

3D Numerical Modelling of Contraction Scour under Steady Current Using the Level Set Method

M.S. Afzal

Department of Marine Technology

Norwegian University of Science and Technology, Trondheim, Norway

H. Bihs & Ø.A. Arntsen

Department of Civil and Transport Engineering

Norwegian University of Science and Technology, Trondheim, Norway

ABSTRACT: Contraction scour is a type of general scour that occurs due to a reduction in the channel cross-section. This is observed where the flow is constricted due to the placement of structures like bridges abutments and other onshore/offshore structures. The flow accelerates in constrictions, which increases the bed shear stress and increases the turbulence associated with it. Development of contraction scour could lead to the failure of the structure if too much sediment is eroded. Accurate predictions of scour processes are necessary to assist the design engineers in monitoring and correcting the aforementioned problems before the structures fail or become unsafe. A three-dimensional computational fluid dynamics model is used to calculate the scour and the deposition pattern in a contraction. The CFD model solves Reynolds-Averaged Navier-Stokes (RANS) equations in all three dimensions. The location of the free surface is modeled with the level set method, which calculates the complex motion of the free surface in a very realistic manner. The level set method is also used for representation of the sediment-water interface. The numerical results for the contraction scour prediction are compared with physical experiments. The numerical model predicts the general evolution (geometry, location and maximum depth) of scour, deposition height and its location accurately with very minor differences compared to the physical experiments.

Keywords: *Sediment Transport, CFD, Scour, Level Set Method, Reynolds-Averaged Navier-Stokes (RANS), Bed Load Transport, Suspended Load Transport, Morphology*

1 INTRODUCTION

The process of scouring is removal of sediments from river bed caused due to moving water or waves. Scour can be classified into two broad categories: General scour and local scour. Contraction scour is a type of general scour which occurs due to a reduction in channel cross-sectional area. Local scour on the other hand occurs due to the direct effect of an obstruction on the flow field. Contraction scour is observed where the flow is constricted due to the placement of structures like bridges etc. The flow accelerates in constrictions which increases the bed shear stress and the turbulence associated with it. The development of contraction scour is noted when the critical shear stress of the bed materials is overcome by the bed shear stress. This could lead to the failure of the structure if too much sediment is eroded near it. Accurate prediction of scour processes is necessary to assist the design engineers in monitoring and correcting the aforementioned problems before the structures fail or become unsafe. In literature, the generalised scour due to channel constriction is not very well documented. Among the one dimensional models, HEC-18 [3] has been used to estimate the mean water velocity in the contracted channel by using the velocity in the un-contracted channel and the contraction ratio. Here an empirical analytical formula was used for estimation of scour parameters. Among the two dimensional models Weise in 2002 [13] and Marek and Dittrich in 2004 [8] deserve to be mentioned. They used 2D numerical morpho-dynamic models to simulate the details of the erosion process in the open-channel contraction laboratory experiment. They concluded that the flow in the contraction domain had dominant 3D flow effects hence, a 2D model could not accurately reproduce the flow. Later Bihs and Olsen in 2007 [2] and Duc and Rodi in 2008 [4] modelled the contraction scour case using their respective 3D models. They used the same experimental data for validation which is utilised in the current study.

The current paper presents the application of the REEF3D sediment transport module by modelling a contraction flow situation in an alluvial laboratory channel with non-cohesive bed, for which experiments have been carried out at the Federal Waterways Engineering and Research Institute Karlsruhe (Bundesanstalt für Wasserbau BAW) [1]. The same study has been used by Weise in 2002 and Marek and Dittrich in 2004 for their 2D numerical modelling study. Later this study was used by Bihs and Olsen and Duc and Rodi in their paper for 3D numerical modelling study.

2 NUMERICAL MODEL

2.1 Governing Equations

The CFD code uses the continuity and the incompressible Reynolds-averaged Navier-Stokes (RANS) equations as the governing equations for mass and momentum conservation.

