Uncertainty in design water levels due to uncertain bed form roughness in the river Rhine

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ABSTRACT: Hydrodynamic river models are applied to design and evaluate measures for purposes such as safety against flooding. The modeling of river processes involves numerous uncertainties, resulting in uncertain model results. Knowledge of the type and magnitude of these uncertainties is crucial for a meaningful interpretation of the model results. The aim of this study is to quantify the uncertainty due to the hydraulic roughness predictor for the river bed in the main channel and assess its effect on the uncertainty in the modelled water levels under design conditions. The uncertainty in the bed roughness consists of the uncertainty between the different roughness predictors and the uncertainty within the roughness predictor itself. We compared five roughness predictors, which determine the hydraulic resistance of the river bed in the Dutch river Rhine, based on measurements of bedform characteristics and water levels for different discharges. Generalized Extreme Value distributions have been fitted through the predicted roughness values and extrapolated to the design return period. The results show that the 95% confidence interval for the Generalized Extreme Value distributions the predicted Nikuradse roughness values under design conditions ranges from 0.32 m to 1.03 m. This uncertainty is propagated through the model by means of a Monte Carlo Simulation. The Monte Carlo analysis shows that the uncertainty in the hydraulic roughness results in a 95% confidence interval for the water depth that range from 11.7 m to 12.4 m.

Keywords: Uncertainty; Monte Carlo Simulation; Hydraulic roughness; Hydrodynamic modeling; Generalized Extreme Value distribution; River Rhine

1 INTRODUCTION

Hydrodynamic river models are applied to design and evaluate measures for purposes such as safety against flooding. These numerical models are all based on a deterministic approach. However, the modeling of river processes involves numerous uncertainties, resulting in uncertain model outcomes. Knowledge of the type and magnitude of uncertainties is crucial for a meaningful interpretation of the model results and the usefulness of results in decision making processes.

The Dutch flood defenses along the river Rhine are designed to withstand a flood with a probability of exceedance of 1/1250 per year. The water levels that occur during such a flood are computed with the deterministic hydrodynamic model, WAQUA (Rijkswaterstaat, 2005), using a design discharge as input. However, uncertainty resides in all parts of this model. To quantify the uncertainty in the design water levels we want to esti-

mate the uncertainties in these parts of the model that have the largest contribution to the uncertainty in the design water levels.

Elicitation of expert opinions showed that the uncertainty in the design discharge and the hydraulic roughness formulation of the main channel contributes mostly to the uncertainty in the model outcomes (Warmink et al., 2009). Also, the studies by Chang et al. (1993), Bates et al. (1996), Hall et al. (2005) and Pappenberger et al. (2008) show that the hydraulic roughness of the river bed is one of the main uncertainties in river models.

The hydraulic roughness in the main channel of the Dutch river Rhine is dominated by the resistance due to bed forms that develop on the river bed and increase in height with increasing discharge. Different predictors exist that relate flow and dune characteristics to hydraulic roughness (e.g. Van Rijn, 1984; Vanoni and Hwang; 1967). In previous research the differences between results of different roughness predictors were found

to be large (Van Rijn 1993, Julien et al. 2002). Noordam et al. (2005) showed that different roughness predictors resulted in substantial different roughness values for a series of measured bed forms and flow characteristics in flumes. Also, Van der Mark (2009) showed that different roughness predictors resulted in different roughness predictions for uniform and alluvial flume data. The results from the study by Julien et al. (2002) show that also for field measurements in the Dutch river Rhine, different roughness predictors result in different roughness values. Furthermore, they showed that there is a strong hysteresis effect between bed form roughness and discharge.

The studies by Noordam et al. (2005) and Julien et al. (2002) only give an indication of the uncertainty in the bed roughness for measured discharges. However, we want to go a step further and quantify the uncertainty in the bed roughness under design conditions. Furthermore, both studies only take three roughness predictors into account, while many different roughness predictors exist that are valid to predict hydraulic bed roughness of lowland rivers. Next to the uncertainty in the roughness itself, also the effect of the uncertain roughness of the water levels is not taken into account.

Therefore, the aim of this study is 1) to quantify the uncertainty in the bed roughness of the main channel under design conditions, by extrapolation of field measurements and 2) to quantify the contribution of this uncertain roughness to the uncertainty in the water levels at design discharge.

