

# Study of flow resistance in open channels

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**ABSTRACT:** Flow resistance in rivers plays an important part in river engineering and has been studied for many years (Yen, 2001). Yet, it still presents challenge to researchers and engineers, particularly in relatively steep gravel bed rivers. Better understanding and knowledge of the flow resistance in such rivers can greatly improve our prediction of the flow conveyance capacity in rivers, thus reducing/preventing river flooding. In this paper, we present data collected from both the large laboratory flume experiments and field measurements to investigate the flow resistance caused by the bed form. The data cover a broad range of flow, bed slope and sediment conditions. The divided hydraulic radius approach was applied to analyze the data. The results show that the relative roughness (the ratio of the water depth over the characteristic size of the bed sediment) plays a significant role in the bed form resistance. New formulas incorporating the relative roughness are then proposed. Comparing with existing formulas, the proposed formulas in this study better estimate the bed form resistance.

*Keywords: Flow Resistance, Bedforms, Roughness, Divided hydraulic radius*

## 1 INTRODUCTION

River flow resistance has significant influence on river's conveyance capacity and sediment transport. Accurate estimation of river flow resistance is of importance to predict the flow-stage relation in rivers, thus to evaluate the likelihood of river flooding and issue warning of flooding. Due to its importance, the river flow resistance has been extensively studied in the past several decades. The classical experiments of Nikuradse (1933) revealed that a relationship between flow resistance coefficient and flow Reynolds number existed. Colebrook (1938) conducted similar experiments using non-uniform grains. Rouse (1965) proposed a similar expression to that of Colebrook for open channel flows with rigid beds. Similar laboratory and numerical research work has been carried out by other researchers (e.g. Shen (1962); Carter et al. (1963); Engelund (1966); Garde & Ranga Raju (1966); Gladki 1979; Griffiths (1981); Brownlie (1983); Pender et al. (1998); Guo (2005); Guo et al. (2008); and Pender et al.(2007) among others).

With the development of more advanced flow measurement techniques, laboratory experiments on the flow over various bed forms can be carried

out with more accurate measurements. Lyn (1993) measured the turbulence of flow over artificial bed forms. McLean et al. (1994, 1999) performed experiments to investigate the flow over two-dimensional bed forms as well as bed-load sediment transport of flows over fixed ripples and dunes. Willette, Pender and McEwan (1998) studied the transport of graded sediment using laboratory experiments. Cao et al. (2006) investigated the flow resistance in open channels. Guo et al. (2008) carried out laboratory and numerical simulation of flow over rough beds for a wide range of flow conditions. Using double-averaging methodology and experimental data of McLean et al. (1994), McLean & Nikora (2006) showed that there exist two distinct regions of the double averaged velocity distribution for rough bed flows, namely a linear region below roughness tops and a logarithmic region above them. These studies have greatly improved our knowledge and understanding of the flow over rough bed or fixed bed forms.

Though these studies have demonstrated some features of the flow resistance in open channel flows with rough bed; accurate estimation of flow resistance in fluvial rivers is still a challenge task.

In particular, our knowledge on the flow resistance in relatively steep gravel bed rivers is grossly inadequate. In such rivers, sand dunes may form, which alter flow resistance (Simons & Richardson (1961); Yen (2002); ASCE Task Committee on Flow and Transport over Dunes, (2002)). As such, traditionally used flow resistance formulas such as the Darcy-Weisbach, Chezy, and Manning equations and some formulas proposed by aforementioned researchers may not be valid for estimating the overall flow resistance. In such situation, flow resistance consists of two components: resistance due to skin friction and resistance due to form drag induced by bed forms (sand dunes and ripples, etc). Therefore, some new approach is needed to treat the river flow resistance with bed forms, particularly in relatively steep gravel bed rivers which has received relatively less attention. In this study, both laboratory experiments and field measurements, together with the data of other researchers, are carried out to investigate the flow resistance in mobile gravel bed flows. The divided hydraulic radius approach (Einstein & Barbarossa (1952)) is applied to evaluate the flow resistance with bed forms in open channel flows.

