River Flow 2010 - Dittrich, Koll, Aberle & Geisenhainer (eds) - © 2010 Bundesanstalt für Wasserbau ISBN 978-3-939230-00-7

Bed roughness at high bed shear in open channels and pressurized pipes

V. Matoušek & J. Krupička
Dept. Of Hydraulics and Hydrology, Faculty of Civil Engineering, Czech Technical University in Prague, Czech Republic

ABSTRACT: The roughness is discussed of gravel bed streams at high bed shear caused by a flood flow carrying a substantial amount of sediments. An analogy is described with flows eroding a stationary deposit in enclosed pressurized pipes. Field measurements were carried out in two gravel-bed streams in the north-east part of the Czech Republic after a disastrous flood in June 2009. The measurements indicated that at the peak discharges the values of the gravel-bed roughness were considerably higher than predicted for gravel-bed streams in the literature. It is suggested that the increase in bed roughness is due to additional friction exerted by transported sediments. A mechanism of friction due to grains transported as bed load is demonstrated on the results of flows in the upper-plane-bed regime through enclosed pipes. A relatively big uncertainty in bed-roughness determination from field measurements reconstructing flood conditions in gravel-bed streams suggests that analogous tests could be carried out in more controlled conditions in laboratory pipes.

Keywords: Sediment transport, Hydraulic transport, Slurry flow, Bed load, Flood

1 INTRODUCTION

Any reliable suggestions for flood control measures require a sufficiently accurate estimation of the relationship between the discharge and the water stage under a flood condition in a river. The relationship is strongly affected by channel flow resistance, and hence by the roughness of a channel bed. The problem is particularly complicated if the stream bed is mobile (erodible).

In the last decade, gravel-bed streams in several regions of the Czech Republic experienced an increased number of extreme flood events that caused damages to the stream channels and their floodplains. Usually, the flooding was accompanied with an intense transport of sediments. It is believed, that the intense transport of sediments considerably increased channel resistance and hence the water stage during the flood events.

2 FLOOD FLOW RESISTANCE OF GRAVEL-BED STREAMS

Information on flow resistance of gravel-bed streams (streams with beds composed of grains of the mass-median size between 2 mm and 256 mm) is rather limited. This is particularly the case for flows at high bed shear, i.e. flood flows.

2.1 *Concept for fixed-bed roughness*

Flow resistance is associated with a velocity profile along a flow depth. A gravel bed is a hydraulically rough boundary for which the velocity profile can be determined using a logarithmic (or more simply a power-law) relationship between the dimensionless velocity u/u_{*b} (u = local velocity of flow at certain vertical position y above bed, u_{*b} = bed shear velocity) and the relative position of the local velocity y/k_s (y = vertical distance of local velocity position above velocity-profile origin, k_s = equivalent roughness height of bed). Integrating and rearranging lead to the relationship for the bed friction coefficient

$$\frac{v}{u_{*b}} = \sqrt{\frac{8}{\lambda_b}} = \frac{1}{\kappa}\ln\left(\frac{B_s R_b}{k_s}\right) \quad (1)$$

where v = average velocity over flow depth, λ_b = friction coefficient for bed, R_b = hydraulic radius of part of flow discharge area that is associated with bed, κ and B_s are constants (the Kármán constant $\kappa = 0.4$, B_s may vary with flow conditions, typical value for open channel flows is 11).

The roughness k_s is defined as the equivalent sand roughness size in the original Nikuradse logarithmic formula (law of the wall) for a hydraulically rough boundary. The formula was calibrated using measured data from enclosed pipes with artificial roughness simulated by sand particles glued to an inner wall of a test pipe. Thus, it makes sense to assume that in case of gravel bed streams, the bed is a hydraulically rough boundary with k_s related to a certain characteristic size of the bed provided that the bed is fixed (not movable). Assuming a fixed bed, i.e. flow conditions at which the relative bed shear stress (bed Shields number

$$\theta_b = \frac{u_{*b}^2}{(\rho_s / \rho - 1) gd} \tag{2}$$

where ρ_s = density of grains, ρ = density of liquid, g = gravitational acceleration, d = grain size) exerted to the bed surface by the flowing liquid does not exceed the threshold value at incipient motion of bed grains, a linear relation has been proposed between the bed roughness size and the size of a characteristic grain of a bed, $\alpha = k_s/d_x$ (see e.g. a survey in Table 2-1 in García et al. 2008). The most often proposed characteristic size $d_x = d_{84}$, i.e. the grain diameter for which 84 per cent by weight of the bed material is finer.