2 DATA

We used the data from the studies by Julien et al. (2002) and Wilbers (2004). The data from Julien et al. (2002) were measured during the 1998 peak discharge in the river Rhine. Longitudinal and cross-sectional profiles of the bed elevation were recorded by single and multi-beam echosounding. Bedform data were recorded about twice a day for ten days during the flood wave and about every three days for the next twelve days. The data were processed and classified into primary and secondary dunes using the procedure described by Ten Brinke et al. (1999). Next to the bedform measurements also the water level, bed material and flow velocities were recorded.

The data from Wilbers (2004) were measured during the flood waves of 1995, 1997 and 1998 using a single-beam echosounder for the 1995 discharge, a single and multi-beam echosounder for the 1997 discharge and a multi-beam echosounder for the 1998 discharge. Both measurements were conducted in the river Rhine near Lo-

bith. Wilbers used a program called DT2D (Wilbers, 2004) to calculate the dune characteristics from the measured bed elevation profiles. To minimize the effect of hysteresis only the data from the rising limb of the flood waves are taken into account.

Julien et al. (2002) and Wilbers (2004) used different methods to calculate dune characteristics from measured bed elevations. This means that the absolute values from both methods are not comparable. Therefore, the data from Julien et al. (2002) are used to compute the performance of the roughness predictors and the extrapolation of the roughness predictors is carried out using the data of Wilbers (2004).

3 METHODS

To quantify the uncertainty in the bed roughness under design conditions, we consider three sources of uncertainty. Firstly, the uncertainty due to the choice for the roughness predictors. This source of uncertainty is further referred to as the *between roughness predictor uncertainty*. Secondly, we consider the uncertainty due to the extrapolation of the extreme value distribution. This assumes that the fitted distribution is appropriate to describe the data. Thirdly, the uncertainties due to the simplifications within the roughness predictors are considered. The uncertainties in the measurements of the dune and flow characteristics are included in this source of uncertainty. This uncertainty is further referred to as the *within roughness predictor uncertainty*. We quantified all three sources of uncertainty and combined them to quantify the total uncertainty in the hydraulic roughness under design conditions.

A Generalized Extreme Value (GEV) is fitted through the predicted roughness values and extrapolated to design conditions. To quantify the second source of uncertainty, we compute the uncertainty due to the extrapolation of each roughness predictor. The uncertainty within each roughness predictor is computed based on their performance for the field measurements and, finally, the three sources of uncertainty are combined and propagated through the WAQUA model to yield the uncertainty in the water levels.

3.1 Selection of roughness predictors

Many different roughness predictors exist to compute bed roughness based on characteristics of the bed, bedforms and flow characteristics. These can be classified in analytical, semi-analytical and empirical roughness predictors. The analytical predictors (Yalin, 1964; Engelund, 1966) are

based directly on the mass and momentum conservation laws. The semi-analytical roughness predictors (Engelund, 1977; Karim, 1999; Van der Mark 2009) are based on the conservation laws, but are calibrated to fit measured (flume) data. The empirical roughness predictors (Vanoni-Hwang, 1967; Van Rijn, 1984, 1993; Engelund-Hansen, 1967, Haque-Mahmood, 1983; Wright-Parker, 2004) are empirical relations between bed-form and flow characteristics and measured bed roughness.

The data from Julien et al. (2002) are used to compare these roughness predictors for the 1998 peak flood event in the river Rhine. The measured roughness is computed using the Darcy-Weisbach friction factor:

$$f = \frac{8ghS_b}{V^2} \qquad (1)$$

where g = gravity acceleration, h = water depth at river axis, S_b = bed slope, and V = flow velocity.

3.2 Fitting the extreme value distribution

The design discharge is a condition that has never occurred, therefore, we need to extrapolate the predicted roughness values to determine the uncertainty due to the differences between the roughness predictors under design conditions. We fit a statistical extreme value distribution through the data, because we do not know the characteristics of the bed forms under design conditions. Although, the studies by Julien et al (2002) and Wilbers (2004) suggest that bed forms might flatten and secondary bed forms might develop, there is little knowledge on how this affects the hydraulic roughness. Therefore, we extrapolate the predicted hydraulic roughness to design conditions and in this way only include the development of the bed forms as far as visible in the available measurements.

We used the historically observed discharges of the last 100 years for the river Rhine to generate an independent series of roughness values. These discharges are the annual maximum observed values at the location where the Rhine enters The Netherlands.

