measurements in the lower part, we see comparable discrepancies independently of the model used. In some parts the classical model is better, in other parts the subgrid model shows better agreement with measurements. But the major difference between the two models resides in the computational efficiency. Table 1 summarizes some characteristics and performance of both models. The classical model spends 5 hours and 30 minutes to perform 14 days simulation, whereas the subgrid model needs only 22 minutes using the same numerical settings and the same computational resources (16 CPUs of one HPC-System node). This represents roughly a speed-up factor of 15.

Table 1: Characteristics and computational performance of classical and subgrid models for a 14 days simulation. Enclosed in parentheses are the number of subgrid edges and subgrid polygons.

	classical	subgrid
Nr. of edges	194453	28369 (293680)
Nr. of polygons	120124	12393 (584609)
vertical resolution (m)	1	1
time step (s)	100	300
real cpu time (min)	330	22
speed-up vs real time	61	916

6 CONCLUSIONS

The subgrid technique offers the possibility to take into account the bathymetric information at subgrid scale allowing model bathymetry to be accurate up to measurement accuracy independent from the resolution of the computational grid. The subgrid technique shows less restrictions on grid generation: e.g. easy fit of lateral boundaries; potential one-dimensional approximation of tributary rivers; use of homogeneous, flow aligned unstructured grid (quadrilaterals, triangles if needed); and an accurate representation of flow area and volume for every water level. Consequently, simulation of wetting and drying of tidal flats can be performed more accurately.

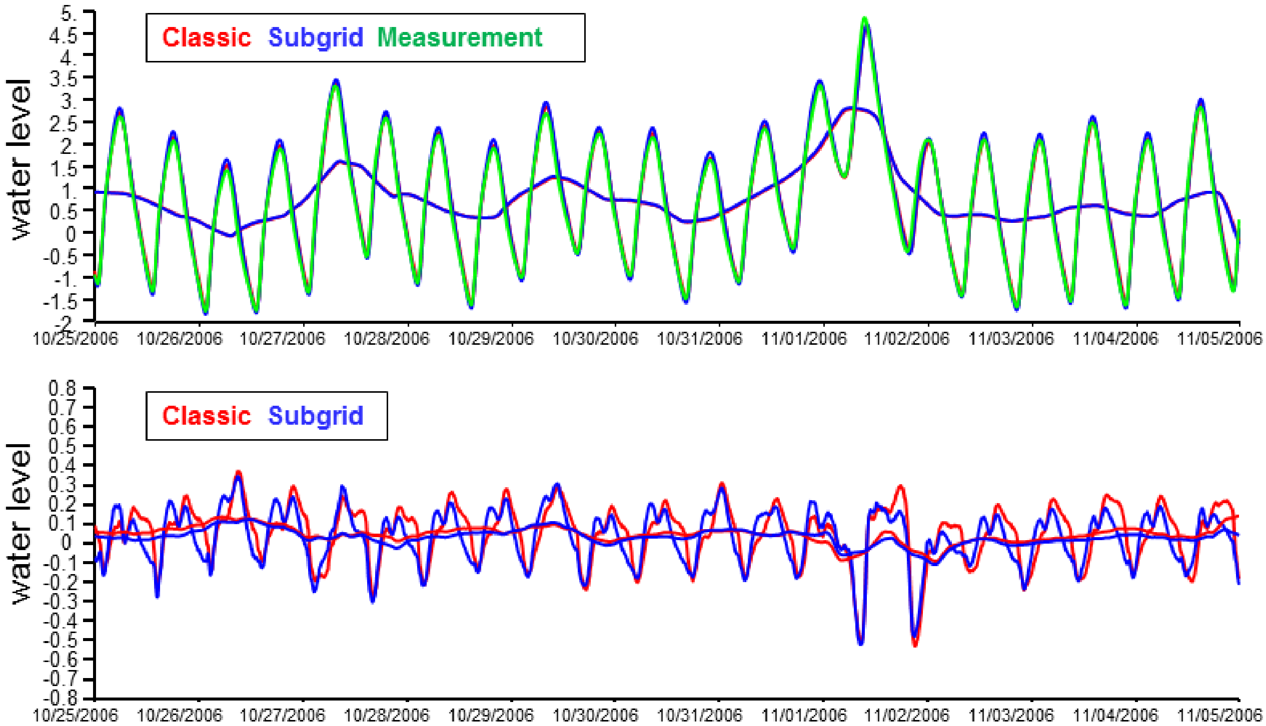

Figure 8. Water level at Hamburg *St. Pauli* for 14 days simulation. Top: comparison between measurements (green), classical (red) and subgrid (blue) results. Bottom: Differences between simulations and measurement. The second set of curves are tidally averaged results (over 12h 25min).

Comparison of simulation results for the Elbe Estuary obtained with a model based on the subgrid technique and a classical model showed good agreement for both long term simulation of tidal dynamics and short term operational forecasts. The computational speed-up reached through the use of subgrid was about a factor of 20.

