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Resistance Prediction for Streams under Low Flow Conditions

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ABSTRACT: Flow resistance under low flow conditions has been studied by many researchers because of its importance in practical applications, and different modifications of resistance coefficients such as Manning's *n*, Chézy *C* and the Darcy-Weisbach *f* for application under appropriate conditions have been proposed. The strong variation of these coefficients with flow depth suggests that the form of these equations is not appropriate for shallow flows. An alternative equation form for resistance prediction under low flow conditions is proposed. It distinguishes between the influences of large-scale and intermediate-scale roughness on flow resistance by a coefficient that depends on the relative submergence. The equations performance is confirmed by comparison of predicted and measured velocities using published experimental and field data.

Keywords: Flow resistance, Roughness, Resistance prediction, Low flows

1 INTRODUCTION

Flow resistance describes the process in streams by which the physical shape and bed roughness of the channel control the depth, width and velocity of flow. Theoretical aspects of open channel flow resistance are documented by, for example, Leopold et al. (1960), Rouse (1965), Bathurst (1982), and Yen (2002). Successful prediction of flow resistance depends on an understanding of flow resistance phenomena as well as application of appropriate formulas accounting for them.

Flow resistance is a term used to describe the net effect of the forces driving and resisting water movement, and is commonly represented by the ratio of the bed shear velocity, V^*, to the mean flow velocity, V. When the flow depth, y, is much higher than the height of the bed material, h, the flow resistance can be considered to result from an effective friction of the material forming the surface of the boundary, and can be described by well known friction coefficients such as the Chézy C, Manning's n and the Darcy-Weisbach f:

$$(V/V^* = (8/f)^{1/2} = (C^2/g) = R^{1/3}/(gn^2) \qquad (1)$$

where R = hydraulic radius; g = gravitational acceleration.

From laboratory experiments (Bayazit 1976) found that flow resistance depends on the ratio of flow depth, y, to height of the bed roughness, h, and once relative submergence (y/h) is less than about 4, the resistance is higher than predicted by the logarithmic resistance equations. According to the relative submergence, the roughness has been divided into three scales: large, intermediate and small (Bayazit 1976, Bathurst et al. 1981). The roughness is large-scale if the roughness elements affect the free surface, a condition when relative submergence, y/h is less than about 1. The roughness is intermediate if the relative submergence lies between 1 and 4. When the relative submergence is higher than 4, the roughness is small.

Flow resistance under low flow conditions has been studied by many researchers because of its importance in practical applications (e.g. Bathurst et al. 1981, Griffiths 1981, Jarrett 1984, Thorne & Zevenbergen 1985, Lawrence 1997, Jonker et al. 2001, Bathurst 2002, Smart et al. 2002). Furthermore, components of flow resistance and physical variables contributing to overall flow resistance have been documented (e.g. Bathurst 1978, Bray & Davar 1987, Lawrence 1997, 2000) Various different equations and resistance coefficients related to low flow conditions have been developed (e.g. Jonker et al. 2001, Nikora et al. 2001, Stone & Shen 2002). Some of these are very complicated and require comprehensive field data (e.g. Bathurst 1978), while others are based on the relative submergence and require consideration only of the bed grain roughness (e.g. Bathurst 2002, Jonker et al. 2001).

In most cases, development of equations was based on laboratory or field data representing the large and intermediate roughness scales. For example, Bathurst (2002) proposed two new resistance relationships for $(8/f)^{0.5}$ (where f is Darcy-Weisbach friction factor) as functions of the relative submergence y/D_{84} (D_{84} is the size of the median axis of the bed material which is larger than 84% of the material). The relative submergence of the field data ranged from 0.37 to 11.4, covering all three roughness scales, large, small and intermediate.

Other researchers described three types of flow regime that related to the relative submergence, viz. flows with high relative submergence, flow with small relative submergence, and flow over a partially inundated rough bed (Lawrence 1997, 2000, Nikora et al. 2001, Smart et al. 2002). Different modifications of the Chézy C, Manning's n and the Darcy-Weisbach f for application under appropriate conditions were proposed.

