

A numerical study of turbulence influence on saltating grains

R.J. Bialik, P.M. Rowiński & W. Czernuszenko

Institute of Geophysics, Polish Academy of Sciences, Warsaw, Poland

V.I. Nikora

School of Engineering, University of Aberdeen, Aberdeen, Scotland, United Kingdom

ABSTRACT: The purpose of this study is to investigate the influence of the turbulence structure on the trajectories of saltating particles in open-channel flows. We assume that the flow is 2D, steady, and the instantaneous longitudinal velocities at any point represent a random, stationary and ergodic field. A special procedure has been designed to generate velocity fields. The method is based on the assumption that the spectrum of turbulent kinetic energy is known at any distance from the bed. The time series of the instantaneous longitudinal velocities at any point are reproduced from the known energy spectrum that consists of four ranges: (1) the range corresponding to the largest eddies; (2) the range of the intermediate eddies; (3) the range of the relatively small eddies; (4) the viscous subrange. Then, using the Inverse Fast Fourier Transform, the time series of turbulent velocities are obtained. The mean velocity field is obtained with the use of an additional assumption that the logarithmic law holds in the near-bed flow region. The proposed approach allows to study the behaviour of saltating particles under the influence of turbulence for a broad range of the sizes of sediment particles. The analyzed distributions of dimensionless saltation lengths and heights show that with increase in turbulence intensities the particle movement becomes more unpredictable and the analyzed dimensional saltation lengths and heights change up to 50% depending on particle Reynolds number.

Keywords: Lagrangian approach, Particle-turbulence interaction, Saltation, Sediment transport, Sediment particles trajectory

1 INTRODUCTION

It is common knowledge that sediment in rivers consists of bed-load, suspended load and wash-load. In the bed-load layer sediment may be transported via three modes: sliding, rolling and saltation and the last one is considered as the most typical motion of sediment grains (Bagnold, 1956, Wiberg and Smith, 1989, Nino and Garcia, 1994, Hu and Hui, 1996) and as such has been the main subject of this investigation.

Various aspects of this phenomenon have been studied over the last two-three decades (Wiberg and Smith, 1985, Sekine and Kikkawa, 1992, Lee and Hsu, 1994, Nino et al., 1994, Nino and Garcia, 1994, 1998b, Rowiński and Czernuszenko, 1999, Lee et al., 2002, Lukerchenko et al., 2006). It is interesting to note that only in a few works the turbulent structure of the flow was taken into account to show its influence on the dynamics of a

saltating particle. Rowiński and Czernuszenko (1999) included a special term responsible for the drift force generated among others by turbulent fluctuations. Nino and Garcia (1998a) used a random walk model for flow velocity fluctuations and then showed that neglecting turbulence leads to the overestimation of the mean values of the dimensionless saltation length and height. Nikora et al. (2001, 2002) showed that the particle motion occurs within at least three ranges of scales: local, intermediate and global.

The focus of the present paper is on the analyses of the influence of the turbulence structure on the trajectories of saltating particles in open-channel flows. Such aim may be realized through the investigation of a relevant model in which turbulence plays a significant role. In the first part of this paper the numerical model of the movement of spherical solid particles in an open channel flow is briefly described. This model, based on the

Lagrangian approach, reflects the balance of basic forces exerting on a saltating particle. In practice, it is an extremely rare case that the turbulent structure of the sediment laden flow is known and therefore a special procedure has been designed to generate turbulent velocities. For the purpose of this study we modified Nikora et al's (2001) approach by introducing quasi-Lagrangian velocity time-series instead of the series based on the Eulerian velocity spectra. The needed information on the turbulence structure and particularly the spectrum of turbulent kinetic energy is assumed to be a priori known at any distance from the bed. This information is used to reproduce time series of the instantaneous longitudinal velocities at any point which are obtained before the calculations of a particle trajectory with the use of the saltation model. In addition, Taylor's frozen turbulence hypothesis is employed to obtain the quasi-Lagrangian velocity time series. All considerations are made under the assumption that the flow is 2D, steady, and the instantaneous longitudinal velocities at any point represent a random, stationary and ergodic field.

