

Application of Multi-Site Stochastic Daily Climate Generation to Assess the Impact of Climate Change in the Eastern Seaboard of Thailand

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ABSTRACT: In the assessment of climate change impacts on future meteorological regimes, downscaling of large-scale climate/weather variables from GCMs is usually applied. Depending on the GCM, the predictors are either available on the monthly or on the daily scale, wherefore, for obvious reasons, the monthly predictions of a GCM are considered to be more reliable for long-term climate impact studies. Nevertheless, in many instances, it is desirable to have predictors on a daily scale, e.g. for the study of short-term seasonal climate fluctuations and extreme events. This requires the rescaling of monthly predictor data to a daily series. Here we present a novel daily weather (climate) -generator (DWG) to do this properly. The new DWG employs various statistic and stochastic techniques to synthesize daily climate from several ensembles of daily series from different climate sites, respecting the relevant statistical attributes of the various monthly climate series, but also their spatial correlation properties between the different sites (multi-site approach). This multi-site/-realization of the synthetic daily climate can exhibit a broad spectrum of climate variability that can be useful in a practical climate assessment, as this approach provides also some uncertainty measure. The DWG proposed here processes the daily precipitation- and temperature- series separately, wherefore for the former both the monthly downscaled rainfall intensity and the probability of rainfall occurrence are employed. For past observed meteorological data in the study region, which is the eastern seaboard of Thailand, the stochastic properties of the daily multi-realizations are conditioned on the observed time series. The performance of the new DWG is compared with those of other classical downscaling methods and shows some advantages.

Keywords: Stochastic daily weather generator; Multi-site; Downscaling; Climate impact study

1 INTRODUCTION

The assessment of climate change impacts on future meteorological and/or hydrological regimes usually requires the downscaling of large-scale climate/weather predictors from GCMs (Wilby *et al.*, 1998, 2002). Depending on the GCM used, the predictors are either available on the monthly or on the daily scale, where the use of the latter is of particular interest, when studying impacts related to shorter-term behavior, e.g., storms and/or floods. However, the direct use of daily climate predictions from one GCM is usually not reliable enough to represent the full variability of the climate variable's time series, namely, its extreme behavior. Notwithstanding that daily climate predictors are available for some GCM-models, their reliability is considered lower than that of monthly GCM predictors. For this reason, downscaling of monthly predictor data may be more recommendable. However, the subsequent step to generate daily series from such a downscaled monthly climate series becomes then a tricky task (Wilks, 1998).

In the present paper a novel or daily weather (climate) -generator (DWG) is presented which regenerates daily from monthly climate data, such that it will render changes in the daily sequencing of an observed series, while still reflecting the intra-month variability of the observed climate event series in a statistically responsible manner (Maurer and Hidalgo, 2008). The basic technique used in this DWG is similar to spatial climate downscaling, where finer-scale variables are generated from larger-field data by following the data sample's statistical properties. With a DWG low-resolution climate projections can be rescaled to a broader spectrum of long-term predictions of daily climate and their effects on the hydrolo-

gy and the water supply in a region be studied than is possible with a regular (monthly-scale) downscaling approach (Wilson *et al.*, 1992, Wilby *et al.*, 1998; Bejranonda, 2014).

Stochastic daily climate generation has been widely used in impact assessments, because of their advantage of easily generating multiple climate ensembles which are useful for statistical risk analysis (Wilby, 1994; Wilby *et al.*, 2002). In this stochastic approach, also known as weather classification, the major statistical attributes of the observed climate time series at a particular site are provided to replicate the persisting climate by multi-realizations of the local weather (Wilby, 1994; Wilks and Wilby, 1999).

