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A Coupled Sediment Transport Model for Flow over Movable Bed

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ABSTRACT: A numerical approximation for bed load sediment transport due to shallow-water flows is studied. Flows over an erodible bed are solved by assuming that the dynamics of the bed load problem is described by two mathematical models: the hydrodynamic model, represented by the depth integrated shallow-water equations, and the morphological model: Exner equation. The model solves the coupled shallow-water and Exner equations, with the interface fluxes evaluated by a Harten-Lax van Leer-Contact (HLLC) approximate Riemann solver for hydrodynamic part and HLL scheme for morphological part. The domain is discretized by triangular grid system which facilitates representing complex domains. A special treatment of the source terms by surface gradient method makes the scheme stable and physically balanced. The model employs Godunov- type finite volume algorithm, with second-order accuracy in space and time. Spatial second-order accuracy is guaranteed by employing multidimensional gradient reconstruction method; and predictor-corrector scheme is used to ensure a second-order accurate solution in time. Finally, some numerical tests are presented to verify the applicability and accuracy of the proposed numerical model.

Keywords: Sediment transport, Dam-break, Finite volume method, Coupled model, Approximate Riemann Solvers

1 INTRODUCTION

The study of morphological flow is important in the field of water resources to understand the behavior of sediment movement under different flow conditions. The level of risk posed by dam-breaks to the society, economy and environment, makes it important to understand and study the underlying hydrodynamics and morphological phenomena associated with dam-breaks. Understanding the water flow mechanism and downstream bed evolution can help mitigate the damage.

As scouring and deposition are two major problems encountered by engineers, various numerical models have been developed to understand these processes. Water reclamation projects and design of hydraulic structures are rendered unreliable unless the assessment of scour pattern is carried out for them. Due to lower costs and less computational time 2-dimensional numerical modelling is preferred over laboratory experiments and 3-dimensional modelling, to give the quick insight into the sediment-transport pattern.

In this paper the mathematical model of morphodynamics is based on shallow water equations and a transport equation for sediment (Exner equation). The set of these equations could be modelled numerically either by a coupled approach or by solving the hydrodynamic flow first to get the flow velocity and then updating the sediment continuity equation. The former approach is used in case of rapid flow variations, and the latter in case of slow flood propagation (Hudson et al., 2005; Liang, 2011). In this paper a coupled finite volume scheme is employed to focus on two dimensional morphodynamics of bedload transport in channels.

To solve coupled equations numerically on a triangular grid system in a single step, we employed a cell-centered Godunov-type finite volume method. According to (Liu et al., 2008), 2-dimensional models can give quick assessment of the scour pattern and relatively accurate maximum scour depth. The cell centered average state variables are extrapolated to the center of the interface by multidimensional gradient reconstruction method proposed in (Jawahar and Kamath, 2000), to get second-order spatial accuracy. Temporal second-order is achieved by employing predictor-corrector solution steps. Approximate Riemann solvers (Toro) are used to compute the fluxes at cell interfaces. HLLC and HLL schemes are employed for computation of hydrodynamic and morphological fluxes respectively.

The objective of this work is to develop a numerical model that can accurately predict channel scour, and predict the position of hydraulic jump in case of dam-break flows without causing any discontinuities in the solution.

2 GOVERNING EQUATIONS

2.1 *Shallow water equations*

The hydrodynamic component of the flow is modelled by non-linear shallow water equations, which in 2-dimensions can be expressed as

$$\begin{aligned} &\frac{\partial}{\partial t}(h)+\frac{\partial}{\partial x}(hu)+\frac{\partial}{\partial y}(hv)=0 \\ &\frac{\partial}{\partial t}(hu)+\frac{\partial}{\partial x}\left(hu^2+\frac{1}{2}gh^2\right)+\frac{\partial}{\partial y}(huv)=gh(S_{ox}-S_{fx}) \\ &\frac{\partial}{\partial t}(hv)+\frac{\partial}{\partial x}(huv)+\frac{\partial}{\partial y}\left(hv^2+\frac{1}{2}gh^2\right)=gh(S_{oy}-S_{fy}) \end{aligned} \tag{1}$$

where *h, u*, and *v* represent water surface elevation, velocity in *x*-direction, and velocity in *y*-direction respectively. S_{ox}, S_{oy} are bed slope source terms, and S_{fx} and S_{fy} are friction terms in in *x* and *y* directions respectively.

$$S_{ox}=-\frac{\partial b}{\partial x},\ S_{oy}=-\frac{\partial b}{\partial y} \tag{2}$$

$$S_{fx}=\frac{n^2u\sqrt{u^2+v^2}}{h^{4/3}},S_{fy}=\frac{n^2v\sqrt{u^2+v^2}}{h^{4/3}} \tag{3}$$

where *b* is bed elevation and *n* is the Manning's roughness coefficient.