2 MODEL FOR SALTATING PARTICLES

The sediment particle motion can be described based on the Lagrangian methodology involving a solution of Newton's equations representing the balance of forces acting upon a moving spherical particle in a turbulent boundary layer. It is proposed that in the case of the bed-load this equation may take the following form (Czernuszenko and Bialik, 2009):

In longitudinal direction x

$$\frac{du_s}{dt} = \frac{3C_L\rho}{4d(\rho_s + C_m\rho)} \left(|u_r|_T^2 - |u_r|_B^2 \right) \left(\frac{v_s}{u_r} \right) + \frac{3C_D\rho}{4d(\rho_s + C_m\rho)} |u_f - u_s| (u_f - u_s) + \frac{(\rho - \rho_s)g \sin \alpha}{(\rho_s + C_m\rho)} \quad (1)$$

In vertical direction z

$$\frac{du_s}{dt} = \frac{3C_L\rho}{4d(\rho_s + C_m\rho)} \left(|u_r|_T^2 - |u_r|_B^2 \right) \left(\frac{u_f - u_s}{u_r} \right) + \frac{3C_D\rho}{4d(\rho_s + C_m\rho)} |u_f - u_s| (-v_s) + \frac{(\rho - \rho_s)g \cos \alpha}{(\rho_s + C_m\rho)} \quad (2)$$

where: u_s and v_s are the time-averaged longitudinal and vertical velocities of particles, u_f is time-averaged longitudinal velocity of water, $u_r = u_f - u_s$ is the relative water and particle velocity, t is time, g is gravity, α denotes the slope of the channel bed, d is the particle diame-

ter, C_D is the drag coefficient, $C_L = 0.85C_D$ is the lift coefficient, C_m is the virtual mass coefficient (for spherical particles it equal to 0.5), ρ_s and ρ are the sediment and water intrinsic densities, respectively. Index "T" denotes the top of a particle and index "B" stands for the bottom of the particle.

The system of the above equations has to be supplemented with the following trajectory equations:

$$\frac{dx}{dt} = u_s \quad \text{and} \quad \frac{dz}{dt} = v_s. \quad (3)$$

Measurements of particles movement in a water stream by Abbott and Francis (1977) indicate an average initial longitudinal and vertical velocity to be of approximately $2u_x$ where u_x is the shear velocity. Based on their investigations equations (1-3) can be solved numerically with the following initial conditions $u_s(t_0) = 2u_x$, $v_s(t_0) = 2u_x$, $x(t_0) = 0$ and $z(t_0) = 0.5d$. The fourth-order Runge-Kutta method was applied in the computations. Boundary conditions describing particle collisions with the channel bed are used in the same form as proposed by Czernuszenko and Bialik (2009) and based on impulsive equation for colliding spherical particles.

3 MODEL FOR FLOW VELOCITY FIELD

The flow velocity u_f is generally decomposed into the mean flow velocity $\overline{u_f}$ and the velocity fluctuation u' such that:

$$u_f = \overline{u_f} + u' \quad (4)$$

The mean flow velocity in the longitudinal direction is obtained assuming that the logarithmic law holds in the near-bed flow region and it could be described in the following form:

$$\frac{\overline{u_f}(z)}{u_*} = \frac{1}{\kappa} \ln \frac{z}{k_e} + B, \quad (5)$$

where: z is a distance from the bed, $\kappa = 0.41$ is von Karman constant, $k_e = d$ is the effective roughness of the bed, $B = 8.5$ is a constant.

Velocity fluctuations may be deduced from the assumed energy density spectra obtained for a given mean flow velocity profile, the vertical distribution of the relative turbulence intensities and the shape of the spectral functions.

Turbulence in the longitudinal direction plays a dominant role and therefore, for simplicity reasons, fluctuations in the vertical and transversal directions were disregarded in the considerations.