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(v + v_t) \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] + g_i \quad (2)$$

where U is the velocity averaged over time t , ρ is the fluid density, P is the pressure, v is the kinematic viscosity, v_t is the eddy viscosity and g is the gravity term. The turbulence modelling approach in this study makes use of the RANS-equations where the eddy viscosity v_t in the RANS-equations is determined through the two-equation the $k-\omega$ model [14]. The pressure gradient term in the RANS-equations is modelled using SIMPLE method[10]. SIMPLE stands for Semi-Implicit Method for Pressure Linked Equations. The pressure correction equation is solved together with the momentum equations successively each time step. The Poisson equation is solved using the fully parallelized Jacobi-preconditioned BiCGStab solver [11].

The fifth-order WENO (Weighted Essentially Non-Oscillatory) scheme as proposed by Jiang and Shu [7] is used to discretize the convective terms of the RANS equations in a conservative finite difference framework. The conservative WENO scheme is used for the treatment of the convective terms for the velocities U_i , while the Jacobi-Hamilton version is used for the variables of the free surface and turbulence algorithms. The time treatment is dealt with using an implicit time scheme.

The Level Set Method (LSM) is employed to model the free surface. This method was proposed by Osher and Sethian[9] for computing and analysing the motion of an interface between two phases in two or three dimensions. This method is employed in the current study to model the interface between water-air and water-sediments. The location of the interface is represented implicitly by the zero level set of the smooth signed distance function $\phi(\sim x, t)$. The level set function gives the closest distance to the interface in every point of the modelling domain. The phases are distinguished by the change of the sign. The main advantage of using the level set method to calculate the interface between the fluids is that $\phi(\sim x, t)$ is smooth across the interface which makes it differentiable at the interface and avoids numerical instabilities.

2.2 Sediment Transport Modelling

Turbulent viscosity based bed shear stress formulation [15] is used for calculation of bed shear stress.

$$\tau = \rho(v_t + v) \frac{\partial U}{\partial z} \quad (3)$$

The sediment transport rates in the bed cells are calculated with Engelund and Fredsøe's bed load formula [6].

$$\begin{aligned}
q_{b,i}^* &= 0; \tau^* < \tau_{c,i}^* \\
q_{b,i}^* &= 18.74 (\tau^* - \tau_{c,i}^*) (\tau^{*0.5} - 0.7 \tau_{c,i}^{*0.5}); \tau^* > \tau_{c,i}^* \\
\tau_{c,i}^* &= \frac{\tau_{c,i}}{(\rho_s - \rho) g d_i}
\end{aligned} \tag{4}$$

Here ρ_s is the density of the sediment, ρ the density of the water, g is the gravity, d_i the sediment particle diameter. $q_{b,i}^*$ is the dimensionless bed load transport rate also called Einstein number[5] and $\tau_{c,i}^*$ is dimensionless critical shear stress.

Van Rijn formula for the suspended load [12] is used to account for the suspended load transport. The level-set method is employed in the current numerical model to track the movable sediment surface. The evolving bed is represented implicitly by the zero level set of the smooth signed distance function $\phi(\sim x, t) = 0$. This is essentially an eulerian approach and re-gridding at the water-sediment interface is not necessary. The evolution of $\phi(\sim x, t)$ corresponding to the motion of the interface is adjusted by simply moving the level set up and down depending upon the erosion or accretion of the bed cells.

Here z_b is the local bed surface elevation. The velocity of the sediment surface F is determined by the mechanism of sedimentation and erosion due to bed load and suspended load transport, which particularly is dependent of the local wall shear stress. This is known as Exner equation (as in 5)

$$(1 - p') F = -\frac{\partial q_{B,x}}{\partial x} - \frac{\partial q_{B,y}}{\partial y} - E + D \tag{5}$$

F is the propagation velocity of the interface along its vertical direction, given by:

$$F = \frac{\partial z_b}{\partial t} \tag{6}$$

Here p' is the porosity of the bed layer, E is the erosion rate caused due to external actions and D is the corresponding deposition rate. The terms on the right hand side of equation 5 are evaluated in the direction normal to the sediment surface. In the model, the flow field is first calculated by solving the Navier-Stokes equation. Then the obtained flow field and the turbulent eddy-viscosity are employed to solve for the bed load and suspended load. The position of the sediment surface, indicated by the zero level set $\phi(\sim x, t) = 0$ is updated for each morphological time step.