Generally, Chézy C, Manning's n and the Darcy-Weisbach f all apply to uniform flow and can be related. There is therefore no clear advantage of one coefficient over the others. The Darcy-Weisbach f for regular canals can be estimated from the Moody diagram, and the most common sources for Manning's n are books of Barnes (1967) and Hicks & Mason (1998). There is no generally recognized source for Chézy C coefficient (Yen 2002).

Under low flow conditions, resistance is largely the consequence of the drag forces imposed by individual roughness elements; with high relative submergence the resistance effects of the elements can be treated as for a distributed bed shear stress. The intermediate roughness represents a state of flow in which the influence of the roughness elements on flow resistance is manifest as a combination of both element drag and boundary shear, or friction. Under such conditions flow resistance expressed by the Darcy-Weisbach or Manning's equations requires their coefficients to vary significantly with the flow depth, suggesting that these equations are inappropriate for the intermediate roughness scale in their original form.

In this paper we propose different equation forms for resistance prediction for large-scale and intermediate scale roughness that have been developed from experimental work under controlled and idealized situations. The new resistance equations distinguish between the influences of large-scale and intermediate-scale roughness on flow resistance. The equations have been tested using published experimental and field data, and show good results.

2 RESISTANCE PREDICTION

When the channel bed material is large relative to the water depth, the flow resistance is exerted by the roughness elements' drag rather than boundary shear, or friction.

Assuming that skin drag is not significant, the resisting force of N independent roughness elements in the considered bed area is

$$F_d = \frac{1}{2} C_d \rho V^2 N A_p \tag{2}$$

where N = number of roughness elements per unit area and A_p = projected cross-sectional area of the individual roughness element, given as

$$A_p = yD \tag{3}$$

where y = flow depth and D = roughness element diameter.

The weight component of the flow balanced by the resisting force under steady uniform flow conditions is given by

$$W = \gamma S(1 \times 1 \times y - N V_{r.el}) \tag{4}$$

where γ = unit weight of water, S = energy gradient, N = number of roughness elements per unit area, and $V_{r.el}$ = submerged volume of an individual roughness element.

The component in brackets in Equation (4) is the volume of overlying water per unit plan area of bed and is known as the volumetric hydraulic radius, R_V (Kellerhals 1967). Equating Equations (2) for unit plan area and (4), with (1x1x $y - N V_{r.el}$) = R_V gives

$$V = \sqrt{\frac{1}{C_D N A_p}} \sqrt{R_V} \sqrt{2gS} \tag{5}$$

Equation (5) can be written on more general form as (Jordanova and James, 2007; Jordanova, 2008)

$$V = \frac{1}{F} \sqrt{R_v} \sqrt{2gS} \tag{6}$$

where F = resistance coefficient.

Because the drag coefficient C_d depends on a number of variables such as the Froude number, the Reynolds number, the roughness element shape and the relative depth, estimating its value is not easy (Flammer et al. 1970). Lawrence (2000) found that experimental drag coefficient values were not only significantly higher than values estimated for an isolated cylinder and sphere in a free stream, but also exceeded reported values for free surface flows around isolated hemispheres. Estimation of C_d aside, it is debatable whether a drag type model in general is appropriate for these conditions (Smart et al. 2002). To obviate the necessity for estimating C_d, experimental data were used to estimate the resistance coefficient F directly from Equation (6).

When the relative submergence lies between one and four, the roughness scale is intermediate. This regime represents a state of flow in which the influence of the roughness elements on flow resistance is manifest as a combination of both element drag and boundary shear equal to or friction.