In principle relations among turbulent intensities in various directions given by Nezu and Nakagawa (1993) may readily be incorporated in the considered model. The vertical distribution of the relative turbulence intensities in the longitudinal direction $\sigma_x(z)$ for the uniform 2D open-channel may be assumed based e.g. on the formulae proposed by Nezu and Nakagawa (1993):

$$\frac{\sigma_u(z)}{u_*} = 2.3 \exp(-(z/H)), \quad (6)$$

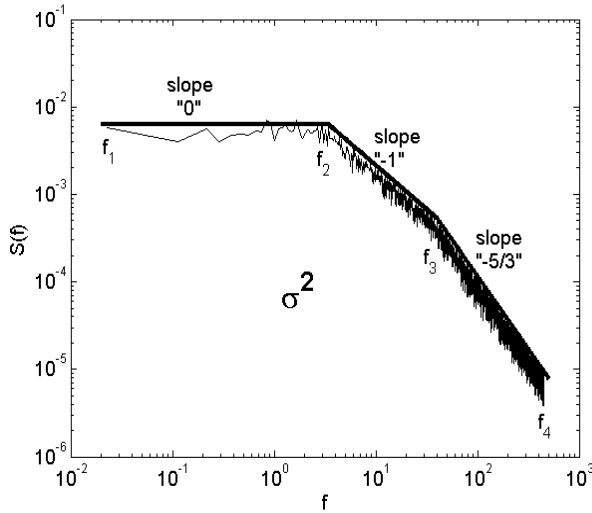


Figure 1. Log-log plot of an example of velocity spectrum used for the turbulent velocity simulations.

In recent studies it has been shown that velocity spectra in an open-channel flow may consist of four ranges (e.g., Nikora, 1999, Nikora and Goring, 2000, Nikora 2005; fig. 1):

1. The range corresponding to the largest eddies,

$$k < (a_1 H)^{-1}, \text{ with } S_{uu}(kz) = C_{1uu} u_*^2 (z/H)^{-1};$$

2. The range of the intermediate eddies,

$$(a_1 H)^{-1} < k < (a_2 z)^{-1}, \text{ with}$$

$$S_{uu}(kz) = C_{2uu} u_*^2 (zk)^{-1};$$

3. The range of the relatively small eddies

$$(a_2 z)^{-1} < k < (a_3 k_e)^{-1}, \text{ with}$$

$$S_{uu}(kz) = C_{3uu} u_*^2 (zk)^{-5/3}$$

4. The viscous subrange where k is the wave number $k=2\pi f/u_f$, f is frequency, $S(kz)$ is the wave number auto-spectrum, H is the flow depth, $C_{1uu}=1.6$, $C_{2uu}=0.9$, $C_{3uu}=0.9$, a_1, a_2, a_3 are constants (Nikora, 2005). The spectrum described above is used to generate the turbulent velocity time series. Having known the shape of the spectral function some realization of the time series may be obtained using the Inverse Fast Fourier Transform.

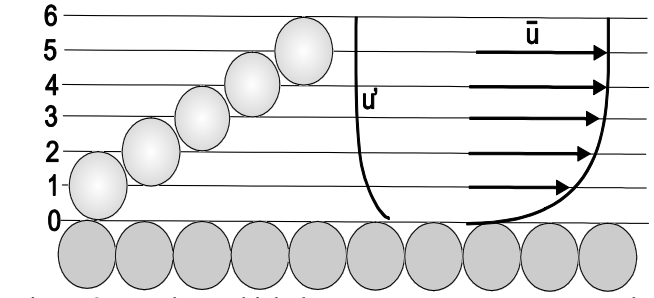


Figure 2. Levels at which the energy spectra are generated.

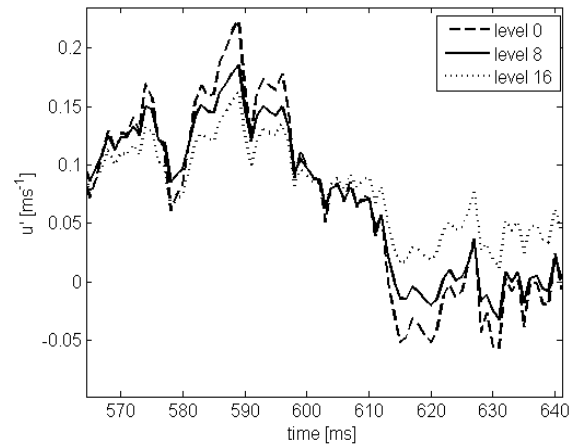


Figure 3. An example of turbulent velocity time series at three consecutive levels

The flow velocity field is then simulated using the Monte Carlo method. The flow is divided into layers (fig. 2) of $\delta z=d/2$ thick. Turbulent velocity time series in the longitudinal direction are created for each layer separately. The correlation between turbulent velocity time series on different levels is simulated using the same spectral phases. An example of turbulent velocity time series at three selected levels for particles of $d=2.47\text{mm}$ and for $u_x = 0.074\text{ms}^{-1}$ are shown in fig. (3).