## 2 LABORATORY EXPERIMENTS

Laboratory experiments were conducted in a flume of 24m long and 7m wide, consisting of four consecutive sections, namely (from upstream) a sediment feeding section (2m), a flow development section (8m), a measurement section (8m) and an exit section (6m). The flume bed was initially paved with the sediment that is the same as that added to the flow during the experiments. Two types of natural sediments were used in the experiments: fine sands with  $D_{35} = 0.56\text{mm}$ ,  $D_{50} = 0.88\text{mm}$  and  $D_{65} = 1.1\text{mm}$  and fine gravels with  $D_{35} = 1.7\text{mm}$ ,  $D_{50} = 2.3\text{mm}$  and  $D_{65} = 3.3\text{mm}$ , respectively, where  $D_{35}$ ,  $D_{50}$  and  $D_{65}$  are sediment sizes of which 35%, 50%, and 65% by weight is finer, respectively. Both sediments have a specific weight of  $2650\text{ kg/m}^3$ . The experiments were carried out using the constant flow rate and sediment transport for a broad range of bed slope, flow rate and sediments. Fifty-two experiments were conducted with the first 30 runs using coarse gravels and the last 22 runs using fine sands. The experimental parameters investigated in this study were: flow rate per unit width ( $q$ ):  $0.006\text{ m}^3/\text{s}\cdot\text{m}$  to  $0.025\text{ m}^3/\text{s}\cdot\text{m}$ ; bed slope ( $J$ ): 0.5%, 1.0%, and 1.5%; average velocity ( $U$ ):  $0.273\text{m/s}$  to  $0.694\text{m/s}$ ; and water depth ( $h$ ):  $0.015\text{m}$  to  $0.045\text{m}$ .

Bed forms are formed for almost all experiments. Figure 1 is the typical bed forms observed

in the experiment for coarse bed materials with water depth of  $0.031\text{ m}$ , average flow velocity of  $0.552\text{ m/s}$  and bed slope of 1%. Water levels were measured at four cross-sections spacing 2m apart in the measurement section. Water surface slope was calculated using the measured water level. Water depth and flume bed topography at the measurement section were measured with a specially designed device. Six equally spaced point measurements were made along each cross-section, i.e., neighboring verticals were spaced 1m apart. In total, twenty-four measurements were made in the measurement section and the average was computed accordingly to represent the section. More details of the experiments can be found in Yang et al. (2009).

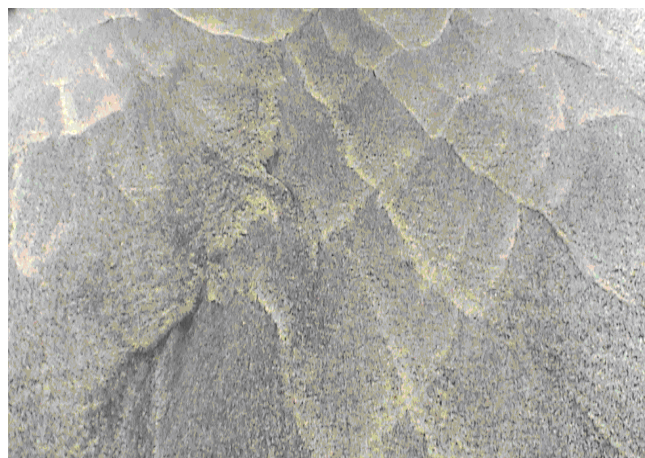


Figure 1 Typical bed forms observed in the experiment for coarse bed materials:  $h = 0.031\text{ m}$ ,  $U = 0.552\text{ m/s}$  and  $J = 1\%$

## 3 FIELD MEASUREMENT

The Hutubi River, a typical gravel-bed river at the North of Xingjiang in the north-west of China, was chosen for field experiments (Yang et al. (2009)). The bed slope of the river is 0.9-1.4% and the width of the river at the measurement reach is about 240 m. A total of 26 measurements were made in August 2002, covering broad range of flow: average velocity:  $1.46\text{m/s}$  to  $2.82\text{m/s}$ , water depth:  $0.255\text{m}$  to  $0.583\text{m}$  and bed slope: 0.93% to 1.43%. Bed materials were measured with the characteristics being  $D_{35} = 20.0\text{ mm}$ ,  $D_{50} = 33.2\text{ mm}$ ,  $D_{65} = 44.9\text{ mm}$ , and  $\gamma_s = 2680\text{ kg/m}^3$ . Water depths and velocity were measured using a typical wading rod and a propeller-type current meter, respectively. For such shallow river, the 0.6-depth method was used to represent the vertical mean velocity. The average velocity and water depth of the cross-section were calculated based on the measurements spacing 2m apart. The flow discharge was calculated using the measured water

depth and flow velocity. Figure 2 shows the measurement section at the Hutubi River.



Figure 2. Photo of field observation at Hutubi River.