2.2 *Movable-bed roughness*

A determination of the roughness of a mobile bed can be complicated by a presence of bed forms that produce the drag roughness additional to the skin roughness. It is difficult to evaluate whether or not bed forms are present and contribute to total roughness in gravel-bed streams conveying flood discharges (i.e. bank-full- and higher discharges). It can be assumed that gravel-bed streams are much less sensitive to bed undulation than sand-bed streams due to a broad grain size distribution of the surface layer of a gravel bed. Furthermore, no generally validated methods are available for bed shear stress decomposition in to skin friction and form drag.

2.3 *Field tests in gravel-bed streams after floods*

In July 2009, a few weeks after devastating floods in the north-east part of the Czech Republic, field tests were carried out on two gravel-bed streams affected by flooding in the region. One selected stream was relatively big (river Bělá, see parameters in Tab. 1), the other was significantly smaller (Javornický potok, see Tab. 1). In each stream, one representative reach was selected that was straight, relatively flow-uniform and flat-bed. The grain size distribution was very similar in both reaches (Tab. 2). The flood peak discharge was very similar to the bank-full discharge, i.e. water did not spill out onto a floodplain. A relatively intense transport of sediments was witnessed in both streams during the flood. However, no evidence of bed forms of any significance was found in the reaches after the flood event.

Table 1. Parameters of channel and flow

	Bělá	Javornický p.
Longitudinal slope of bed [-]	0.005	0.01
Flow rate [m^3/s]	150	34
Discharge area [m^2]	59.74	15.23
Width in water surface [m]	28.4	9.3
Width in bed [m]	15.5	5.1
Depth [m]	3.0	2.5
Hydraulic radius [m]	2.6	1.3

Table 2. Grain size distribution of channel bed

Size[mm]	d_{16}	d_{25}	d_{50}	d_{84}	d_{90}	d_e
Bělá	35	45	110	230	260	129.3
Javornický p.	60	65	90	160	190	111

2.4 *Friction conditions evaluated*

For each reach, the geometry of a channel was determined (3 channel cross sections, the longitudinal slope of the bed and both banks) together with marks that water surface at and near the peak discharge left on the stream banks. The peak discharge was measured in a stream gauge station not far above the test reach at river Bělá. At Javornický potok no gauge station was available. The peak discharge was determined from the water surface marks on the bed-drop object just above the reach.

Table 3. Parameters of friction

	Bělá	Javornický p.
Froude number [-]	0.55	0.56
Bed shear velocity [m/s]	0.36	0.36
Shields number for d_{16} [-]	0.225	0.133
Shields number for d_{84} [-]	0.034	0.050
Shields number for d_{90} [-]	0.030	0.042
Manning's n [s/m$^{0.33}$]	0.053	0.054
k_s/d_{84} [-]	7.4	7.6
k_s/H [-]	0.57	0.49

An average value of the Manning's roughness coefficient *n* for the reach was determined for the measured parameters processed by the software HEC-RAS considering a steady non-uniform flow. The obtained values of *n* are in Tab. 3 together with other evaluated flow friction parameters (the bed shear velocity, the equivalent roughness of bed). Very similar values of the bed shear velocity as from HEC-RAS for a one-dimensional non-uniform flow can be obtained for the observed reaches also using the momentum equation for one-dimensional normal (steady uniform) flow provided that R_b lower than H (H = stream depth) and very similar to R (R = hydraulic radius of entire discharge area) is used in the equation. The use of R_b smaller than H is justified by the fact that the observed stream reaches are relatively narrow with regard to the flood flow depth. Hence, stream bank effects on the total flow resistance cannot be neglected.