To estimate the flow resistance under such condition the following hypothesis was applied:

- If the relative submergence is equal or greater than four, then friction resistance dominates, and velocity can be estimated as

$$V = \sqrt{\frac{8g}{f}}\sqrt{R}\sqrt{S} \quad (7)$$

- If the relative submergence is equal to or less than one, the drag effect of individual roughness elements on flow resistance will dominate and the proposed Equation 6 should be used.
- As the relative submergence increases from one to four, the dominant resisting effect changes gradually from element drag to friction. The velocity can be estimated by

$$V = \left(\frac{1}{F}\right)^{a}\left(\sqrt{\frac{4}{f}}\right)^{(1-a)}\sqrt{2\,g\,R_v}\sqrt{S} \quad (8)$$

where a = coefficient related to the relative submergence and varies from 1 to 0. When the relative submergence is equal to one, the roughness is large-scale and Equation (8) reduces to Equation (6). With a equal to 0 Equation (8) will take the form of Equation (7) for small-scale roughness.

Application of the proposed Equation (8) requires specification of the coefficient a as a function of the relative submergence. Laboratory experiments were carried out to determine a suitable relationship form for the coefficient a.

3 LABORATORY INVESTIGATION

An experimental programme was carried out in laboratory flumes under controlled and idealized situations to establish the effects of roughness elements on flow resistance under different hydraulic conditions determined by bed slope, S, and discharge, Q, and to develop and test resistance prediction methods. Experiments were carried out using different sizes and different densities, λ, of roughness elements. Roughness elements were simulated by hemispheres formed of concrete. Two series of laboratory experiments were conducted (Jordanova and James, 2007). The experimental data related to the intermediate-scale roughness are summarized in Table 1.

3.1 *Series 1.1 experiments*

Series 1 experiments were conducted in a 0.38 m wide, 15.0 m long, glass-sided tilting laboratory flume. A tailgate at the downstream end of the flume was used to control the flow depth in the channel to ensure uniform flow. Water was supplied to the flume through a closed circulation system, and two valves situated in the supply pipe at the head of the experimental flume were used to control the discharge. The discharge was varied by opening and closing these valves and measured using a V-notch, which was installed at the downstream of the flume and an electronic flow meter in the supply water pipe. All experiments were carried out under uniform conditions (Table 1)

Table 1. Experimental conditions

Series	Test	D	λ^*	Slope	y/h	Q
		mm	%			l/s
Series 1.1	1	47	82	0.0011	1.4-3.6	0.4-5.3
	2	47	82	0.0021	1.0-4.0	0.4-10.9
	3	47	47	0.0011	1.0-3.6	1.4-4.2
	4	47	30	0.0011	1.0-3.6	0.9-4.3
	5	47	22	0.0011	1.0-3.2	0.4-4.0
Series 2.1	1	116	15	0.001	1.0-1.2	11.0-17.4
	6	116	15	0.001	1.0-1.4	13.8-27.3
	7	54	3	0.001	1.0-1.9	7.0-27.6
	8	54	3	0.001	1.0-2.0	5.3-28.3
Series 2.2	1	108	55	0.0005	1.0-3.5	1.6-55.7
	2	108	22	0.0005	1.0-3.0	3.3-54.7
	3	108	12	0.0005	1.1-2.4	8.2-55.2
	4	108	6	0.0005	1.1-2.2	14-55.2
	5	72	3	0.0005	1.0-3.0	6.6-55.2
	6	72	10	0.0005	1.0-3.5	2.8-60.0
	7	72	24	0.0005	1.4-4.1	4.1-55.2
	12	46	4	0.0005	1.0-4.65	3.6-55.7
	15	108	75	0.0005	1.0-2.3	1.2-25.0
	16	108	63	0.0005	1.0-2.1	1.4-21.4

[*] Per cent areal roughness concentration (density).

3.2 *Series 2 experiments*

The Series 2 experiments were conducted in a 2.0 m wide, 15.0 m long, tilting laboratory flume. Series 2 experiments were conducted with bed slopes of 0.001 (Series 2.1) and 0.0005 (Series 2.2) and different sizes of hemispheres (Table 1).