## 4 RESULTS AND DISCUSSION

### 4.1 Estimation of $R''_b$ and $u^*$

For flow resistance comprising skin friction and bed form resistance, one possible approach is to separately estimate them. Many approaches could be used for this purpose. Among them, the divided hydraulic radius approach (first introduced by Einstein & Barbarossa, (1952)) is the one which has been widely used and is adopted in this study to investigate the flow resistance consisting of skin friction and bed form resistance. This approach linearly divides the hydraulic radius  $R_b$  into  $R'_b$  and  $R''_b$ , related to the grain friction and bed form resistance, respectively, i.e.,  $R_b = R'_b + R''_b$ , or  $R''_b = R_b - R'_b$ . Therefore, if  $R_b$  and  $R'_b$  can be determined, the hydraulic radius related to bed form resistance  $R''_b$  can then be evaluated.

The method proposed by Einstein (1942) is applied to estimate  $R_b$ . The approach is to divide the Manning coefficient  $n$  into two parts,  $n_b$  and  $n_w$ , related to the channel bed and bank, respectively:

$$n = R^{2/3} J^{1/2} / U \quad (1)$$

$$n^{3/2} P = n_w^{3/2} P_w + n_b^{3/2} P_b \quad (2)$$

where  $P$  is wetted perimeter ( $= P_w$  (bank) +  $P_b$  (bed)). The value of  $n$  can be calculated using velocity formulas for uniform flows. Assume  $n_w = 0.010$ ,  $P_w = 2h$ , and  $P_b =$  channel width, and  $R_b$  can be obtained by first solving Eq. (2) for  $n_b$  and then substituting  $n_b$  into the following equation (Yang et al. 2009):

$$R_b = \left[ \frac{U n_b}{J^{1/2}} \right]^{3/2} \quad (3)$$

The hydraulic radius  $R'_b$  related to the grain friction can be calculated using the following equation:

$$\frac{U}{\sqrt{g R'_b J}} = 5.75 \log \left( \frac{12.27 \chi R'_b}{K_s} \right) \quad (4)$$

where  $g$  is acceleration of gravity;  $K_s$  is equivalent roughness size and  $\chi$  is a coefficient which relates to  $K_s/\delta$  ( $\delta = 11.6\nu/u^*$ ) (where  $\delta$  is the thickness of laminar layer in vicinity of boundary,  $\nu$  is kinematic viscosity of water and  $u^*$  is shear velocity) as following (Chien & Wan, (1999)):

$$\chi = \begin{cases} 1.61 - [\eta - 1]^2 & , \eta \leq 1 \\ 1.61 - 0.25[\eta - 1]^2 & 1 < \eta \leq 2 \\ 1.8[\eta]^{-0.5} + 0.056\eta - 0.001318\eta^2 & 2 < \eta < 10 \\ 1.0 & , \eta \geq 10 \end{cases} \quad (5)$$

where  $\eta = K_s/\delta$ . Once  $R'_b$  and  $R_b$  are determined,  $R''_b$  and  $u^* = (gR''_b J)^{1/2}$  can be evaluated.

### 4.2 New Formulas

One of the main objectives of this study is to accurately estimate the flow resistance induced by bed forms. To this end, in addition to the laboratory experiments and field measurements of the authors', a total of 279 laboratory experiments and field observations of other researchers (Einstein & Barbarossa (1952); Vanoni & Nomicos (1960); Simons & Richardson (1961); Shen (1962); Simons & Senturk (1977); Brownlie (1983); and Graf & Suszka (1987)) are collected for the analysis. These data cover a broad range of flow and sediment conditions:  $0.049\text{m/s} < U < 2.82\text{m/s}$ ;  $0.015\text{m} < h < 16.7\text{m}$ ;  $0.002\% < J < 2.5\%$ ;  $0.016\text{mm} < D_{35} < 20\text{mm}$ ,  $0.019\text{mm} < D_{50} < 33.2\text{mm}$ ;  $0.021\text{mm} < D_{65} < 44.9\text{mm}$ ; and  $4.0 < h/D_{50} < 55670$ . The analysis of data shows that the relative roughness  $D_{65}/h$  plays an important role in the bed form resistance. Therefore, the traditional relationship of Einstein & Barbarossa's (1952) between  $U/u^*$  and flow parameter  $\psi' (= (\gamma_s - \gamma) D_{35} / (\gamma R'_b J))$  needs to modify to include  $D_{65}/h$ .