3 EXPERIMENTAL AND NUMERICAL SETUP

The experimental data is obtained from physical experiments of the contraction case conducted at the laboratories of the BAW (Federal Waterways Engineering and Research Institute) in Karlsruhe, Germany [1]. The flume was 16.50 m long and 1 m wide. The contraction was 0.5 m wide. The left side wall (in stream wise direction) was an alternation of glass and concrete. The average size of the sediments in the flume was $d_{50} = 5.5$ mm. The bed sediment layer was 20 cm thick. The experiments were run with three different discharges: 80 l/s, 130 l/s and 150 l/s. There was no erosion with the 80 l/s discharge. With the discharges of 130 l/s and 150 l/s erosion occurred with different intensities. In the current section the scour pattern obtained due to the discharge of 150/s is compared with the numerical model results. The numerical model setup for contraction scour case is as shown in figure 1.

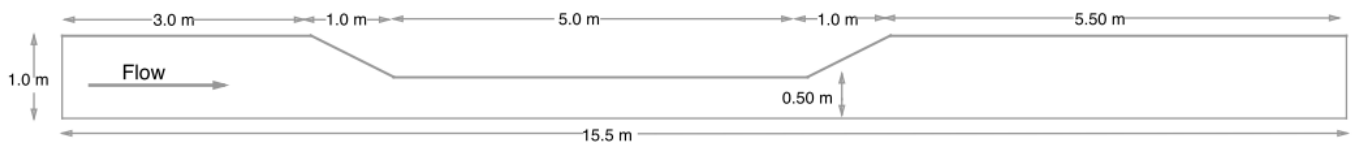


Figure 1. Contraction: Numerical Setup.

A three dimensional tank 15.5 m long, 1.0 m wide, 0.5 m contraction and 0.8 m deep with constant water depth of 0.268 m at exit for 80 l/s discharge case and 0.275 m water depth at exit for 150 l/s discharge case is used. A 0.20 m thick sediment layer has been used in the numerical model setup. The numerical model has a mesh size of 0.05 m.

4 RESULTS

The visual observations in the laboratory have shown that no sediment transport occurred at flow discharge of 80 l/s. Since no sediment transport occurred in the physical experiment, this run was used investigate the ability of the numerical model to compute the hydraulic situation in the contraction geometry. To this effect, the velocity at free surface of the model is compared against the experimental observations. This test case is used to validate the numerical model. Figure 2 and 3 show free stream velocities measured in the experiment and calculated by the numerical simulations until steady state.

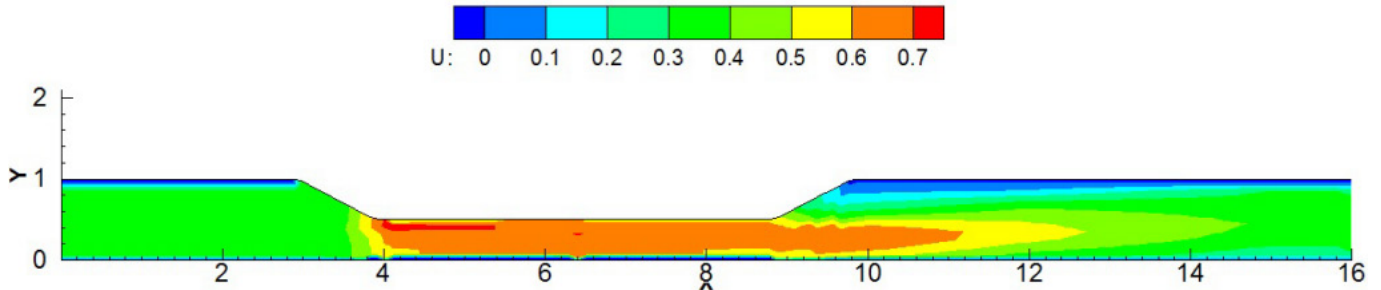


Figure 2. Contraction: Mean velocity at free surface for $Q=80$ l/s (Experiment from [2]).