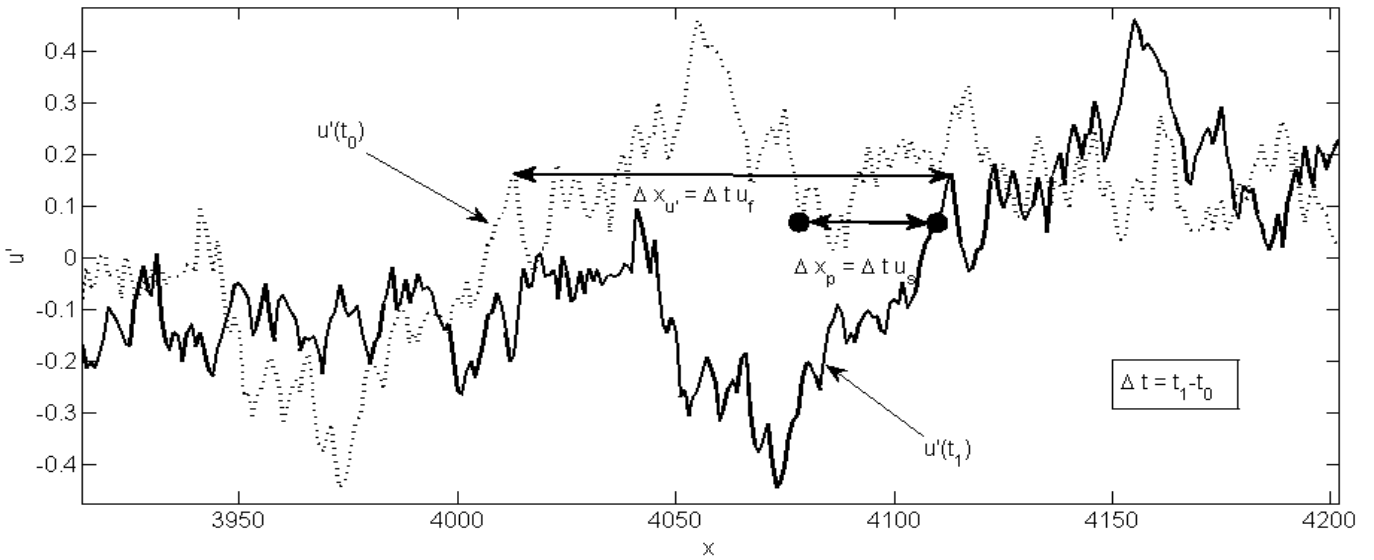


Figure 4. An illustration of Taylor's frozen turbulence hypothesis.

It is important to note that eqs. (1-2) describing the saltation process in the open-channel require the knowledge of the Lagrangian velocity series. Hence eq. (4) takes the following form:

$$u(x, z, t) = \bar{u}(z) + u'(x, z, t), \quad (7)$$

In order to analyse the turbulent velocity time series let us assume that Taylor's hypothesis is valid, i.e. the eddies are frozen in the mean flow and they do not change considerably as they are advected downstream. Let us consider the situation schematically represented in fig. (4). Saltating particles usually move slower than water itself ($u_n < u_f$). At the same time interval $\Delta t = t_1 - t_0$ particles cover the distance $\Delta x_n = u_n \Delta t$ while water shifts over the distance of $\Delta x_{ut} = u_f \Delta t$. This difference has to be taken into account when so-called quasi-Lagrangian fluctuation velocity time series is built up.

The comparison between the "quasi-Lagrangian" spectrum based on the obtained flow velocity fluctuations and the mean Eulerian spectrum used for the generation of the turbulent velocity time series is shown in fig. (5). The slope of "-2" of the obtained quasi-Lagrangian spectrum confirms that the proposed method fits well the theoretically based Lagrangian spectrum (Monin and Yaglom, 1971).