From equations (3), (4) and (5),  $R'_b$  and  $R_b$  can be calculated. Thus,  $R''_b$  and  $u^* = (gR''_b J)^{1/2}$  as well as  $\psi'$  can be estimated and the regression best fit curves for the upper and lower flow regimes are:

For the lower flow regime:

$$\frac{U}{u_*''} = \frac{17.5}{0.15\sqrt{h/D_{65}} + \log(0.2\psi')}, \text{ for } h/D_{65} \leq 150 \quad (6a)$$

$$\frac{U}{u_*''} = \frac{16.8}{0.10\sqrt{h/D_{65}} + \log(0.2\psi')}, \text{ for } h/D_{65} \leq 150 \quad (7a)$$

$$\frac{U}{u_*''} = \frac{17.5}{0.15 \times \sqrt{150} + \log(0.2\psi')}, \text{ for } h/D_{65} > 150 \quad (6b)$$

$$\frac{U}{u_*''} = \frac{16.8}{0.10 \times \sqrt{150} + \log(0.2\psi')}, \text{ for } h/D_{65} > 150 \quad (7b)$$

and for the upper flow regime:

The upper flow regime corresponds to  $U/(\rho_s - \rho)gD_{50}/\rho)^{1/2} \geq 6$ .

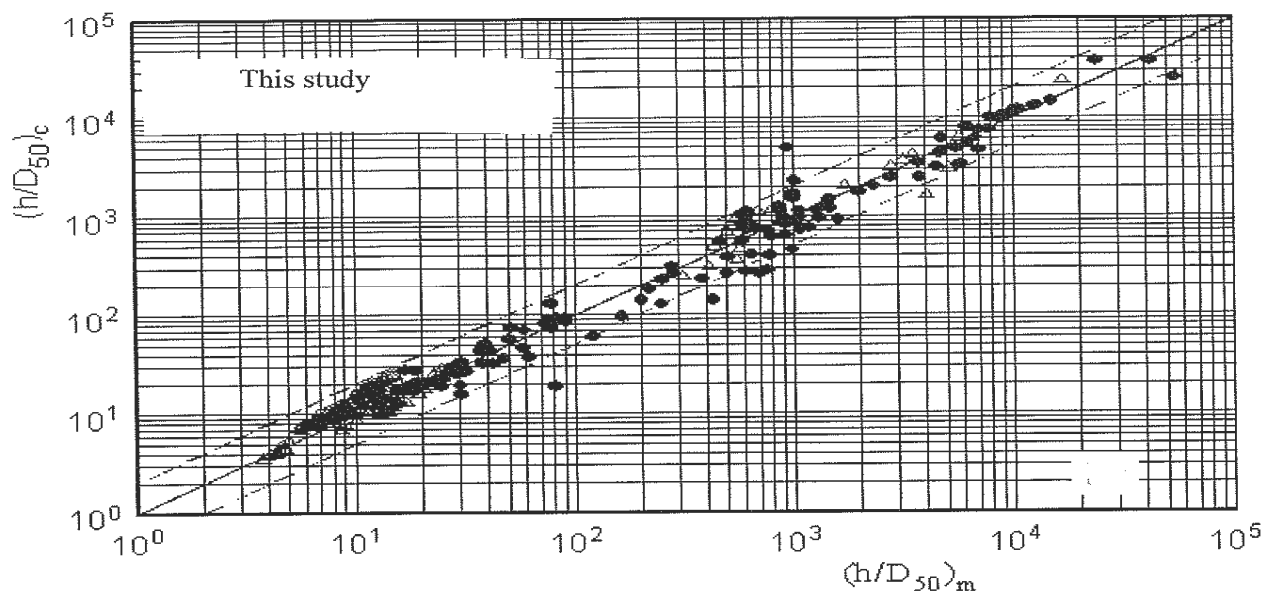


Figure 3. Comparison of calculated and measured relative water depth using the formulas of this study. The dotted lines in this figure and figures 4-6 indicate the interval of  $0.5 < \text{calculated value}/\text{measurement} < 2$ . Majority (98.5%) of  $h_c/h_m$  is within the interval of [0.5, 2].