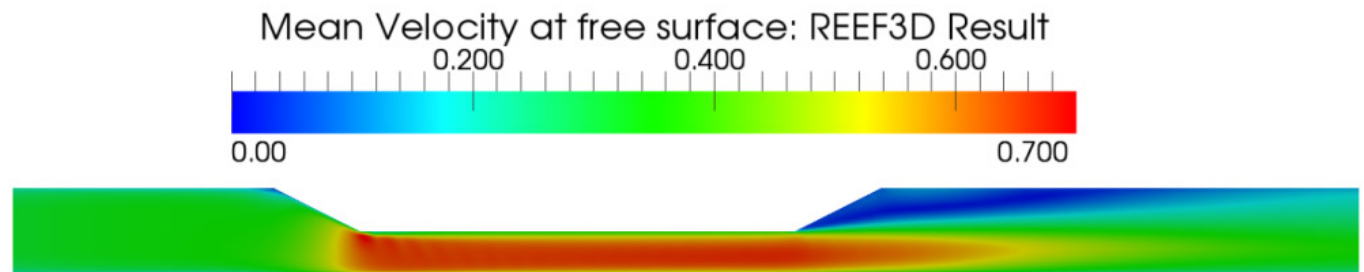


Figure 3. Contraction: Mean velocity at free surface for $Q=80$ l/s (REEF3D).

The computed flow pattern is quite similar to that observed in the laboratory experiment. As expected, the velocity increases in the contracted channel since water elevation and width decrease. The water slows down in the expansion. High velocities up to 0.70 m/s is noticed in the numerical simulation which is very close to the one obtained in the experimental results. However, at this flow discharge, the bed shear stress does not exceed the critical shear stress of the channel bed, so that no scour occurs in the channel. A recirculation zone was present during the numerical simulation at the free surface and at the bed. The area of high velocities due to the contraction is identical to the experimental result.

Figure 4 shows the bed elevation changes after 150 minutes of the experiment for a discharge of 150 l/s. The color legends of the figure 4 are similar to those of figure 5. The scour occurs first at the beginning of the contraction and then develops downstream, along the contraction part of the channel. The eroded sediment from the scour hole settles down as it moves downstream into the expansion region with lower velocity and bed shear stress to form a deposition area downstream of the scour hole. The observed maximum scour hole in the experiment extends along the contracted bank.

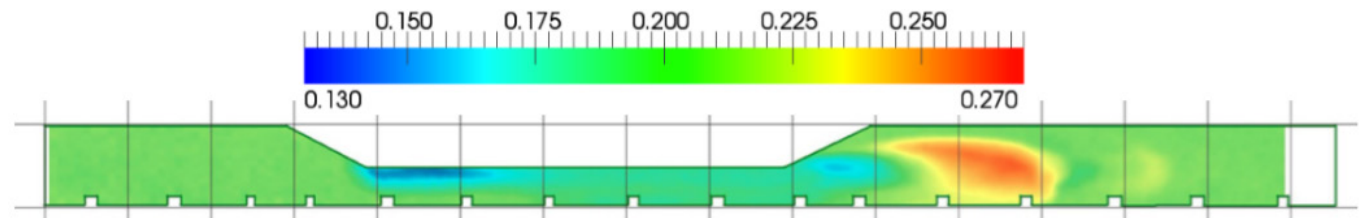


Figure 4. Contraction: Bed elevation changes after 150 mins for $Q=150$ l/s (Experiment (from [1])).