2.2 *Bed evolution equation*

$$\frac{\partial b}{\partial t}+\frac{1}{1-p}\frac{\partial q_{sx}}{\partial x}+\frac{1}{1-p}\frac{\partial q_{sy}}{\partial y}=0 \tag{4}$$

where *p*, q_{sx}, and q_{sy} represent sediment porosity and sediment discharge in *x* and *y* directions respectively. The sediment discharge in this study is approximated by Grass (Grass, 1981) equation.

$$q_{sx}=Au(u^2+v^2)^{\frac{c-1}{2}},q_{sy}=Av(u^2+v^2)^{\frac{c-1}{2}} \tag{5}$$

where *c* is related to the properties of sediment; for fine sand (selected in our study) it's value is 3. *A* is a constant varying between 0 and 1 representing the intensity of the interaction between flow and sediment particles, with 1 standing for the strongest interaction. We chose *A* equal to 0.01 in our study for all test cases.

2.3 *Coupled Model*

The equations for coupled numerical model are expressed as

$$\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}+\frac{\partial \boldsymbol{G}}{\partial y}=\boldsymbol{S} \tag{6}$$

where $\boldsymbol{U}$ presents vector of conserved variables; fluxes in the x- and y-axes are represented by $\boldsymbol{E}$ and $\boldsymbol{G}$ respectively; and $\boldsymbol{S}$ is a vector of source terms. In vector form Eq (6) can be expressed as

$$\boldsymbol{U}=\begin{bmatrix} h \\ hu \\ hv \\ b \end{bmatrix}, \boldsymbol{E}=\begin{bmatrix} hu \\ hu^2+1/2gh^2 \\ huv \\ \frac{A}{1-p}u(u^2+v^2) \end{bmatrix}, \boldsymbol{G}=\begin{bmatrix} hv \\ huv \\ hv^2+1/2gh^2 \\ \frac{A}{1-p}v(u^2+v^2) \end{bmatrix}, \boldsymbol{S}=\begin{bmatrix} 0 \\ gh(S_{ox}-S_{fx}) \\ gh(S_{oy}-S_{fy}) \\ 0 \end{bmatrix} \tag{7}$$

For a control volume with area A and boundary Γ, Eq. (6) can be expressed in integral form as

$$\int_A \frac{\partial \boldsymbol{U}}{\partial t} dA + \oint_\Gamma \boldsymbol{F}(\boldsymbol{U})\cdot \boldsymbol{n}\, d\Gamma = \int_A (\boldsymbol{S}_o + \boldsymbol{S}_f)\, dA \tag{8}$$

where **n** is the unit outward normal, and $\boldsymbol{F}$ is a flux vector. When summed over all the edges m of the control volume, Eq. (8) is expressed as

$$\frac{\partial \boldsymbol{U}_i}{\partial t} = -\frac{1}{\Delta A_i}\sum_{m=1}^{3} \boldsymbol{F}_m(\boldsymbol{U}_m)\cdot \boldsymbol{n}_m l_m + \boldsymbol{S}(\boldsymbol{U}_i) \tag{9}$$

where i is the index of cell under consideration, and l_m is the length of interface. Solution is obtained in two steps; the predictor step in which the solution advances by half a time step and fluxes are computed locally is given as

$$\boldsymbol{U}_i^{n+1/2} = \boldsymbol{U}_i^n - \frac{\Delta t}{2}\left[\sum_{m=1}^{3} \boldsymbol{F_m}(\boldsymbol{U}_m)^n / A\cdot \boldsymbol{L_m} + \boldsymbol{S}^n\right] \tag{10}$$

In corrector step the Approximate Riemann solvers are employed to calculate the fluxes at the interface and solution is advanced to a full time step.

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \Delta t\left[\sum_{m=1}^{3} \boldsymbol{F_m}(\boldsymbol{U}_m^L, \boldsymbol{U}_m^R)^{n+1/2} / A\cdot \boldsymbol{L_m} + \boldsymbol{S}^{n+1/2}\right] \tag{11}$$