4 ESTIMATION OF COEFFICIENT

The experimental data (Table 1) were divided into two sets. One set of data (Series 1.1, experiments 1, 4 and 5, Series 2.1, experiments 6 and 8, and Series 2.2, experiments 1, 3, 5, 7, 14 and 16) was used for development of a suitable functional relationship of the coefficient a as a function of the relative submergence. The remaining data constituted a set used for confirmation of the relationship. Equation (8) was applied to evaluate a from the measured V and R_v to each experimental run. Application of Equation (8) required input of the resistance coefficient F and friction factor f. These values were calculated from the experimental data for the relevant flow conditions. Experimentally-derived values of the coefficient a together with the related relative submergence are plotted in Figure 1. A suitable relationship form of the coefficient a as a function of the relative submergence was fitted as

$$a = -0.67\, Ln\left(\frac{y}{h}\right) + 0.992 \qquad (9)$$

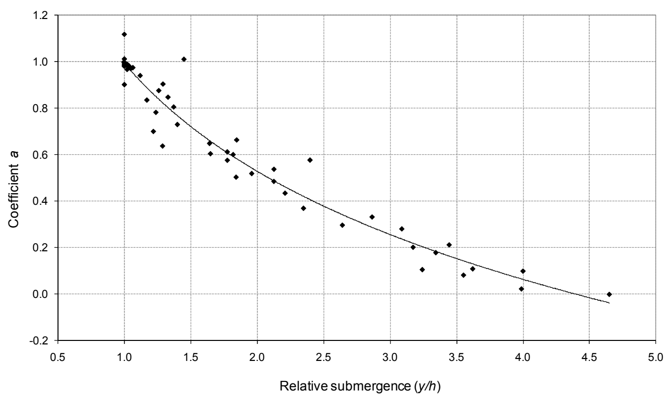

Figure 1. Functional relationship of relative submergence and coefficient a

5 EQUATION CONFIRMATION

The performance of the proposed Equation (8) with a given by Equation (9) has been assessed by comparison of measured and predicted values of velocity for the second set of the experimental data (Series 1.1, experiments 2 and 3, Series 2.1, experiments 1 and 7, and Series 2.2, experiments 2, 4, 6 and 15). Measured and predicted velocities, together with the perfect fit line and 15 % accuracy limits for Series 1 and 2 experiments are plotted in Figure 2.

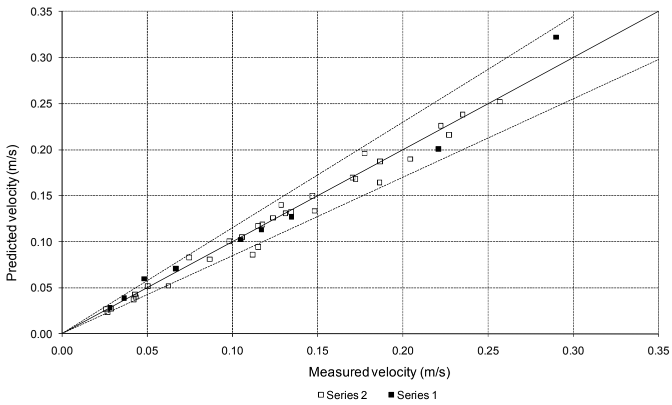

Figure 2. Measured and predicted (Equations 8 and 9) velocities with 15 % accuracy limits

The calculated average absolute errors in velocity prediction by application of Equations (8) and (9) for Series 1.1, 2.1 and 2.2 experiments are listed in Table 2.

Table 2. Average absolute errors in velocity prediction by application of Equations (8) and (9)

Series	Average (%)	Std (%)
1.1	7.55	6.30
2.1	1.79	1.68
2.2	7.49	6.13

5.1 *Verification of proposed Equations (8) and (9) with Bathurst et al. (1981) published experimental data*

Published experimental data of Bathurst et al. (1981) were used for further verification of Equations (8) and (9). Bathurst's experiments were carried out at Colorado State University in a flume with a length of 9.54m and a width of 1.168m width. The resistance of five bed materials with roughness heights 12.7, 19.5, 38.1, 50.8 and 63.5mm were tested. Experiments were performed with 3 flume slopes of 0.02, 0.05 and 0.08. Experimental conditions used for verification of proposed Equations (8) and (9) are summarised in Table 3.