An uncertainty in a field determination of Manning's n for gravel-bed streams at flood discharges is relatively high. This is due primarily to the uncertainties in a determination of the parameters measured *ex post* in a channel as are the maximum flow depth (or rather the maximum water surface profile in a reach) and the corresponding peak discharge during a flood event. For the measurements in the two gravel-bed reaches discussed here the total uncertainty in the determined values of n is estimated as high as approximately ± 15 per cent.

2.5 *Bed roughness evaluated*

The n values obtained from the field measurements (Tab. 3) are considerably higher than those suggested by the predictive formula for gravel-bed streams (e.g. Limerinos 1970). The selected reaches did not exhibit evidence of significant sources of form resistance (big roughness elements, big bed forms, bends, flow non-uniformities) that could be accounted for the difference between the observed roughness and the predicted roughness. The values of the relative submergence (the flow depth to particle size ratio) were high enough (usually > 10) to avoid additional form drag associated with wake effects produced by large roughness elements (as described by Limerinos). No evidence of regular bed forms was observed in the channel after the flood event. In the particular reaches no pool-step or pool-riffle sequences were present.

Hence, a major part of the total bed shear stress can be considered a product of the skin friction in the observed streams. It is likely that transport of sediments of different grain fractions in the flood discharge accounts for a considerable increase in the skin roughness of the gravel bed. A comparison of values of Shields number for different fractions of transported grains (Tab. 3) with the threshold value of Shields number at incipient motion of grains ($\theta_{b,cr} \approx 0.045$) suggests that all grains finer than say d_{84} were in motion in both observed gravel-bed streams during the flood culmination.

In general, gravel-bed streams are almost invariable bed-load dominated. However, gravel-bed streams usually transport a lot of sand-size grains as well. Inspection of the stream reaches after the flood indicated that this was indeed the case. A contribution of particular fractions of transported gravel/sand grains on flow resistance is difficult to identify, but it can be assumed that coarser fractions contribute more to total resistance than finer fractions. Coarser particles are more subjected to interparticle collisions and hence contribute through the particulate bed shear stress more to the total bed shear stress than particles suspended in the carrying liquid that remain virtually contactless and do not contribute to the particulate bed shear stress.

Table 2-1 in García et al. (2008) shows a considerable scatter in values of $\alpha = k_s/d_{84}$ (from 1.6 to 5.1) by different investigators with an average value near 3. Limerinos proposed $\alpha = 2.8$ for gravel-bed streams (Table 2-1 in García et al. 2008).

From our measurements, the values of k_s in Tab. 3 are obtained using

$$k_s = \frac{B_s R_b}{\exp\left(\dfrac{\kappa R_b^{1/6}}{n\sqrt{g}}\right)} \tag{3}$$

for $\kappa = 0.4$, $B_s = 11$, and $R_b \approx R$. The values of $\alpha = k_s/d_{84}$ are more than twice higher than suggested by Limerinos, namely 7.4 for Bělá and 7.6 for Javornický potok (Tab. 3). These values are extremely high for fixed beds and are very high even for mobile beds.

3 FLOW RESISTANCE AT HIGH BED SHEAR IN PRESSURIZED PIPE

Laboratory pipe loops manufactured to convey slurries are appropriate for testing phenomena related to friction at the top of a granular bed. This is because a broad range of flow conditions can be installed in the loop. Moreover, it is easy to control the conditions and to measure required quantities. Our tests were focused to eroded beds in the upper-plane bed regime, i.e. to flows with Shields number θ_b that exceeded say 0.6. At high shear stress, a flow erodes the top of the mobile bed and prevents a development of bed forms. As a result the bed is flat and intense transport of solid particles takes place. The transported particles influence friction conditions at the top of the bed. Our tests are described in more details e.g. in Matoušek & Krupička (2009). The eroded-bed roughness size k_s is determined from the measured quantities using Eq. (1) with B_s = 14.8 for circular-pipe flows.