The generation of a daily climate series is based on some conditioning of the climate properties and the weather states, i.e. the occurrence of wet or dry conditions (Katz, 1996; Semenov and Barrow, 1997; Wilks, 1998; 1999a;b). This approach was originally proposed by Richardson (1981) who used a first-order Markov chain process to define the occurrences of wet and dry states, based on the distributions of the observed rainfall sequences. In addition, various theoretical statistical distributions, e.g. exponential, gamma, mixed-exponential and log-normal distributions, have further been applied to fit the observed precipitation distributions (Liu *et al.*, 2011). Many daily weather generation models developed over the last few decades, e.g. WGEN (Richardson, 1981), SIMMETEO (Geng, 1988), WXGEN (Hayhoe and Stewart, 1996; Hayhoe, 2000), MARKSIM (Jones and Thornton, 2000) and MODAWEC (Liu *et al.*, 2009) are based on these few fundamental concepts.

All of the above mentioned daily weather generators are fundamentally based on “single-site” weather which is not practical for assessing climate at the regional scale. Thus, extensions of this single-site climate generation by means of an integration of the spatial correlation pattern (Cliff and Ord, 1981; Hubert *et al.*, 1981; Upton, 1985) of the distributions of climate data at different locations have been proposed (e.g. Wilson *et al.*, 1992; Hughes and Guttorp, 1994; Charles *et al.*, 1999; Wilks, 1998; 1999a; Wilby *et al.*, 2003; Brissette *et al.*, 2007; Khalili *et al.*, 2007; 2009). Such a multi-site DWG will also be developed in the present paper and applied to the study region.

2 STUDY AREA AND DATA

Thailand’s eastern seaboard (EST) industrial zone, located in the Chonburi and Rayong provinces in the eastern coastal zone of that country, has been promoted to become a major area for industrial and tourist development over the last two decades. Thus it is of no surprise that the concomitant increasing water demand has led to significant stress on the water resources in the EST in recent years (Bejranonda, 2014). This became particularly imminent during the multi-seasonal drought in year 2005, which brought the industrial production in the area partly to a hold. There is now sufficient evidence that the named extreme weather conditions of 2005 occurring in that part of Thailand are not a singularity, but might be another signal of recent ongoing climate change in that country as a whole. In fact, this situation is bound to be aggravated over the whole 21st -century, as indicated by the results of an analysis of downscaled GCM-climate predictions of Bejranonda and Koch (2010) and Bejranonda (2014).

Data used in the present analysis and particularly for the calibration and validation of the DWG are records of daily maximum and minimum temperatures between 1971-2006 at four sites and of daily precipitation at 24 sites (see Fig.1).