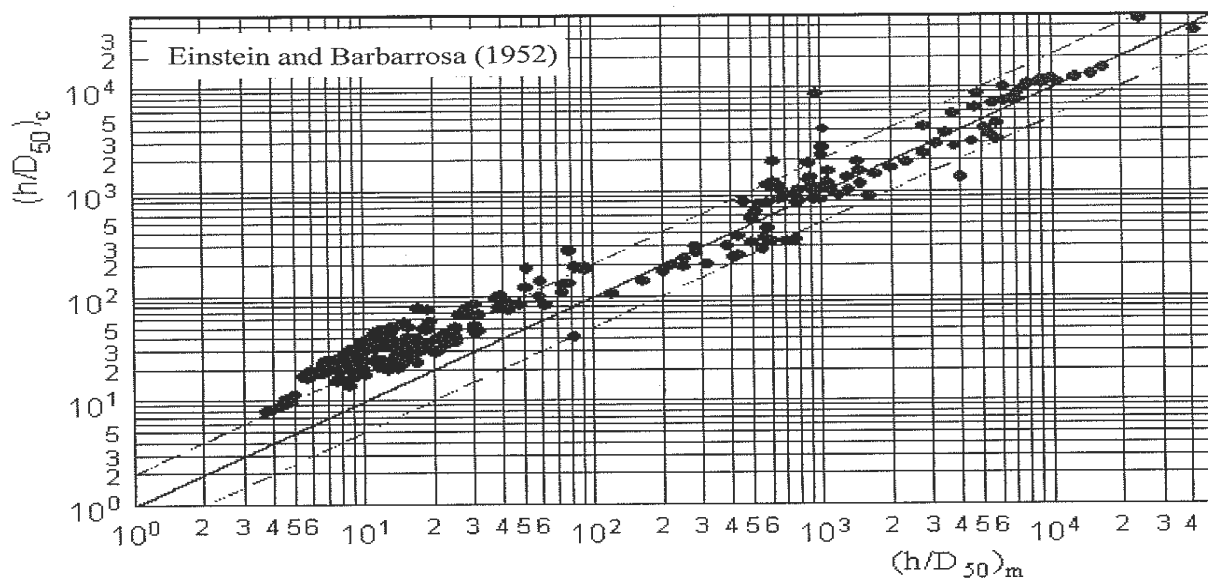


Figure 4. Comparison of calculated and measured relative water depth using the formulas of Einstein & Barbarrosa (1952). Only 52% of  $h_c/h_m$  is within the interval of [0.5, 2].

The prediction of water depth by above formulas is demonstrated in Figure 3. To examine the accuracy of the above formulas, the prediction of water depth by Einstein & Barbarosa (1952), Shen (1962) and Brownlie (1983) are shown in Figures 4, 5 and 6, respectively. In Figures 3-5, water depth  $h = R'_{b+} R''_{b-}$ . The calculation and figures show that the percentages of the ratio of the calculated ( $h_c$ ) over measured ( $h_m$ ) flow depths by various formulas falling into a certain interval are different. Only 52% (Einstein & Barbarosa's formula) and 77.4% (Shen's formula) of  $h_c/h_m$  are within the interval of [0.5, 2], respectively. Brownlie's formula for the upper flow regime gives

very good prediction of water depth with 99% of  $h_c/h_m$  being within the interval of [0.5, 2], however, his formula for the lower flow regime only gives reasonable prediction with only 77.7% of  $h_c/h_m$  falling within the interval of [0.5, 2]. In comparison with those formulas, the prediction accuracy of above equations (6a, 6b, 7a and 7b) is greatly improved with 98.5% of  $h_c/h_m$  being within the interval of [0.5, 2]. Figure 4 also shows that the formula of Einstein and Barbarosa generally overestimates the flow depth.