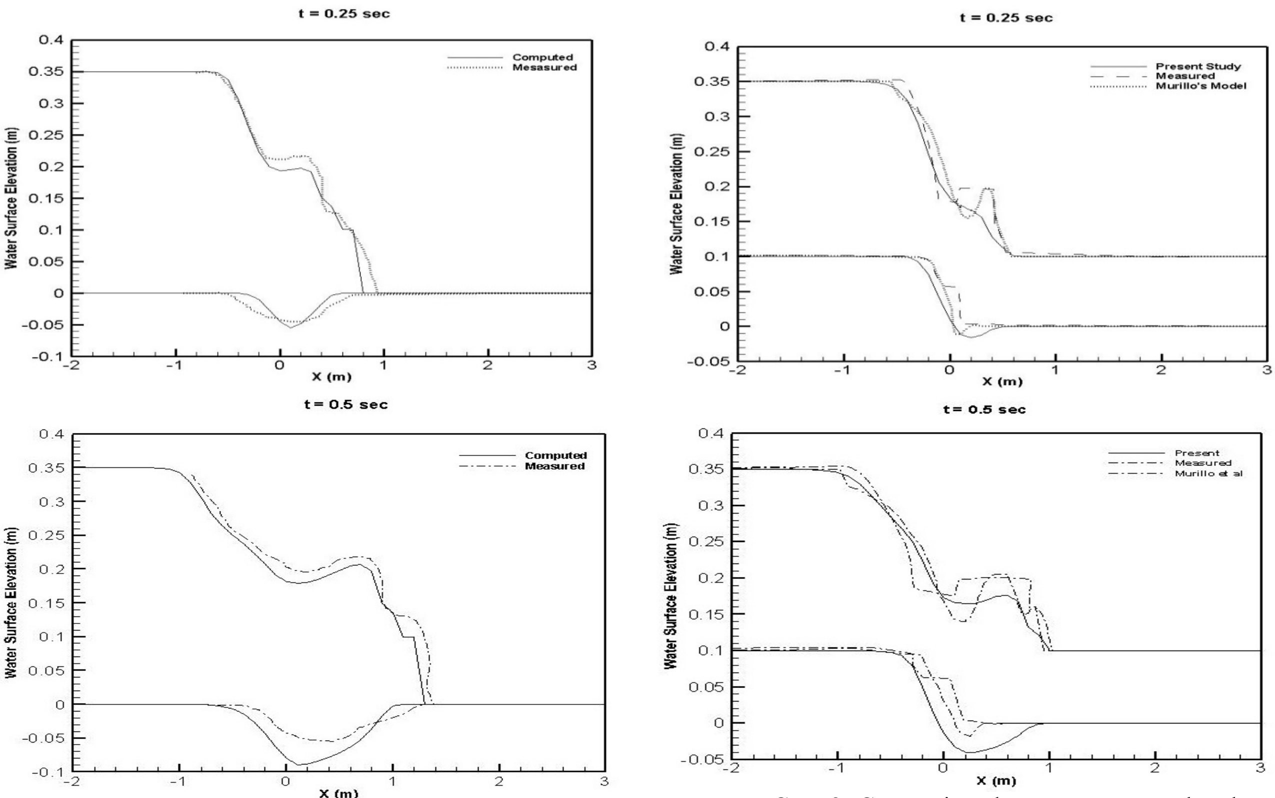
Wave speeds used in flux calculation are approximated by eigenvalues of Jacobian matrix for flux components shown in Eq. (7). Estimated eigenvalues and the procedure to calculate them is explained in (Soares-Frazão and Zech, 2011). Because of the explicit nature of the proposed scheme, the time step is limited by CFL condition. Solid and open boundary conditions are applied at the side walls and downstream boundary respectively.

3 TEST CASES

3.1 *Sediment-Transport for Dam-Break Flow*

We tested the validity of our model for experiments conducted at UCL Civil Engineering Department lab Belgium (Spinewine and Zech, 2007). A flume of length 6 m, 0.25 m wide, and 0.70 m high was used in experiments. The study considers an idealized dam-break over an erodible bed. A dam-break is simulated by removing a thin gate in the middle of the flume. In our study we simulate the model for two cases shown in Figure 1. The domain in all cases is discretized by 2-dimensional triangular mesh, and a CFL number of 0.6 is considered. The values of calibration constant (A), sediment porosity (p), and Manning's coefficient (n) are selected equal to 0.01, 0.47, and 0.0165, respectively, in both cases.

As the initial conditions for case 1 and 2 the water depth upstream of the dam is equal to 0.35 and 0.25 respectively; for case 1, water depth and downstream of dam is 0.0, and 0.1 for case 2. Initial sediment layer of thickness is equal to 0.125 for case 1 throughout the length of channel, while for case 2 the sediment thickness is 0.1m higher upstream of the dam. Figure 1 shows the comparison of our results with results in literature (Li and Duffy, 2011; Murillo and García-Navarro, 2010; Wu and Wang, 2007). Hydraulic jump is formed on the site of dam-break due to greater rate of bed erosion. Present study shows greater rate of bed erosion in both cases, while the position of hydraulic jump shows satisfactory agreement with other results. The comparison in Figure 1 shows fairly good agreement, implying that the proposed finite volume scheme is able to capture hydraulic jump and predict bed erosion for dam-break flows.