The first step is to determine a relation between the observed discharge and the predicted roughness based on the Wilbers (2004) data. The predicted roughness is expressed as a Nikuradse roughness coefficient, k_s, because this coefficient is independent of the water depth. This parameterization is carried out by fitting a linear or first order power relation to the predicted roughness values given the corresponding measured discharge. Next, for all historically observed discharge val-

ues, a roughness value is computed based on the parameterized relation.

Secondly, a Generalized Extreme Value (GEV) distribution is fitted to the generated series of independent roughness values for each roughness predictor (Coles, 2001).

$$GEV(x) = \exp\left\{-\left[1 + \kappa\left(\frac{x - \mu_{GEV}}{\sigma_{GEV}}\right)\right]^{-1/\kappa}\right\} \qquad (2)$$

where μ_{GEV}, is the location parameter of the GEV distribution, σ_{GEV} is the scale parameter and κ is the shape parameter. The GEV distribution is the general case for extreme values. The three types of the GEV distribution for $\kappa > 0$, $\kappa < 0$ and $\kappa = 0$ (Gumbel distribution) have distinct forms of tail behavior. The GEV distribution has three parameters and, therefore, three degrees of freedom during the fitting procedure. The Gumbel distribution has a fixed tail behavior which is linear on a logarithmic x-axis, while for the GEV distribution the data themselves determine the most appropriate type of tail behavior and there is no need to make an a priori judgment about which extreme value distribution to adopt (Coles, 2001).

For each roughness predictor, the optimum values for the parameters of the cumulative GEV distribution are determined. The fitting of the distribution is carried out using the "ORCA routines", a MatLab package for extreme value statistics (Deltares, 2010) using probability weighted moments. The fitted GEV distributions are extrapolated to the design return period of 1250 years.

3.3 Uncertainty due to extrapolation

Next to the uncertainty between the selected roughness predictors also the uncertainty due to the extrapolation of each roughness predictor affects the uncertainty in the hydraulic roughness. The 95% confidence intervals for the GEV distribution are computed by the ORCA-routines using a bootstrap technique, given the variances of the fitted parameters. The confidence intervals therefore assume that the distributions appropriately describe the data. To test if this is a valid assumption we use a Probability Plot Correlation Coefficient test (Stedinger et al. 1993).

3.4 Uncertainty within roughness predictors

To quantify the third source of uncertainty, we included the variability with the roughness predictors. This uncertainty is caused by the simplifications of the roughness predictor and the uncertainties in the measurements of the bed form and flow characteristics. This uncertainty is quan-

tified by the performance of the roughness predictors for the river Rhine data. We assign a weight to the different roughness predictors based on their performance for the Julien et al. (2002) data. The performances of the roughness predictors, i are expressed by the Root Mean Squared Errors (RMSE). The weights are then computed by:

$$w_i = \frac{\left(\dfrac{1}{RMSE_i}\right)^a}{\displaystyle\sum_{i=1}^{5}\left(\dfrac{1}{RMSE_i}\right)^a} \qquad (3)$$

This equation is a normal standardization, in which we take the inverse of the RMSE as a measure for the performance, because the roughness predictors with low RMSE values are assigned a large weight. We assume a to be one so the weights decrease linearly with the value of the RMSE.

3.5 *Propagation of uncertainties to the design water levels*

The final step is to compute the contribution of the uncertain roughness under design conditions to the uncertainty in the computed water levels. Therefore, a Monte Carlo Simulation (MCS) is carried out using a simplified schematization of the WAQUA model, which is used for the computation of the design water levels. The WAQUA model is a numerical, two dimensional, depth averaged river model.

First a model is created with dimensions similar to those of the river Rhine upstream of the first bifurcation in The Netherlands (figure 1). We use grid cells of 40 m and the model represents a straight river reach of 16 km long. The depth of the main channel is 8 m and the dikes are assumed to be infinitely high. This new WAQUA model is used for the Monte Carlo Simulation, because it is faster to simulate than the original WAQUA model for the Dutch Rhine branches.
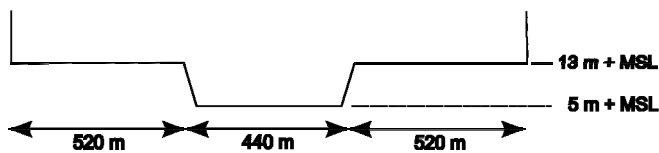

Figure 1. Cross-section of the new WAQUA model. The dimensions are similar to the river Rhine

MCS consists of a large number of deterministic simulations where the uncertain roughness is randomly generated (Morgan and Henrion, 1990). A single simulation of this new WAQUA model requires a lot of computational time, therefore, only 1000 simulations are carried out. The number of

samples (m) for every selected roughness predictor, i, is computed by:

$$m_i = Nw_i \qquad (4)$$

where N=1000 is the total number of samples drawn from the distributions. The number of samples for each roughness predictor are rounded towards the nearest integer, so that the sum of the samples equals N.