Figure 1 shows results of the test with the slurry of a narrow graded fraction of coarse sand (d_{18} = 1.15 mm, d_{50} = 1.36 mm, d_{84} = 1.55 mm) in a circular pipe of the inner diameter 100 mm. At θ_b of about 0.45, α gains a value only slightly higher than 1, i.e. much lower than in the measured gravel-bed streams. However, it must be seen that the solids discharge at this relatively low (from the point of view of pressurized flows) value of Shields number at the lower limit of the upper-plane-bed regime (and i.e. the upper threshold of bed undulation) was very low in the test pipe, definitely much lower than the corresponding solids discharge in the streams during the flood.

In the pipe, the equivalent roughness tended to increase with Shields number (Figure 1). The solids discharge increased with Shields number as well. The relationship between k_s/d_{50} and θ_b was roughly linear and could be approximated by the relationship proposed for sheet flows by Wilson (e.g. 1989, 2005),

$$\frac{k_s}{d_{50}} = \frac{const}{C_{sh}\tan\varphi}\theta_b = 3.3\theta_b \quad (4)$$

where C_{sh} = average volumetric concentration of solids within a shear layer, φ = dynamic friction angle of solids.

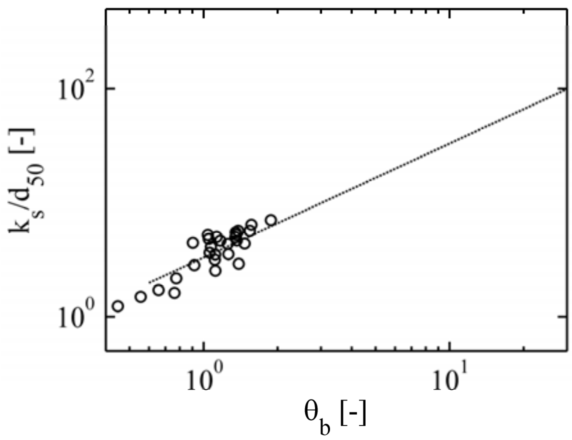

Figure 1. Roughness ratio for eroded plane bed composed of coarse sand grains in circular pipe of inner diameter of 100 mm.

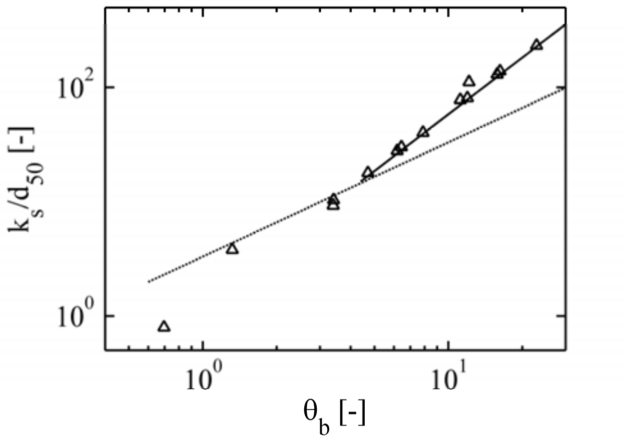

Figure 2. Roughness ratio for eroded plane bed composed of medium sand grains in circular pipe of inner diameter of 150 mm.

However, tests with slurry of a finer sand fraction (medium sand, narrow graded, d_{18} = 0.30 mm, d_{50} = 0.37 mm, d_{84} = 0.46 mm) in a 150-mm pipe indicated that the roughness ratio k_s/d_{50} tended to deviate from the linear relationship with the Shields number at high θ_b values (Figure 2).

Table 4. Experimental data from pressurized pipes

Solids	Conduit[a]	[b]No.	Reference
sand d_{50} = 0.22 mm, S = 2.65	○ 100	20	(3)
sand d_{50} = 0.38 mm, S = 2.65	○ 100	37	(3)
sand d_{50} = 1.36 mm, S = 2.66	○ 100	38	(3)
sand d_{50} = 0.3 mm, S = 2.65	○ 105	6	(9)
sand d_{50} = 0.56 mm, S = 2.65	○ 105	4	(9)
bakelite d_{50}=1.05 mm, S = 1.53	○ 105	8	(9)
sand d_{50} = 0.37 mm, [b]S = 2.65	○ 150	14	(4)
sand d_{50} = 0.354 mm, S = 2.67	□ 98x98	30	(7)
sand d_{50} = 0.55 mm, S = 2.66	□ 98x98	26	(7)
sand d_{50} = 0.7 mm, S = 2.67	□ 98x98	47	(8)
sand d_{50} = 1.1 mm, S = 2.66	□ 98x98	30	(7)
nylon d_{50} = 3.94 mm, S = 1.14	□ 98x98	12	(7)
bakelite d_{50}=0.67 mm, S = 1.57	□ 98x98	13	(7)
bakelite d_{50}=1.05 mm, S = 1.54	□ 98x98	33	(7)
sand d_{50} = 0.13 mm, S = 2.65	□ 300x100	19	(10)