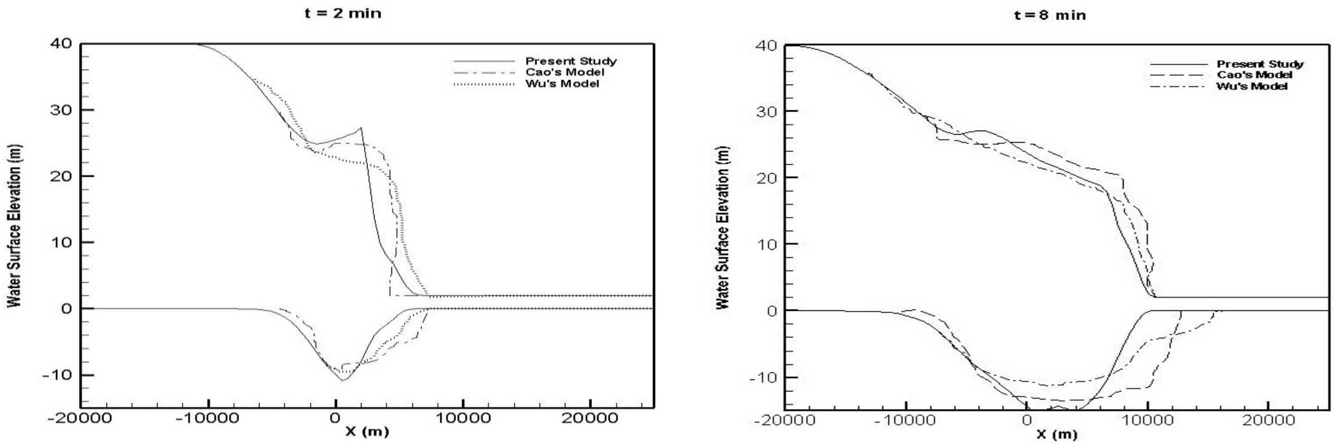
Case 1: Comparison between measured and computed results at two time intervals

Case 2: Comparison between measured and computed results at two time intervals

Figure 1. Laboratory Scale Dam-break Profiles for Two Cases Using Grass Formula

3.2 *Field scale Dam-break test*

In this test a river reach 50 km long, and 1 km wide is considered with a hypothetical wall in the middle of the channel acting as a dam. This test has been previously performed by (Cao et al., 2004; Wu and Wang, 2007) to check the bed erosion and hydrodynamics of flow under large-scale dam-break-flow. Upstream and downstream water depths of 40 m and 2.0 m respectively, are taken as initial conditions. The values of calibration constant (A), sediment porosity (p), and Manning's coefficient (n) are equal to 0.01, 0.4, and 0.3, respectively. Figure 2 shows the water surface and bed profiles for the present study and its comparison with the results of Cao et al. and Wu et al. after 2 min and 8 min of the dam-break. Significant erosion is shown for field-scale dam-break flow. It can be observed from the figure that as the hydraulic jump propagates downstream with time. The difference in comparisons can be attributed to the different formulas of sediment-transport used in (Cao et al., 2004; Wu and Wang, 2007). The increased bed erosion predicted by (Cao et al., 2004) shown in Figure 2 is attributed to the fact that no upper bound was placed on entrainment function which results in over prediction of bed erosion (Wu and Wang, 2007). Overall the comparison of our model results with other studies shows satisfactory agreement.



a) Flow & Bed profile after 2 min of dam-break

b) Flow & Bed profile after 8 min of dam-break

Figure 2. Field Scale Dam-break Flow Simulated with Grass Formula

4 CONCLUSIONS

In this paper scour pattern of the bed under dam-break flows and the formation of hydraulic jump is studied, to understand the wave behavior and bed-erosion pattern. Assessment of maximum water height can help designing safety structures like dykes and banks to avoid flooding of adjacent flood plains. In all cases, the bed friction and calibration constant (A), which represents the intensity of the interaction between water and sediment particles, greatly influenced the stability and accuracy of the model, computation time, and position of hydraulic jump. . We used a second-order finite-volume method of Godunov type to solve the coupled system of shallow-water and Exner equations. The accuracy of the model was tested for a number of benchmark tests highlighting the ability of the model to accurately capture the position of hydraulic jump and magnitude of bed erosion. The observations show the robustness, and efficiency of model under different conditions. The small differences with other studies might be attributed to a number of sediment characteristics that were considered in more complex transport formulas in other models. Overall satisfactory results were obtained for all tests.

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