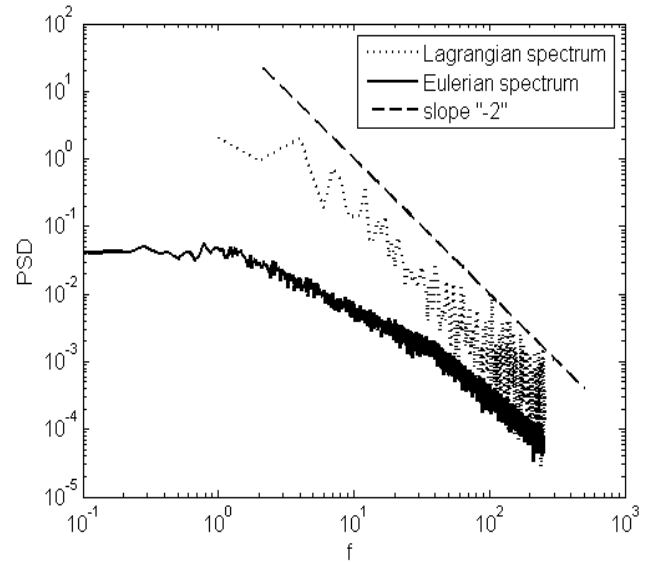


Figure 5. Comparison of Lagrangian and Eulerian energy spectra.

4 RESULTS OF COMPUTER SIMULATIONS

In order to analyze the effect of particle-turbulence interactions, 100,000 simulations of particles' saltations were carried out. The model described above allowed us to vary the relative turbulence intensities and the mean velocity distribution and thus to investigate the behaviour of particles under different flow conditions. Three representative particle diameters: 1, 4 and 8mm were used in simulations. Ten thousand hops of saltating grains were simulated for all considered conditions to secure proper statistics of the saltation parameters. In order to avoid the influence of the initial conditions on the statistics of saltation the first five hops were rejected in estimations. Moreover, the time step for numerical simulation of particle trajectories Δt is related to Nyquist fre-

quency, which is equal to maximum frequency in the velocity spectrum f_{Δ} (fig.1): $\Delta t = 1/2f_{\Delta}$.

Figures (6-8) show the distribution of the simulated particle hops. L_i/d and H_i/d denote the dimensionless saltation lengths and heights of the hops, “i” stands for the number of the hop ($i=1 \dots 10000$). Figures 6(a)-8(a) illustrate the distribution of the simulated hops under laminar flow conditions, namely when the relative turbulent intensity was assumed to be zero. Figures 6(b)-8(b) show the results when the relative turbulence intensity equals to half intensity from the model of Nezu and Nakagawa (1993), whereas figs. 6(c)-8(c) assume the intensity as described by Nezu and Nakagawa (1993). Main statistical characteristics of the considered saltations are shown in tables (1-3).

The computational results clearly demonstrate that the motion of solid particles depends on the values of turbulence intensity and this effect is independent of particles sizes. It is worth noting that inclusion of turbulence in the model leads to the increase of the dimensionless length of the saltation and its height by as much as 35% for particle with diameter $d=1\text{mm}$ and by 50% for particles $d=4\text{mm}$ and 8mm . This effect also depends on the flow conditions and particularly on the shear velocity. The saltation characteristics increase with increase in the shear velocity.

It is quite peculiar (see figures 6-8) that the movement of particles becomes more unpredictable with the increase of the relative turbulence intensities. Note, for example, that in the laminar case the particle with $d=1\text{mm}$ (fig.6a), for the hops of the dimensionless length equal to $35d$ the dimensionless saltation height was from $4.8d$ to $5.4d$ whereas in the turbulent case (fig.6c) for the same dimensionless saltation length $35d$ the dimensionless saltation height varied in a wide range, from $2.9d$ to $6.1d$. All other cases confirm such relation.