The drawn samples from the different distributions give the combined uncertainties in the hydraulic roughness due to the three different sources of uncertainty. Subsequently, every sampled roughness value is assigned to the main channel in the new WAQUA model as a constant Nikuradse roughness, which is independent of the water depth. The Nikuradse roughness for the floodplains is set constant at 0.6 m, which is approximately the average of the vegetated floodplains in the WAQUA model for the Dutch Rhine branches. A simulation is carried out for each of the sampled roughness values with a constant design discharge of 16000 m³/s set as upstream boundary condition (Parmet et al. 2001). A constant water level is set as the downstream boundary condition. We used a 60 km long model, because the downstream boundary condition affects the computed water levels and a significantly long model is required to reduce the effects of the fixed downstream boundary condition.

4 RESULTS

4.1 *Selection of roughness predictors*

In this study we selected 5 roughness predictors that were considered applicable to predict the roughness for the main channel of the river Rhine under design conditions. Figure 2 shows the performance of ten roughness predictors. It is shown that the analytical and semi-analytical roughness predictors perform worse than the empirical predictors. This was expected, because the (semi-)analytical predictors do not account for variability in the bedforms characteristics, because mainly flume data was used for calibration.

Furthermore, figure 2 shows that the Vanoni-Hwang, Van Rijn, Yalin, Engelund66, Engelund77, Wright-Parker and Haque-Mahmood yield the best results. These roughness predictors are calibrated partly on field measurements of lowland rivers, therefore they are assumed to also be able to predict the roughness in other lowland rivers, such as the river Rhine.

We selected the Van Rijn, Vanoni-Hwang, Engelund77, Haque-Mahmood and the Wright-

Parker roughness predictors. The Yalin and Engelund66 roughness predictors are omitted because the assumptions of abrupt flow expansion are not valid under field conditions. Furthermore, these predictors are adapted in the Engelund77 roughness predictor by calibration of the analytical roughness predictors on irregular bed forms. The Wright-Parker roughness predictor is based on the Engelund-Hansen roughness predictor and adapted using field measurements. Therefore, the Wright-Parker predictor is more appropriate to predict bed roughness for the river Rhine. Furthermore, the Karim and Van der Mark roughness predictors are not taken into account, because they are only calibrated on flume data and perform poorly for the data in this case study.
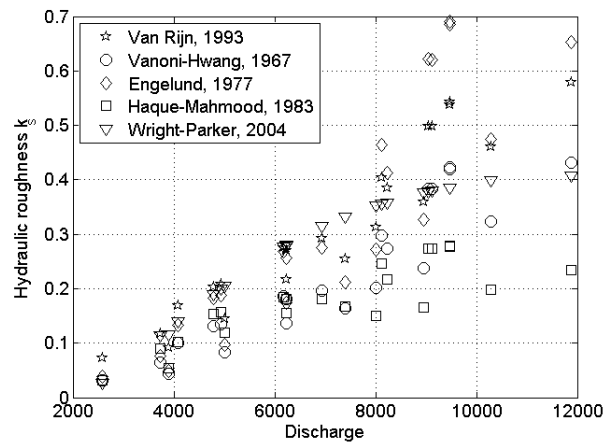
Figure 3. Nikuradse roughness as function of discharge predicted by the five selected roughness predictors.

Figure 3 shows the predicted roughness values for the data from Wilbers (2004) using the five selected roughness predictors. This figure shows that for all roughness predictors, generally, the Nikuradse value increases with increasing discharge. This was also expected, because the bedforms increase in height and length with increasing discharge. Furthermore, there is a large scatter in roughness values, especially for larger discharges. At a discharge near 12000 m^3/s, which is 75% of the design discharge, the predicted roughnesses range from 0.22 for the Haque-Mahmood roughness predictor to 0.65 for the Engelund, 1977 roughness predictor.