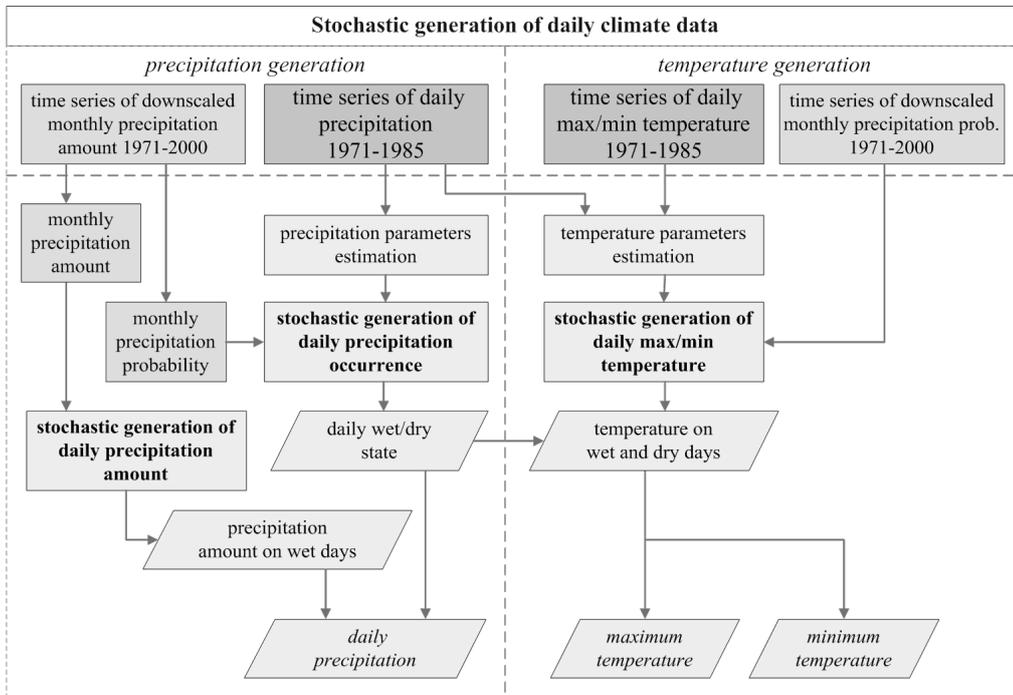


Figure 2. Schematic concept of the daily climate generator developed in this study to reproduce daily values of precipitation and temperature between 1971-2000, wherefore the 1971-1985- time period is used for calibration of the generator.

3.2 Multi-site daily climate generation

The major idea behind the multi-site DWG is that, because of the very high temporal and spatial fluctuations of climate variables, namely, the rainfall, its distribution is very distinct at different site locations, especially, in large-scale watersheds (Wilks, 1998; Srikanthan and McMahon, 2001; Khalili *et al.*, 2009). This means that by taking into account the spatial autocorrelation of the multi-site distributions of the relevant climate variables at different sites, a more reliable outcome is achieved (Cliff and Ord, 1981; Hubert *et al.*, 1981; Upton, 1985). Such a spatial autocorrelation is constructed under the concept of Tobler (1970) “Everything is related to everything else, but near things are more related than distant things”.

The basics of the spatial autocorrelation approach which has been applied for capturing patterns of climate for generating multi-site weather (e.g. Brissette *et al.*, 2007; Khalili *et al.*, 2007; 2009), is an important spatial statistical parameter, the so-called *Moran's I*, defined as (Moran, 1950):

$$I = \frac{\sum_{i=1}^n (x_i - \bar{x}) \sum_{j=1}^n \omega_{ij} (x_j - \bar{x}) / \sum_{i=1}^n \sum_{j=1}^n \omega_{ij}}{\sum_{i=1}^n (x_i - \bar{x})^2 / n} \quad (1)$$

where x_i is the observed value of climate variable x (precipitation or temperature) at location i , \bar{x} is the average of the x_i over n locations, and ω_{ij} are the spatial weights, computed as the inverse of the squared distance d_{ij} between point i and j , wherefore ω_{ij} is normalized by the total sum of weights in a row $\sum_{j=1}^n d_{ij}$, so that the sum of every row equals 1 and for the diagonal member $\omega_{ij} = 0$.

In the stochastic approach, the generation of random numbers is particularly important, as they are used to define the distribution of the synthetic data. These spatially autocorrelated random numbers are generated by applying a spatial moving average process on a set of uniformly distributed random numbers in the form (Cliff and Ord, 1981; Cressie, 1993; Khalili *et al.*, 2007):

$$V = \gamma \times W \times u + u \quad (2)$$

where V is a vector of size $(n, 1)$ of spatially autocorrelated random numbers of n locations, γ is the moving average coefficient which is estimated as discussed in a subsequent section, W is the $n \times n$ weight matrix, consisting of the weighting coefficients ω_{ij} above, and u is a $n \times 1$ vector of n independent and uniformly distributed random numbers in the range $[0, 1]$. The range of the γ -coefficient is defined by the eigenvalues of the weight matrix (Khalili *et al.*, 2007), i.e. lies between $-1/W_{max}$ and $1/W_{max}$, with W_{max} and W_{min} the largest positive and negative eigenvalue, respectively.