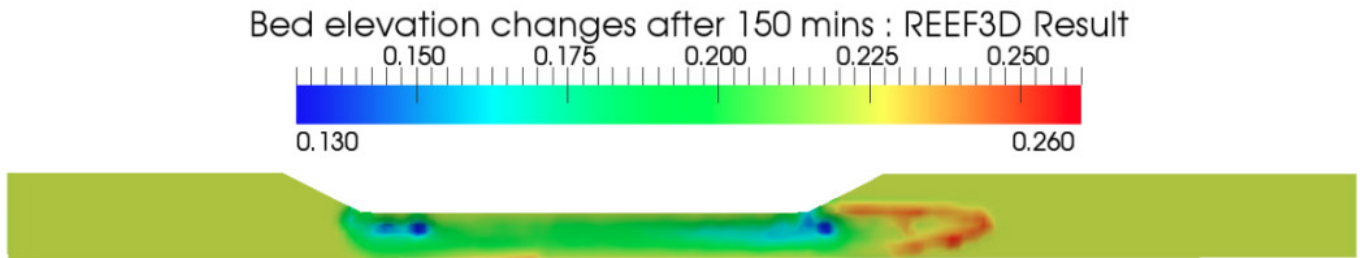


Figure 5. Contraction: Bed elevation changes after 150 mins for $Q=150$ l/s (REEF3D).

Figure 5 shows the bed elevation changes after 150 minutes of the numerical simulation. The predicted maximum scour depth (7 cm) occurs at the location close to and behind the start of the contraction and also near the end of contracting extending to the beginning of the expansion. This corresponds well with the experimental results which has a scour depth of 7 cm. The scour depth after the contracted part is well predicted too (7 cm). In the numerical model the eroded sediment is deposited around 1.5 m after the start of expansion. This behaviour correspond well with the physical experiments where the sediment was transported through the expansion and settles about 1.5 m further downstream. There is slight difference in deposition height. The numerical results predicted deposition of 6 cm compared to experiments (7 cm). The numerical model predicted a recirculation near the bed in the expansion which is similar to the experimental results. Scour depth, its location, deposition height and its location are predicted well with the numerical model.

5 CONCLUSIONS

A study of contraction scour has been performed using the sediment transport module of REEF3D. The numerical model predicts the general evolution (geometry, location and maximum depth) of scour, deposition height and its location accurately with very minor differences. Since these characteristics are most relevant in hydraulic engineering applications, the performance of the numerical is assumed to be satisfactory in prediction of contraction scour. The results predicted by numerical model are better than found by Bihs and Olsen [2]. The numerical model is also able to represent the complex free surface using the level set method. Figure 6 shows the bed elevation changes in combination with representation of the complex free surface under constant discharge conditions in a very realistic manner using a three dimensional plot of the numerical model result after 150 minutes of simulation.