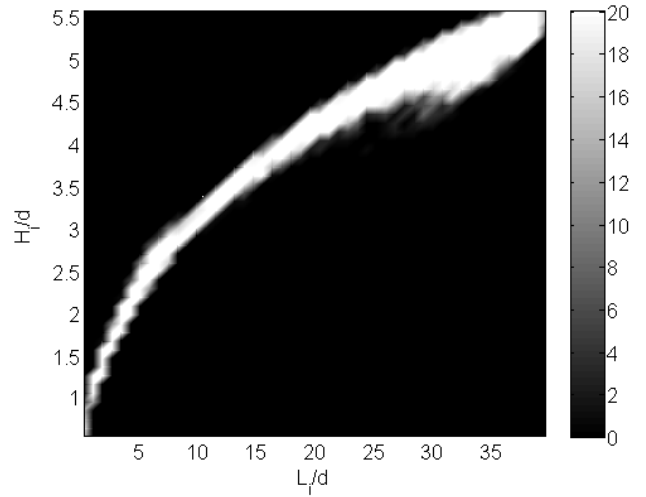


Figure 6a. $\sigma' = 0$, $d = 1\text{mm}$, $u_* = 0.05\text{ms}^{-1}$

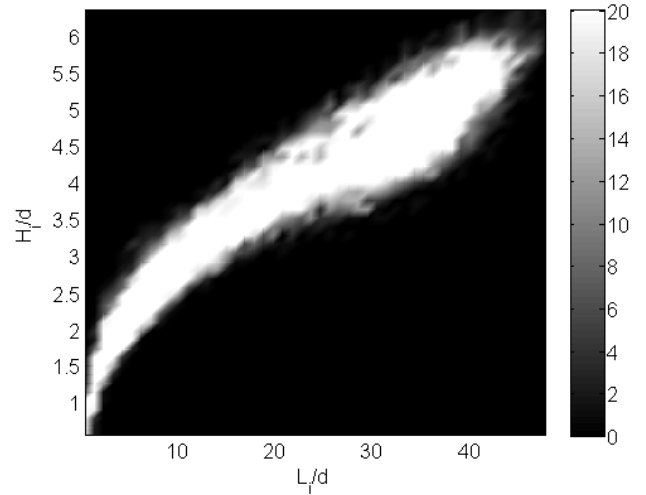


Figure 6b. $\sigma' = 0.5\sigma$, $d = 1\text{mm}$, $u_* = 0.05\text{ms}^{-1}$

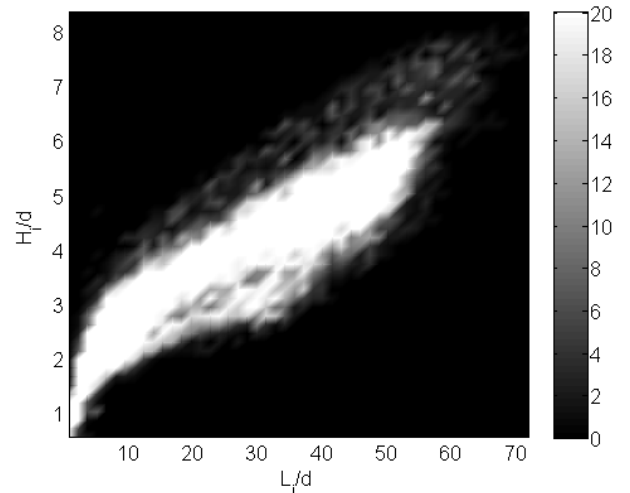


Figure 6c. $\sigma' = \sigma$, $d = 1\text{mm}$, $u_* = 0.05\text{ms}^{-1}$

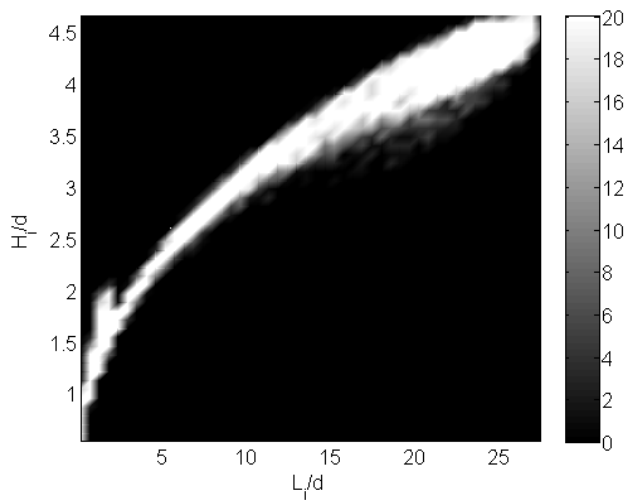


Figure 7a. $\sigma' = 0$, $d = 4\text{mm}$, $u_* = 0.09\text{ms}^{-1}$

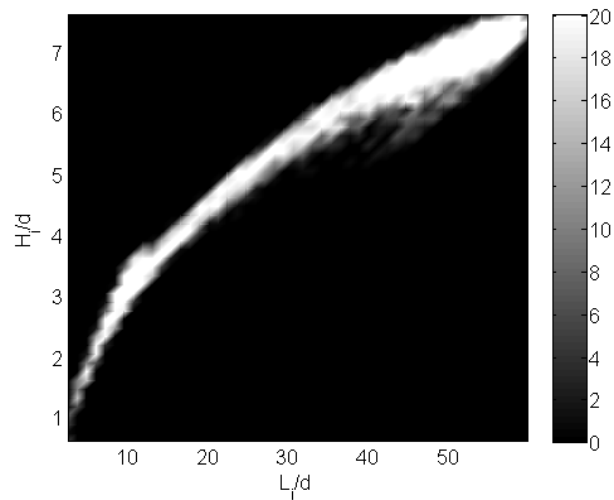


Figure 8a. $\sigma' = 0$, $d = 4\text{mm}$, $u_* = 0.12\text{ms}^{-1}$

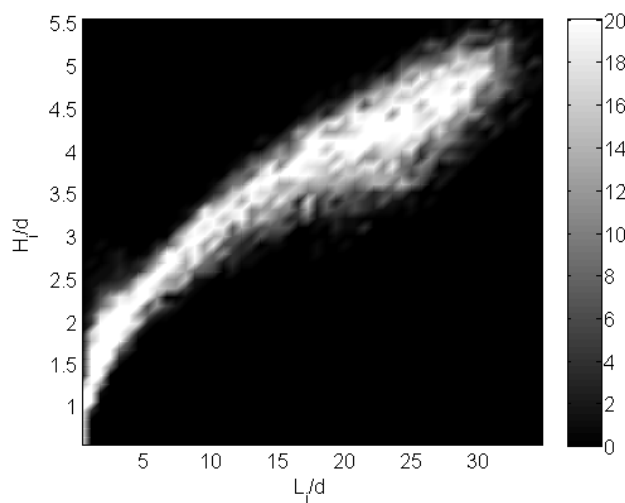


Figure 7b. $\sigma' = 0.5\sigma$, $d = 4\text{mm}$, $u_* = 0.09\text{ms}^{-1}$

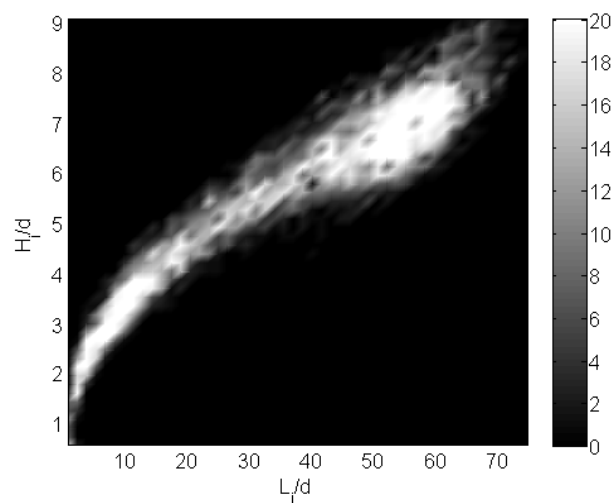


Figure 8b. $\sigma' = 0.5\sigma$, $d = 4\text{mm}$, $u_* = 0.12\text{ms}^{-1}$

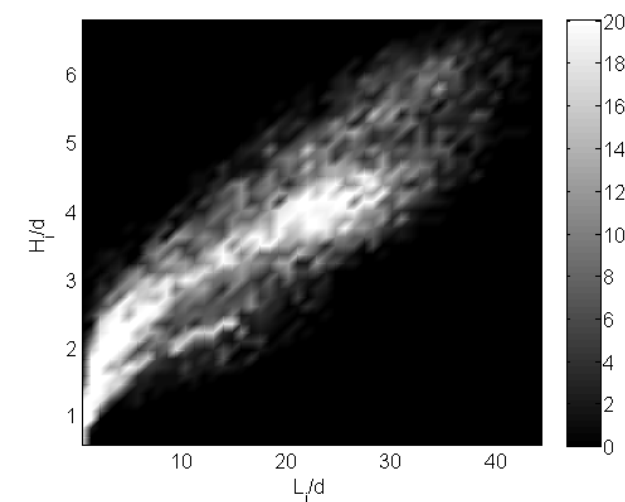


Figure 7c. $\sigma' = \sigma$, $d = 4\text{mm}$, $u_* = 0.09\text{ms}^{-1}$

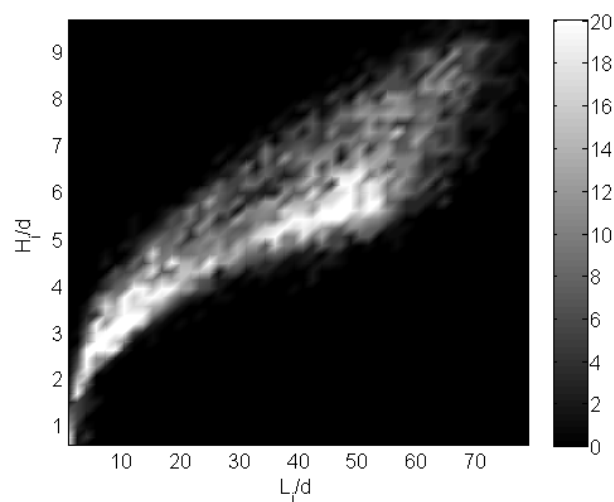


Figure 8c. $\sigma' = \sigma$, $d = 4\text{mm}$, $u_* = 0.12\text{ms}^{-1}$

Table 1. Statistical characteristics of saltation for different relative turbulence intensities for $d=1\text{mm}$ and $u_* = 0.5\text{ms}^{-1}$

	$\sigma' = 0$	$\sigma' = 0.5\sigma$	$\sigma' = \sigma$