As the generated autocorrelated numbers V in Eq. (2) may not be any longer uniformly distributed, the empirical cumulative distribution function (ECDF) is used to convert these back into the $[0, 1]$ range:

$$Vn_{k,i,d,rlz,u} = F_{nml}^k(V_{i,d,rlz,u}) = N(V_{rlz,u=1...1000}[k]) \in [0..1] \quad (3)$$

where $N()$ is the normalized function of the autocorrelated random numbers V , based on the empirical distribution of 1000 realizations of $V_{rlz.u=1\dots 1000}$ at station k . Consequently, the function F_{nml}^k is driven by the spatially autocorrelated random numbers $V_{i,d,rlz.u}$ to provide normalized values $Vn_{k,i,d,rlz.u}$ for month i on day d of realization $rlz.u$ at site k which all lie in the $[0,1]$ range which, after reversing the standardization, are used to generate the amount of precipitation and the temperature values at station k .

3.3 Generation of precipitation occurrence

While the general multi-site procedures outlined above apply for both the generation of the daily precipitation amount and the temperature, for the former the occurrence of the wet/dry conditions must be defined first, as, obviously, rainfall can only occur on a wet day. Among the various approaches used in the scientific literature for the generation of daily rainfall occurrence, the chain-dependent technique, which is based on a first-order, two-state Markov process, has most frequently been applied (e.g. Todorovic and Woolhiser, 1975; Katz, 1977; Waymire and Gupta, 1981; Stern and Coe, 1984; Katz and Parlange, 1995; Qian *et al.*, 2002). In this two-state Markov model wet or dry days are classified, depending on the amount of rainfall for that day, i.e. if the latter is greater than 0.1 mm/day, the day is defined as a wet day, and vice versa. The series of rainfall occurrence on day t at site k is then defined as (Qian *et al.*, 2002):

$$X_t(k) = \begin{cases} 0 & | \text{ dry day} \\ 1 & | \text{ wet day} \end{cases} \quad (4)$$

The next step in the Markov process consists in the definition of the transition probabilities p_{01} and p_{11} between two consecutive days, defined as (Corte-Real *et al.*, 1998; Qian *et al.*, 2002):

$$p_{01}(k) = Pr[X_t(k) = 1, | X_{t-1}(k) = 0]; \quad p_{11}(k) = Pr[X_t(k) = 1, | X_{t-1}(k) = 1] \quad (5)$$

i.e. p_{01} and p_{11} are the probabilities of a wet-day occurrence, when the previous day has been dry or wet, respectively. These probabilities are determined from the observed empirical probabilities (relative frequency) $Pw_i(k)$ of the countable wet days for a particular month i through

$$p_{01}(k) = f_{p01}(Pw_i(k)); \quad p_{11}(k) = f_{p11}(Pw_i(k)) \quad (6)$$

The functions $f_{p01}()$ and $f_{p11}()$ in Eq. (7) are polynomial functions determined from a regression of the observed p_{01} and p_{11} over the observed $Pw_i(k)$. Fig. 3 exhibits these polynomial functions for three rainfall stations 48092 for months September and December which are the months of lowest and highest precipitation in the study region, respectively, using the observed rainfall data between years 1971-2006.

3.4 Generation of precipitation amount

Once a wet day d has been synthesized, as outlined above, the precipitation amount for that day $R_{i,d,rlz}(k)$ is generated by inverting the ECDF of the vector of the normalized spatially autocorrelated random numbers $Vn_{i,d,rlz.u}$ (Eq. 3) - after fitted by an exponential cumulative distribution function (*Efit*) (Khalili *et al.*, 2007) - and scaled appropriately - to ensure the conservation of the monthly precipitation amount - by $Rmean_{k,i}$ the mean monthly rainfall at station k of month i and $Dwet_{k,i,rlz}$, the corresponding cumulative number of wet days, obtained from the precipitation occurrence generation. This results in

$$R_{i,d,rlz}(k) = -\ln(1 - Efit(Vn_{i,d,rlz.u}[k])) \cdot Rmean_{k,i} / Dwet_{k,i,rlz} \quad (7)$$

By using the spatially autocorrelated random numbers of Eq.(3), the synthetic precipitation is generated for 30 realizations ($rlz=1,\dots,30$), to produce an statistical meaningful ensemble set of the precipitation.