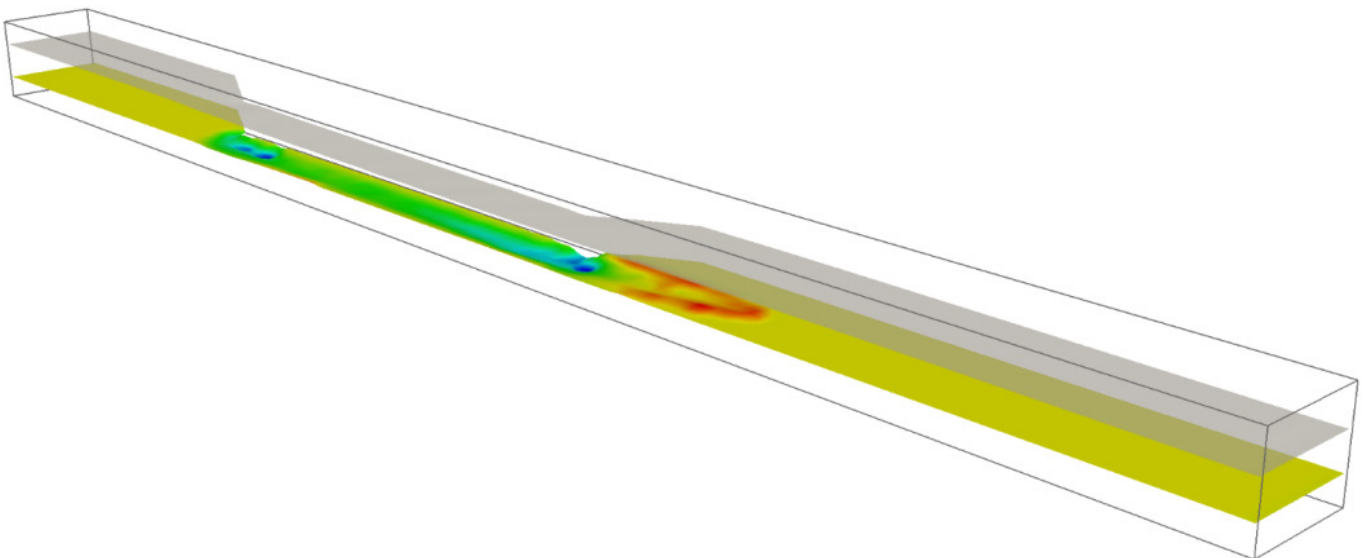


Figure 6. Contraction: 3D Model result under constant discharge showing free surface and topography (REEF3D).

REFERENCES

- [1] BAW. Morphologische versuche an einer rinne mit ein- schnürung mittel- bis feinkiessohle. Company report: Baw-nr: A39530110064-01, 2009.
- [2] H. Bihs and N. R. B. Olsen. Three-dimensional numerical modeling of contraction scour. In Proc., 32st Cong. of IAHR, Venice, Italy, 2007.
- [3] R. Croce, M. Griebel, and M. A. Schweitzer. Evaluating scour at bridges. Preprint, Federal Highway Administration, Hydraulics engineering circular No. 18, Publication FHWA HI-96-031, Washington, D.C, 1995.
- [4] B. Duc and W. Rodi. Numerical simulation of contraction scour in an open laboratory channel. *Journal of Hydraulic Engineering*, 134(4):367–377, 2008.
- [5] H. A. Einstein. The bed-load function for sediment transportation in open channel flows. Technical Bulletin 1026, U.S. Dept. of the Army, Soil Conservation Service, 1950.
- [6] F. Engelund and J. Fredsøe. A sediment transport model for straight alluvial channels. *Nord. Hydrol.*, 7(5):293–306, 1976.
- [7] G. S. Jiang and C. W. Shu. Efficient implementation of weighted eno schemes. *Journal of Computational Physics*, 126:202–228, 1996.
- [8] M. Marek and A. Dittrich. 3d numerical calculations of the flow in an open channel consisting of an expansion and a contraction. In Proc., 6th Int. Conf. on Hydro-Science and -Engineering (CD-ROM), 2004.
- [9] S. Osher and J. A. Sethian. Fronts propagating with curvature- dependent speed: Algorithms based on hamilton-jacobi formulations. *Journal of Computational Physics*, 79:12–49, 1988.
- [10] S. V. Patankar and D. B. Spalding. A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. *International Journal Heat Mass Transfer*, 15:1787–1806, 1972.
- [11] van der Vorst H. BI-GStab: A fast and smoothly converging variant of BI-CG for the solution of nonsymmetric linear systems. *SIAM J. Sci. Stat. Comput.*, 13:631–644, 1992.
- [12] L. C. van Rijn. Sediment transport, part ii: Suspended load transport. *Journal of Hydraulic Engineering*, 110(11):1613–1641, 1984b.
- [13] S. Weise. Verifikation eines zweidimensionalen feststofftransportmodells anhand von hydraulischen versuchen. Master’s thesis, Hochschule für Technik, Wirtschaft und Kultur Leipzig (FH), 2002.
- [14] D. C. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., La Canada, California., 1994.
- [15] J. Zeng, G. Constantinescu, and L. Weber. A fully 3d non- hydrostatic model for prediction of flow, sediment transport and bed morphology in open channels. In n Proceedings of the 31st IAHR Congress, pages 1327–1338, 2005.