Table 3. Summary of Bathurst et al, (1981) experimental data

Bed roughness mm	Discharge (m^3/s)	Depth measured (m)
12.70	0.0019-0.0490	0.012-0.046
19.05	0.0021-0.0546	0.016-0.054
38.10	0.0018-0.0802	0.023-0.101
50.80	0.0025-0.0495	0.041-0.095
63.50	0.0037-0.0497	0.049-0.108

Application of the proposed Equations (8) and (9) required estimation of the resistance coefficient F, friction factor f, and the relative submergence. For each experimental run values of F and f were calculated. For each test, graphs of F and f as functions of the relative submergence were plotted and were extended, if necessary, to relative submergences equal to one for graphs of F and to four for graphs of f. These graphs were used to estimate the values of F and f.

Measured and predicted (Equations (8) and (9)) velocities together with the perfect fit line and 25 % accuracy limits for experiments with five flume beds are plotted in Figure 3.

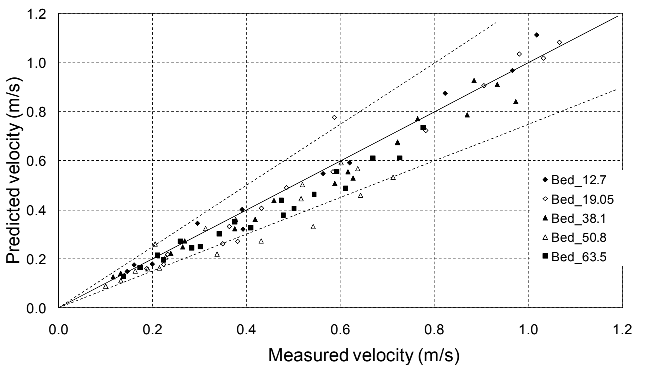

Figure 3. Measured and predicted (Equations 8 and 9) velocities with 25 % accuracy limits experimental data of Bathurst et al., (1981)

Average absolute errors in prediction of flow velocity were calculated for each bed material size and slope, and these are listed together with the standard deviation in Table 4.

Table 4. Average absolute errors in velocity prediction by application of Equations (8) and (9) to experimental data of Bathurst et al. (1981)

Series	Slope	Average (%)	St. deviation (%)
12.70	0.02	2.50	0.23
	0.05	9.25	4.23
	0.08	7.76	6.98
19.05	0.02	19.70	11.52
	0.05	5.26	2.60
	0.08	10.26	10.28
38.10	0.02	8.48	4.40
	0.05	8.85	4.51
	0.08	6.76	3.85
50.80	0.02	7.06	4.66
	0.05	29.49	8.23
	0.08	14.60	8.85
63.5	0.02	14.75	4.71
	0.05	14.24	5.51
	0.08	5.84	1.68

The measured and predicted velocities plotted in Figure 3, and predicted errors listed in Table 4 show that the proposed approach can be recommended for estimation of flow velocity under intermediate-scale roughness conditions.

5.2 *Verification of proposed Equations (8) and (9) with Bathurst (1985) and Hicks and Mason (1998) published field data*

Further verification of the performance of the proposed Equations ((8) and (9)) was carried out by comparison of measured and predicted flow velocities of Bathurst (1985) and Hicks and Mason's (1998) published field data that relevant the intermediate-scale criterion. Conditions for data used for this verification are listed in Table 5.

Table 5. Field data used for verification of proposed Equations (8) and (9)

Data source	River	Mean flow depth (m)	Bed material D_{84} (mm)
Bathurst	South Tyne	0.50	240
	Ettrick	0.21 - 0.47	193
(1985)	Tweed	0.72	183
	Tromie-2	0.40 - 0.89	183
	Findhorn	0.30 - 0.45	140
Hicks and	Waiau Water Race	0.22 - 0.30	80
Mason	Cardrona	0.28 - 0.30	78
(1998)	Hutt	0.42 - 0.67	212
	Clarence	0.38 - 0.77	200
	Forks	0.28 - 0.39	104
	Waipapa	0.39 - 0.41	91
	Flaser	0.31 - 042	208
	Rowallanbum	0.62 - 0.86	250
	Northbrook	0.16 - 0.26	50
	Ruakokapatuna	0.24 - 0.42	119
	Kapoaiaia	0.26 - 0.54	212
	Butchers Creek	0.31 - 0.67	168
	Stanley Brook	0.32	106