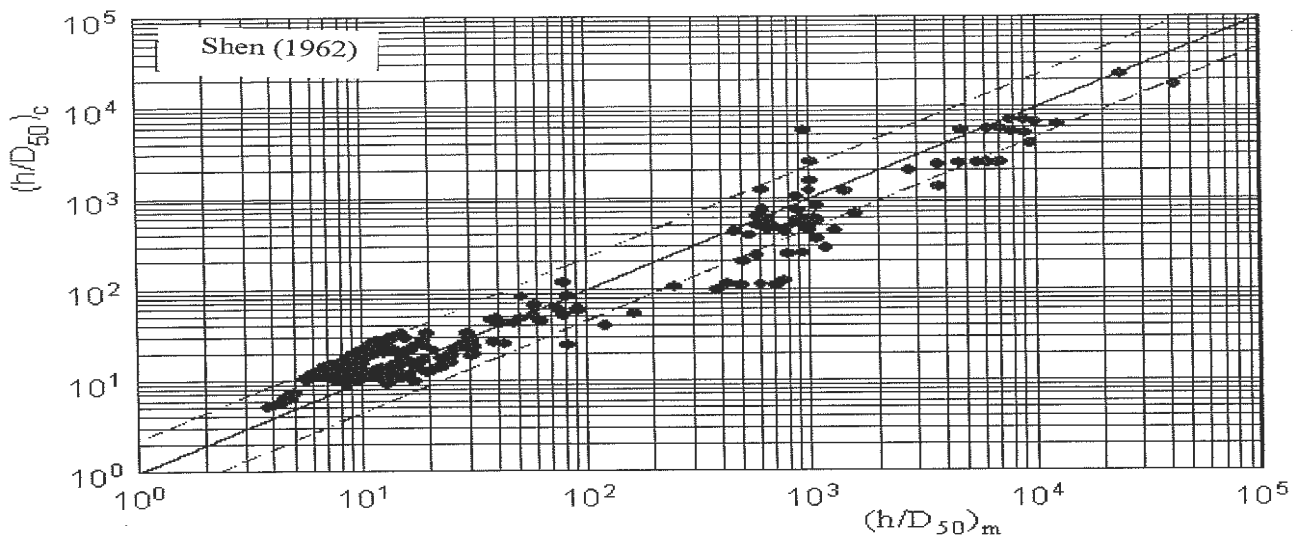


Figure 5. Comparison of calculated and measured relative water depth using the formulas of Shen (1962). Only 77.4% of  $h_c/h_m$  is within the interval of [0.5, 2].

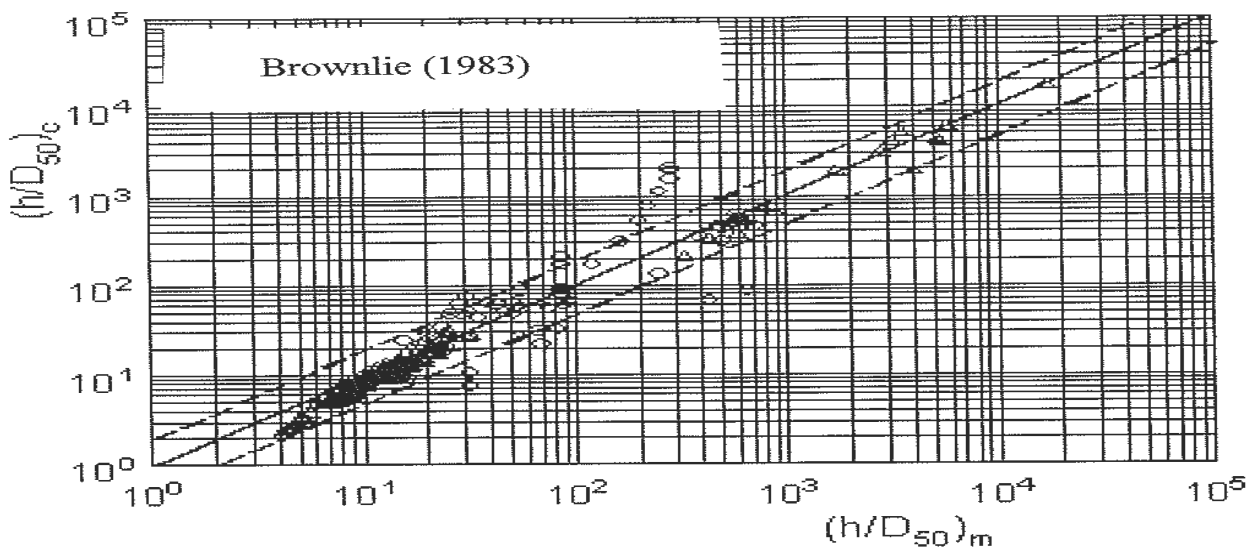


Figure 6. Comparison of calculated and measured relative water depth using the formulas of Brownlie (1983). Though 99% of  $h_c/h_m$  is within [0.5, 2] for upper flow regime, only 77.7% of  $h_c/h_m$  is within the interval of [0.5, 2] for lower flow regime.

## 5 CONCLUSION

Flow resistance in rivers and open channels is of enormous importance in river engineering and dynamics. The results of laboratory experiments and field measurements are presented to predict the water depth in fluvial rivers/open channels. The available laboratory and field data are collected for the analysis. The data cover a broad range of flow and sediment conditions. Major factors influencing flow resistance, such as flow discharge, bed slope, sediment conditions and relative roughness ( $h/D_{65}$ ) have been investigated. The divided hydraulic radius approach has been applied for analyzing data. The proposed formula for estimating the flow resistance give more accurate prediction than existing formulas.

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