L/d	23.62	24.47	31.17
H/d	4.01	4.34	4.93
max(L/d)	38.81	46.16	63.31
max(H/d)	5.47	6.12	7.58
sd(L/d)	6.16	6.72	9.69
sd(H/d)	1.12	1.37	1.38

Table 2. Statistical characteristics of saltation for different relative turbulence intensities for $d=4\text{mm}$ and $u_* = 0.09\text{ms}^{-1}$

	$\sigma' = 0$	$\sigma' = 0.5\sigma$	$\sigma' = \sigma$
L/d	13.63	14.72	18.07
H/d	2.09	2.32	2.63
max(L/d)	25.91	35.76	43.43
max(H/d)	4.56	5.54	6.08
sd(L/d)	5.96	6.32	8.96
sd(H/d)	1.21	1.42	1.61

Table 3. Statistical characteristics of saltation for different relative turbulence intensities for $d=4\text{mm}$ and $u_* = 0.12\text{ms}^{-1}$

	$\sigma' = 0$	$\sigma' = 0.5\sigma$	$\sigma' = \sigma$
L/d	22.29	22.07	21.97
H/d	4.17	4.24	4.27
max(L/d)	58.90	69.22	74.54
max(H/d)	7.07	8.21	9.18
sd(L/d)	9.71	11.02	13.11
sd(H/d)	1.89	2.03	2.38

In Tables 1 to 3, $\overline{L/d}$, $\overline{H/d}$ denote the average dimensionless saltation length and height, respectively. max(L/d) and max(H/d) denote maximum values of saltation characteristics and sd(L/d) and sd(H/d) represent standard deviation of dimensionless saltation length and height. σ' and σ stand for the turbulence intensities using in the model and describing by the formula proposed by Nezu and Nakagawa (1993), respectively.

5 CONCLUDING REMARKS

In this study the newly developed Lagrangian model for the description of sediment grains has been used to study the influence of turbulent intensity upon the particles' trajectories. The time series of turbulent velocities were generated based on specially designed procedure. The presented approach aimed at a simplified representation of the turbulence structure making the computations easy and computationally relatively inexpensive. In contrast to other studies, the proposed model is in fact a simple method based only on the basic characteristics of turbulence, i.e.: shape of velocity spectrum, value of turbulence intensities and vertical distribution of mean flow. The results of the numerical simulations based on the proposed

procedure allows for the verification of the hypotheses regarding the influence of the turbulence structure on the saltating particles. They show that turbulence plays an essential role and influences the particles' trajectories considerably and as a consequence is crucial for the bed-load transport. Moreover, the increase of the relative turbulence intensity leads to larger hops (increase in mean hop length and height) and to reduction in the particles paths predictability.

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