Both the Haque-Mahmood and the Wright-Parker roughness predictor show a concave shaped trend of roughness with discharge, while the other three roughness predictors indicate a more linear increase. Note that only the rising limb data of the flood wave is taken into account.

4.2 *Extreme value distribution fitting and confidence interval*

Figure 4 shows the fitted GEV distributions for the five selected roughness predictors. The fitted GEV distributions are extrapolated to the return period of 1250 years. The roughness values at the design discharge range from 0.36 m for the Haque-Mahmood roughness predictor to 0.92 m for the Engelund roughness predictor. The figure shows that all hydraulic roughness predictors show a clear increase in the hydraulic roughness with increasing return periods. However, the GEV distributions slightly underestimate the predicted roughness values at higher return periods.

The tail behavior of the GEV is determined by the data and not fixed a priori. The trend in the GEV distribution shows a convex shape, which is expected as dunes flatten with increasing discharge and the roughness is reduced. This is in accordance with the expectation that the hydraulic
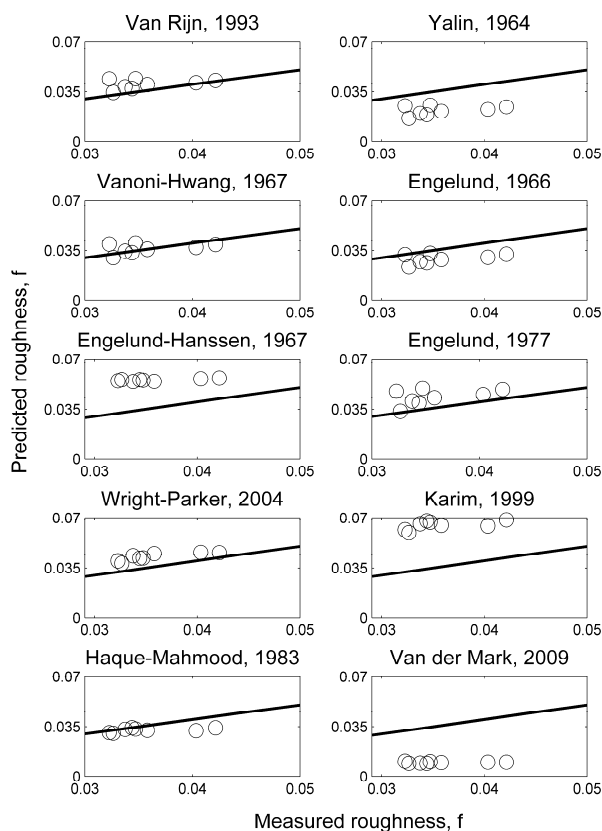

Figure 2. Performance of the different roughness predictors for the rising limb data measured during the 1998 peak discharge in the river Rhine, from Julien et al. (2002). The straight line represents the line of perfect agreement.

roughness does not infinitely grow with increasing discharge (Van Rijn, 1984; Julien et al. 1995).

We carried out a Probability Plot Correlation Coefficient (PPCC) test (Stedinger et al. 1993) to test if the predicted roughness values could have been drawn from the fitted distributions. The analysis showed that for the GEV distributions, there was no reason to reject these distributions at the 95% confidence interval. This means that the samples could have been drawn from a GEV distribution and that the assumption that the data can be described by the GEV is valid.
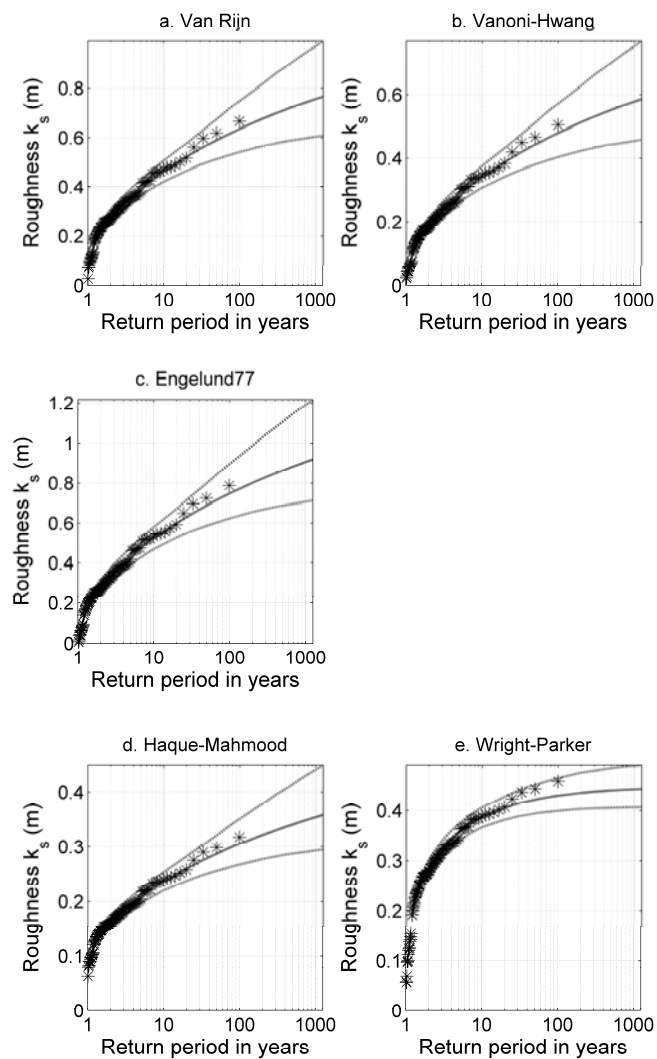