Although the time step used in the subgrid model is three times larger than the one used in the classical model, the fraction of elements where the *CFL* numbers are low ($CFL < 0.5$) is in average larger in the case of subgrid. The fraction of elements where the *CFL* numbers are high ($CFL > 1$), however, is in average lower in the subgrid case. As a result, if we want to obtain comparable external Courant numbers to the classical model, there is still potential to use larger time steps in the subgrid model which means we can go even faster. In other words, the resolution of the computational grid can be coarsened. On the subgrid level, we are using an average resolution of 16 m which is for the investigated coastal domain with large intertidal mudflats sufficient. For other types of models, e.g. where the resolution of flow gradients is of significance, the subgrid resolution should be higher or even go to the limit of the available bathymetric data.

The Subgrid technique improves accuracy if used with the same (high) resolution classical computational grid. On the other hand, comparable results are obtained if the subgrid technique is used even on a much coarser computational grid. Therefore, subgrid models are particularly suitable for simulations where real computation time is an important issue.

More generally, the subgrid technique is a promising framework to perform accurate temporal and spatial large scale simulations of coastal and estuarine flows and transport processes like flooding and drying of large areas at low computational cost. Other potential applications of the subgrid technique are sensitivity studies of computational results with regard to model resolution.

REFERENCES

[1] Balzano A. Evaluation of methods for numerical simulation of wetting and drying in shallow water flow models. Coastal Engineering 1998, 34, 83:107.

[2] Bates PD. De Roo APJ. A simple raster-based model for flood inundation simulation. Journal of Hydrology 2000, 236, 54:77

[3] Bates PD. Development and testing of a subgrid-scale model for moving boundary hydrodynamic problems in shallow water. Hydrological Processes 2000, 14, 2073:2088.

[4] Berg P. Weismann Poulsen J. Implementation details for HBM. DMI, Copenhagen, 2012. DMI Technical Report No. 12-11 (available at www.dmi.dk/dmi/tr12-11.pdf).

[5] Casulli V. Semi-implicit finite difference methods for the two-dimensional shallow water equations. Journal of Computational Physics 1990, 86, 56:74.

[6] Casulli V., Cheng RT. Semi-implicit finite difference methods for three-dimensional shallow water flow. International Journal for Numerical Methods in Fluids 1992, 15, 629:648.

[7] Casulli V., Cattani E. Stability, accuracy and efficiency of a semi-implicit method for three-dimensional shallow water flow. Computers and Mathematics with Applications 1994, 27, 99:112.

[8] Casulli V., Walters RA. An unstructured grid, three-dimensional model based on the shallow water equations. International Journal for Numerical Methods in Fluids 2000, 32, 331:348.

[9] Casulli V., Zanolli P. High resolution methods for multidimentional advection-diffusion problems in free surface hydrodynamics. Ocean Modelling 2005, 10, 137:151.

[10] Casulli V. A high-resolution wetting and drying algorithm for free-surface hydrodynamics. International Journal for Numerical Methods in Fluids 2009, 60, 391:408.

[11] Casulli V., Stelling GS. Semi-implicit subgrid modelling of three-dimensional free-surface flows. International Journal for Numerical Methods in Fluids 2010, 67, 441:449

[12] Doms G. A Description of the Non-hydrostatic Regional COSMO Model. COSMO Consortium 2011. (available at http://www.cosmomodel.org).

[13] Horritt DM, Bates PD. Effects of spatial resolution on raster based models of flood flow. Journal of Hydrology 2001, 253, 239:249

[14] Mortensen N.G., Landberg L., Troen I., Petersen E.L. Wind Atlas Analysis and Application Program (WASP) Vol. 2: User Guide, Risø National Laboratory, Roskilde, Danmark, 1993, 133 pp.

[15] Sehili A.M., Lang G., Lippert C. High resolution subgrid models: background, grid generation and implementation. Ocean Dynamics 2014, Volume 64, Issue 4, 519:535

[16] Stelling G.S., Kernkamp H.W.J., Laguzzi M.M. Delft Flooding System: a powerful tool for inundation assessment based upon a positive flow simulation. Hydroinformatics 1998, 449:456.

[17] Stelling GS. Quadtree flood simulations with sub-grid digital elevation models. Proceedings of the Institution of Civil Engineers ICE. Water Management 2012, Volume 165, Issue 10,567:580

[18] Umlauf L., Burchard H. A generic length-scale equation for geophysical turbulence models. Journal of Marine Research 2003, 61, 235:265(31).

[19] Van't Hof B., Vollebregt E.A.H. Modelling of wetting and drying of shallow water using artificial porosity. International Journal for Numerical Methods in Fluids 2005, 48,1199:1217

[20] Yu D. Lane SN. Urban fluvial flood modelling using a two-dimensional diffusion-wave treatment, part 1: mesh resolution effects. Hydrological Processes 2006a, 20, 7, 1541:1565.
[21] Yu D. Lane SN. Urban fluvial flood modelling using a two-dimensional diffusion-wave treatment, part 2: development of a sub-grid-scale treatment. Hydrological Processes 2006b, 20, 7, 1567:1583.