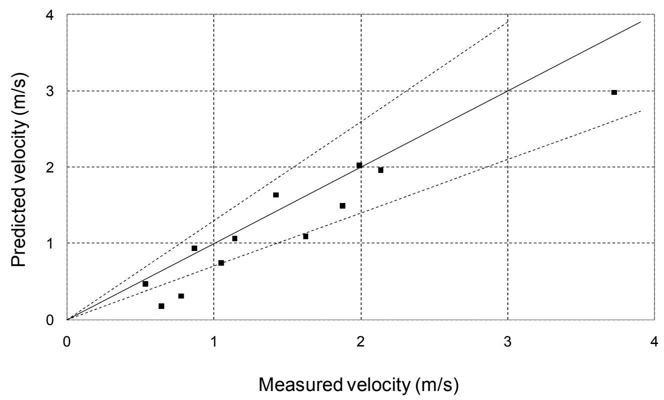

Figure 4. Measured and predicted (Equations (8) and (9) flow velocities with 30% accuracy limits for published field data of Bathurst (1985)

Equations (8) and (9) were applied to the Hicks and Mason (1998) field data. Predicted and measured flow velocities together with the perfect fit line and 30% accuracy limits are plotted in Figure 5.

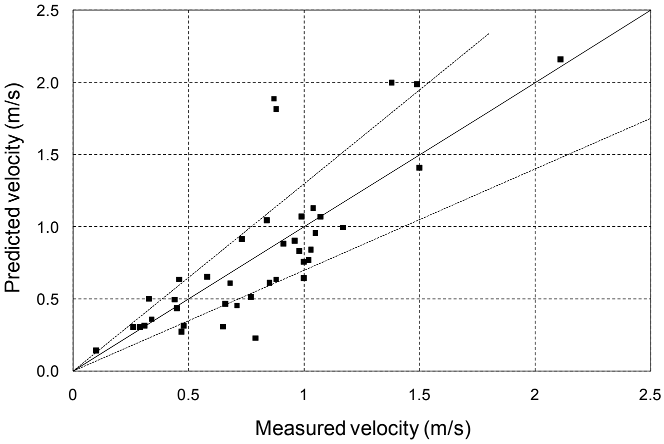

Figure 5. Measured and predicted (Equations (8) and (9) flow velocities with 30% accuracy limits for published field data of Hicks and Mason (1998).

The average absolute error in the predicted velocity is 27.09%. Although the errors are not small, estimation of the resistance coefficients is more reliable and less subjective than when using conventional equations. Using conventional equations requires estimation of a single resistance coefficient that varies significantly with discharge, making its estimation for new situations very difficult. The use of Equations (8) and (9) requires F and f, which are both more constant for a particular channel, and are (at least potentially) easier to estimate from channel characteristics. (There are many formulas for f in terms of bed material size; more work is required to get similar relationships for F). The transition equation therefore provides a better basis for generalizing field observations than a conventional resistance equation.

6 CONCLUSIONS

A transitional formula (Equation (8)) is proposed for describing the flow resistance in channels with bed roughness in the intermediate range, i.e. with relative depths (y/h) between 1 and about 4. The equation provides an estimate of velocity as a weighted combination of equations based on form resistance (applicable for the large roughness range) and surface shear resistance (applicable for small roughness conditions). The weighting variable, a, has been quantified from laboratory experimental results (Equation (9). This approach avoids the use of a single resistance coefficient that varies with flow condition.

The equations perform well against independent laboratory data and satisfactorily against field measurements. Reliability of predictions depends on knowledge of the large-scale and small-scale resistance coefficients (F and f), the former in particular requiring more extensive laboratory and field investigation.

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