Figure 4. Fitted GEV distributions for the five roughness predictors. The dotted lines show the 95% confidence interval

Furthermore, figure 4 shows the 95% confidence intervals for the fitted distributions. This figure shows that the 95% confidence intervals increase with increasing return period. The width of the confidence intervals compared to the extrapolated roughness at the design discharge is about 50% of the extrapolated roughness value for the first four roughness predictors, ranging from 43% for the Haque-Mahmood roughness predictor to 55% for the Engelund77 roughness predictor. The width of the confidence interval for the Wright-

Parker roughness predictor is only 18% of the extrapolated roughness at design discharge.

4.3 *Monte Carlo Simulation*

Figure 5 shows the samples drawn from the distributions at the design return period for the five roughness predictors. The number of samples is determined based on the performance of the roughness predictors for the Julien et al. (2002) data. The number of samples ranges from 121 samples for the Engelund77 predictor to 292 samples for the Vanoni-Hwang roughness predictor, for a total of 1000 samples.

Figure 5 shows that with 95% confidence the roughness under design conditions lies approximately between 0.32 m and 1.03 m with a mean of 0.57 m, which is a range of 71 cm. Furthermore, the smaller 95% confidence intervals for the Haque-Mahmood and Wright-Parker roughness predictors in figure 4 result in high and narrow peaks in figure 5, while the Van Rijn, Vanoni-Hwang and Engelund77 roughness predictors show less high and wider peaks in figure 5.

Figure 6 shows the uncertainty in the water levels at Lobith (the location of the cross-section). This figure shows the sum of five peaks for each roughness predictor. The left peak results mainly from the Haque-Mahmood and Wright Parker roughness predictors. The larger peak on the right is the combination of the other roughness predictors that overlap.
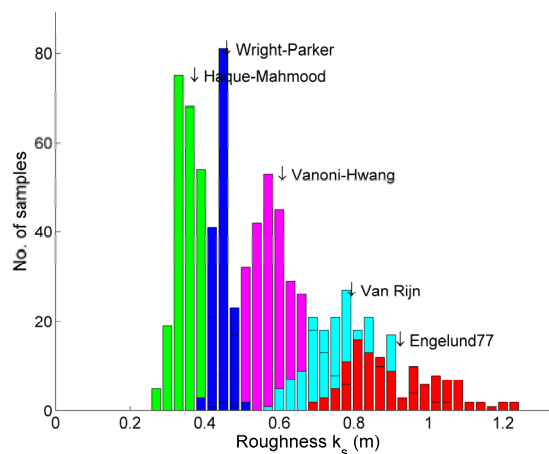

Figure 5. Uncertainty within and between roughness predictors for the GEV distributions. The samples are drawn from the lognormal distribution at the design return period.
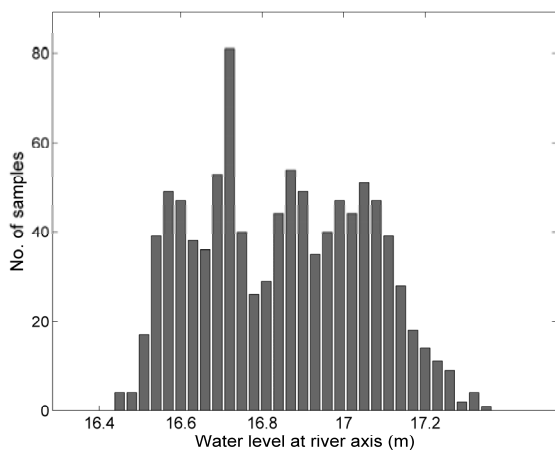
Figure 6. Uncertainty in water levels in the river Rhine due to uncertain roughness for a constant design discharge.