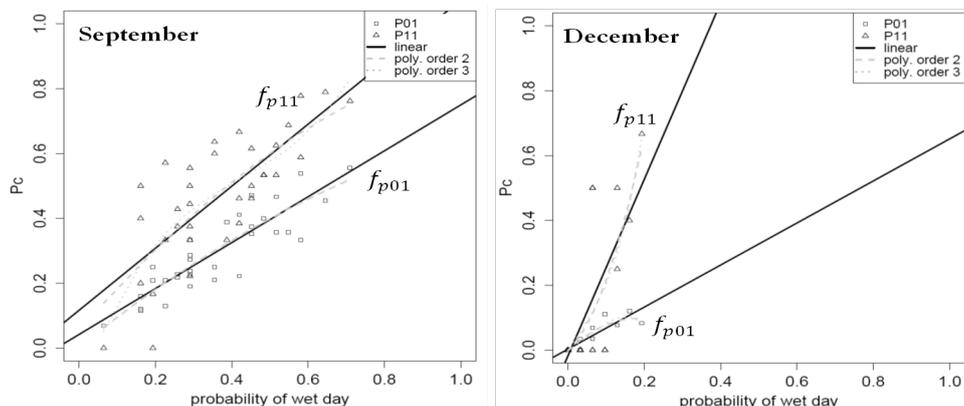


Figure 3. Regressions of transition probabilities p_{01} and p_{11} (abbreviated as p_c on the vertical axis) on the average probability of a wet day (%wet) for rainfall station 48092 for months September and December for all years between 1971-2006.

3.5 Estimation of the moving average parameter from empirical Mohran's I .

In order to condition the generated multi-site rainfall on the observed/predictor data, the moving average coefficient γ in Eq. (2) must be appropriately selected. This is achieved by producing 30 realizations of Eq. (2) with different γ , i.e. 30 rainfall generations (Eq. 7) are done for each month $i=1, \dots, 12$ of the year, from which *Moran's I* (Eq.1) is derived. The tuples (γ, I) define a function $\gamma = f_i(I)$ which is determined by linear or polynomial regression, as shown in Fig. 4 for the months September and December.

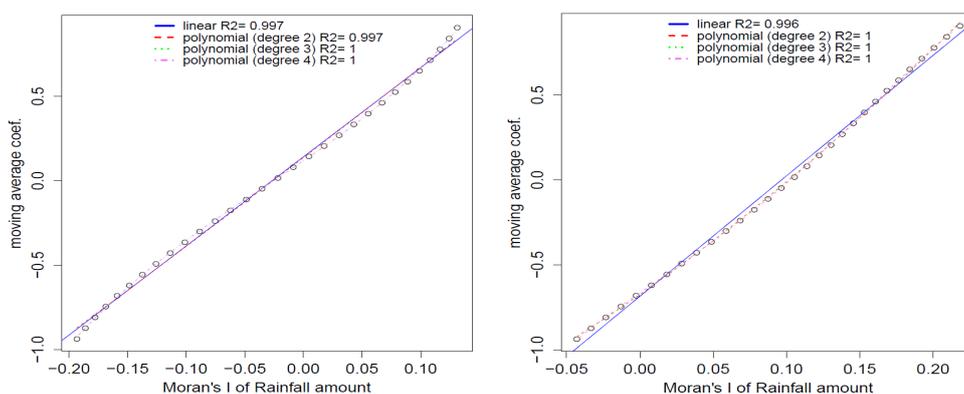


Figure 4. Relationship between *Moran's I* of 24-sites daily precipitation and moving average coefficients in September and December for years 1971-2006 fitted with polynomial regressions.

The average *Moran's I* of a specific day d in month i is then calculated from the 36-year-long, 366-day *Moran's I* series over the observed validation period, to define the *Moran's I* for that day of the month which, employing the associated monthly regression function $\gamma = f_i(I)$, is then used to compute the appropriate moving average coefficient for the final precipitation generation. Fig. 5 shows the empirical daily *Moran's I* for the 1971-2006 rainfall data over the EST and their 36-year averaged values.

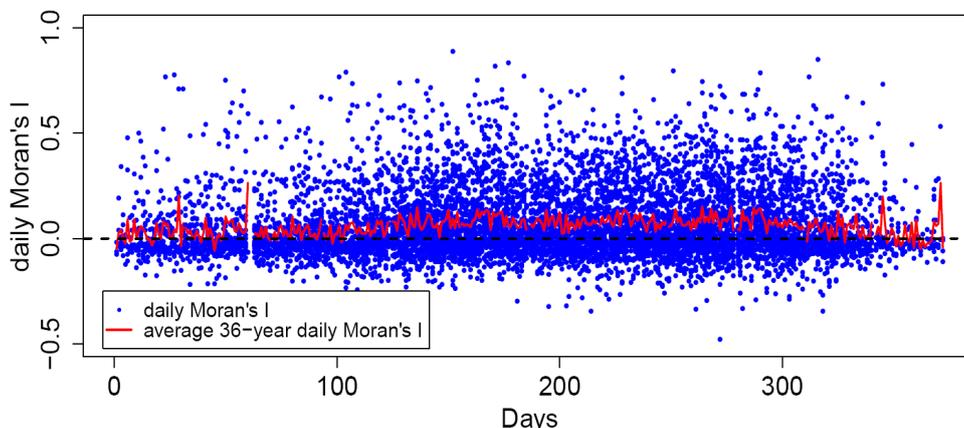


Figure 5. Observed daily *Moran's I* of 24 stations rainfall for the 36 years between 1971-2006 and the 36-year averaged value.

3.6 Generation of the maximum and minimum temperatures

The stochastic multi-site generation of the daily maximum (T_{max}) and minimum (T_{min}) temperature follows pretty much the procedures for the precipitation generation discussed in the previous section. A noteworthy difference is that, in agreement with other studies (e.g. Qian *et al.*, 2002; Khalili *et al.*, 2009; Liu *et al.*, 2009), the two temperatures $T_{i,d,rlz}[k]$ are synthesized by the normal distribution $N(\mu, \sigma^2)$

$$T_{i,d,rlz}[k] = f_{norm}(z_{k,i,d,rlz}, \mu_{k,i}, \sigma_{k,i}) = \mu_{k,i} + z_{k,i,d,rlz} * \sigma_{k,i} \quad (8)$$

where $z \in N(0, 1)$ is a standardized normal random variable, and μ and σ are the empirical means and standard deviations of the corresponding data and are determined by linear regressions with the data itself (see Bejranonda, 2014). Moreover, since both daily T_{max} and T_{min} depend on whether the day is wet or dry (e.g. Richardson, 1981; Qian *et al.*, 2002; Wilks, 2006; Khalili *et al.*, 2009; Liu *et al.*, 2009), Eq. (8) is applied separately for wet and dry conditions. To generate multi-realizations of f_{norm} , z is drawn from the quantile function, $z=Q(p) = F^{-1}(p)$, with $F(x)$ the normal distribution function, for a given probability p , defined by the normalized spatially autocorrelated random numbers, as discussed earlier.

4 RESULTS AND DISCUSSION

4.1 Validation of daily climate generation

The multi-site DWG has been programmed in the R[®] environment. Validation of the DWG model is carried out over the observed data period 1971-2000 in the EST study region. More specifically, the time period 1971-1985 is used for calibration, and the period 1986-2000 for verification.

Results of this exercise, using 30 realizations, are shown in the three scatterplots of Fig. 6 of the simulated over the observed monthly rainfall and the two temperatures T_{max} and T_{min} . As expected, the stochastic generation of the rainfall is more scattered than that of T_{max} and T_{min} . Also, similar to results of Liu *et al.* (2011), the Markov-chain based generated rainfall appears to be slightly underestimated here.