The figure shows that the uncertainty in the water level has a maximum range of 88.3 cm. If we only take a single roughness predictor into account (not shown in the figure), this range is approximately 20 to 40 cm depending on the selected roughness predictor. The uncertainty between the different roughness predictors, therefore, has a large influence on the uncertainty in the modelled water levels. Furthermore, Figure 6 shows that with 95% confidence the design water level at the observed location lies approximately between 16.52 m and 17.22 m above MSL with a mean of 16.85 m, which is a range of 70 cm.

5 DISCUSSION

The study by Julien et al. (2002) shows that hysteresis has a large effect on the relation between roughness and discharge. We corrected for hysteresis by taking only the rising limb data of the flood wave. However, hysteresis is still present in the discharge-roughness relation. There is still a constant lag between discharge and bedform roughness. This may result in a shift of the average roughness values and may affect the absolute values of the extrapolated roughness and water levels. However, the hysteresis effect results in a constant lag and, therefore, does not significantly affect the uncertainty around the mean values. In further research, however it is recommended to account for the hysteresis effect. Furthermore, in this study we only considered a constant discharge thereby omitting the interaction between varying discharge and roughness. However, if in future studies we want to be able to compute the roughness under varying discharge, this hysteresis effect has a significant effect on the bed forms and therefore on the computed water levels (see for example Paarlberg 2008).

The uncertainty in the water levels is sensitive to the selected roughness predictors that are included in the uncertainty analysis and the selected distribution used to extrapolate the hydraulic roughness to the design discharge. The selection of the hydraulic roughness predictors has been carried out with care by using only the predictors that are valid for the river Rhine.

The comparison of the predicted roughness values to the calibrated roughness in the WAQUA model shows that all predicted roughness values are larger than the calibrated roughness. This indicates that the other roughnesses in the model, such as vegetation roughness or roughness due to groynes are overestimated.

The presented uncertainty analysis shows that the uncertainty due to the used roughness predictor significantly contributes to the uncertainty in the design water levels. To reduce the uncertainties, more research is required on the physical processes in lowland rivers that cause the development of bed forms and resulting hydraulic roughness. In future research, we recommend to include more physical information in the extrapolation of the discharge. More measurements are required to give insight in these processes. Effort in an improved estimation of the hydraulic roughness can significantly reduce the uncertainties in the hydraulic roughness especially under design conditions.

6 CONCLUSIONS

The aim of this study is to quantify the uncertainty due to the hydraulic roughness predictor for the river bed in the main channel and assess its effect on the uncertainty in the modelled water levels for design discharge conditions. This uncertainty consists of uncertainty between different roughness predictors, the uncertainty due to extrapolation and the uncertainty within each roughness predictor. To quantify the uncertainty between the selected roughness predictors, we selected the Van Rijn, Vanoni-Hwang, Engelund, 1977, Haque-Mahmood and Wright-Parker roughness predictors that are appropriate to predict bed roughness for the river Rhine, based on a comparison of the performance of these roughness predictors for measurements of bed forms characteristics and water levels under varying discharge. It is shown that different roughness predictors result in a wide spreading of roughness values for the same measurements of bedform and flow characteristics.

The Generalized Extreme Value distribution is used to extrapolate the predicted roughness values for each roughness predictor to design conditions. The Generalized Extreme Values distribution ac-

counts for the fact that we deal with extreme values. The three different sources of uncertainty are quantified and combined to show that the 95% confidence interval of the Nikuradse roughness for the main channel of the river Rhine under design conditions ranges from 0.32 m to 1.03 m, which is a range of 56 cm. A Monte Carlo Simulation shows that this result in a 95% confidence interval for the design water levels with a range of 70 cm. The uncertain hydraulic roughness of the main channel has therefore a significant influence on the modelled water for the Dutch river Rhine. Improving the roughness predictions has the potentials to significantly reduce the uncertainties in modelled (design) water levels.

ACKNOWLEDGEMENTS

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