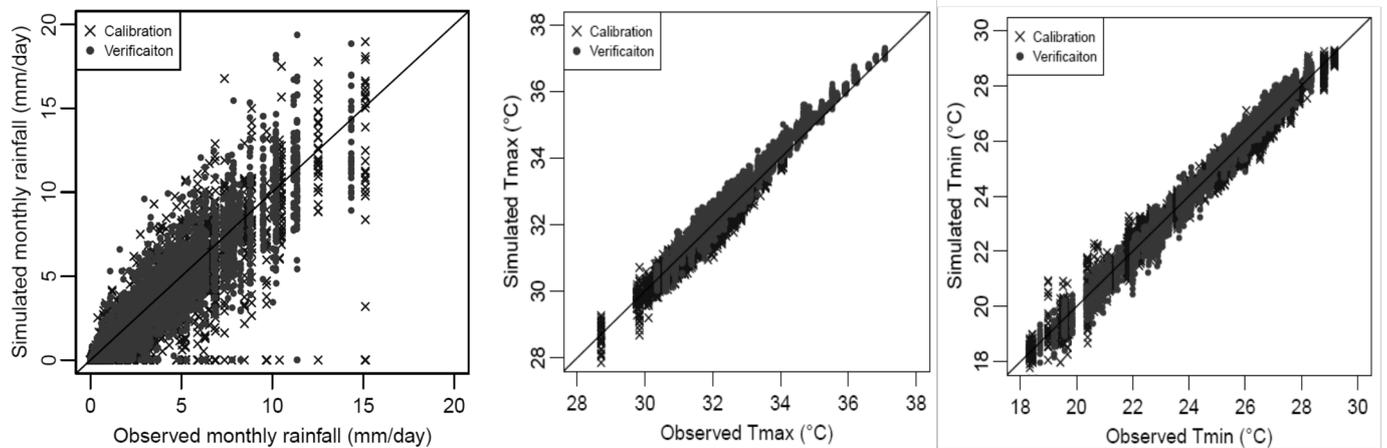


Figure 6. Scatterplots of observed and simulated monthly average rainfall maximum and minimum temperatures at station 48092 by separating data into calibration (1971-1985) and verification (1986-2000) periods of all 30 realizations.

In Table 1 the residual errors for the 30-realization- average of the four predictors, as measured by the mean error (*ME*), the *RMSE* and the Nash-Sutcliffe (*NS*) coefficient, are listed. One may notice that the visual results of Fig. 6 are basically confirmed and, in particular, that notwithstanding that the wet/dry-day occurrence is not so well predicted, the rainfall amount is still regenerated at a high significance level.

Table 1. Residual errors, as measured by the ME, MSE, and the NS of the multi-site generation of monthly wet day and rainfall amount for the calibration period 1971-1985 and the verification period 1986-2000.

Predictor	Calibration: 1971-1985			Verification: 1986-2000		
	residual error		NS	residual error		NS
	ME	RMSE		ME	RMSE	
Wet rate (% wet day)	0.36	3.32	0.71	0.70	2.89	0.80
Rainfall amount (mm/day)	-0.15	0.24	0.99	0.19	0.34	0.99
T_{max}	-0.04	0.07	0.99	0.20	0.24	0.95
T_{min}	-0.01	0.08	0.99	0.08	0.21	0.99

4.2 Application to downscaled GCM-predictors and comparison with other downscaling methods

As discussed in the introduction, for climate prediction on a daily scale, there is the option to use either downscaled predictors from a GCM which provides daily predictors or to use downscaled GCM- monthly predictors and rescale the latter down to daily scale by means of a daily weather generator, such as the one developed here. Bejanonda (2014) has compared and applied various GCM/downscaling combinations for the prediction of the 21th- century climate and its ensuing impact on the water resources in the EST study region. Here we restrict ourselves to a comparison of the present DWG with three other climate prediction methods, when applied to the past (1971-2000) observed climate data, which serves as the reference state for the future climate predictions, as they have been carried out by Bejanonda (2014).

All four GCM/downscaling combinations are based on predictors of the ECHO-G GCM from the Hamburg MPI (Legutke and Voss, 1999), archived on the CMIP3- server (Meehl, 2007), and which come on a daily as well as on a monthly scale. More specifically, SDSM (Wilby *et al.*, 2002) and LARS-WG (Semenov and Barrow, 1997) statistical downscaling models are applied on the monthly predictors, whereas a new downscaling approach of Bejanonda (2014) that uses a multiple linear regression (MLR)-model, similar to a transfer model between atmospheric CGM predictands and observable climate predictors, is applied on daily data (MLR-daily). These three approaches are then compared with a combination of the monthly GCM/MLR-downscaled predictors and the new climate generator (MLR+DWG).

The daily maximum temperature frequency distributions (kernels) obtained with these four downscaling models are exhibited in Fig.7. One may notice from the figure that neither the SDSM- nor the MLR-daily model fit the observed T_{max} - distribution well. The situation is clearly better for LARS-WG, and particularly good for the new MLR+DWG combination which definitely proves its usefulness in climate impact studies, as is further elaborated in Bejanonda (2014).

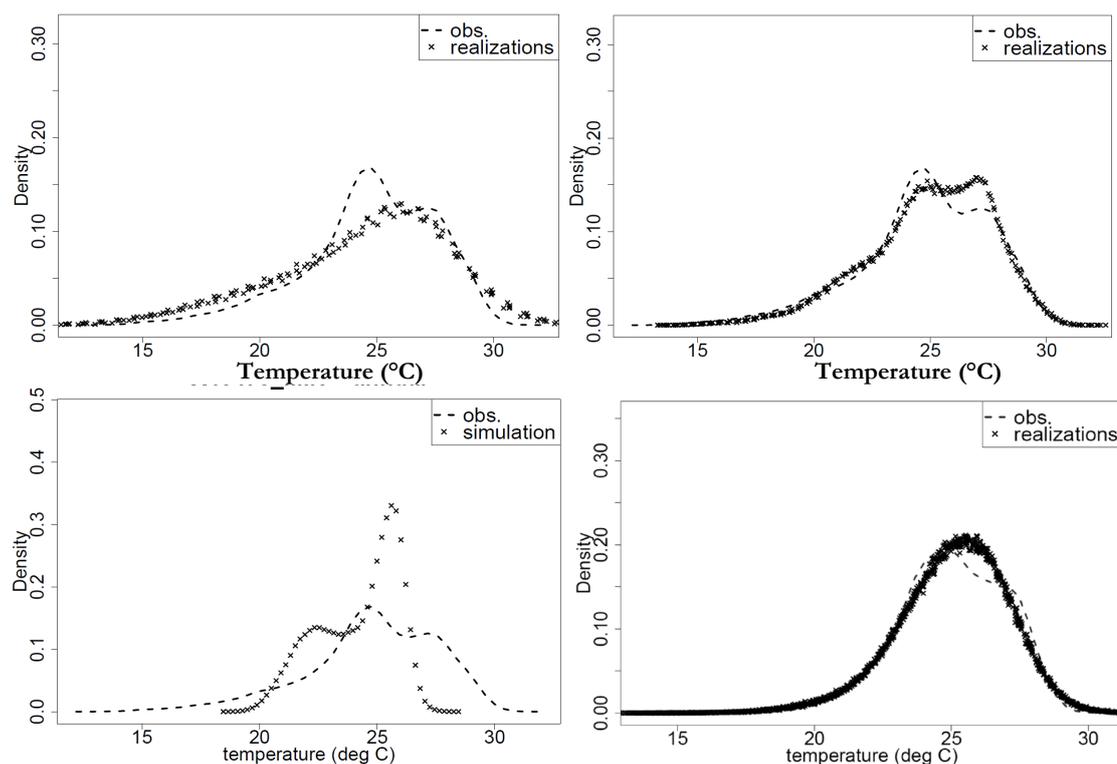


Figure 7. Kernel density estimation of daily maximum temperature at station 48478 for years 1971-2000 using the various downscaling models as indicated (cross points) and the daily observation (dashed line).

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