[a] shape and size in millimeter.
[b] number of experimental data.

Another data from the literature and own tests for a broad range of tested narrow-graded fractions of grains (Tab. 4) confirmed the trend and suggested parameters additional to Shields number that affect the roughness ratio at very high bed shear. Taken all parameters into account a preliminary semi-empirical formula was proposed for the upper-plane-bed regime

$$\frac{k_s}{d} = 1.7\frac{W_{s*}^{1.1}}{Fr_b^{2.3}}\left(\frac{R_b}{d}\right)^{2.5}\theta_b^{1.4} \quad (5)$$

where the dimensionless settling velocity of grain $W_{s*} = \left[\frac{(S-1)}{g\nu}\right]^{1/3} w_t$, the bed Froude number $Fr_b = \frac{v_a}{\sqrt{gR_b}}$, and

S = relative density of grains, ν = kinematic viscosity of liquid, w_t = terminal settling velocity of grain, v_a = average velocity of flow in discharge area above bed.

The data in Table 4 covered bed friction conditions within a range of θ_b from 0.6 to 23. Figure 3 shows that Eq. 5 performs very well within the entire range of k_s/d_{50} and thus θ_b.

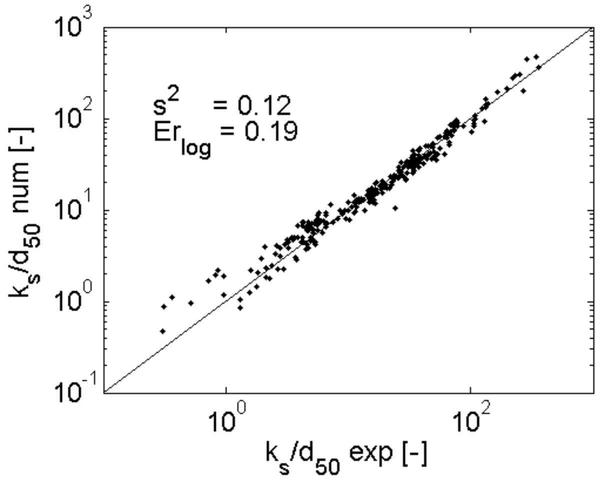

Figure 3. Comparison of k_s/d_{50} from experiments and Eq. 5 for all data in Table 4.

The statistical parameters used for a fit evaluation in Figure 3 are defined as follows:

$$s^2 = \frac{\sum_{i=1}^{N}\left(\frac{X_{predicted,i} - X_{measured,i}}{X_{measured,i}}\right)^2}{N-1} \quad (6)$$

$$Er_{log} = \frac{\sum_{i=1}^{N}\left|log\left(\frac{X_{predicted,i}}{X_{measured,i}}\right)\right|}{N} \quad (7)$$

(X = processed quantity; N = number of processed data).

4 DISCUSSION

4.1 *Relation between bed roughness and solids transport in upper plane bed regime*

The pipe-flow database contained data covering a broad range of solids flows rates. Besides the bed roughness formula, a solids transport formula was tested using the database. The data confirmed an existence of a relationship between the dimensionless solids discharge and the bed Shields number. At high bed shear, the relationship of the Meyer-Peter and Müller (MPM) type could be used provided that the coefficients of the MPM formula were determined as functions of the particle Reynolds number instead of being kept constant for different transported materials (Matoušek 2009). According to the solids transport formula, the solids discharge increases with θ_b. The ratio k_s/d_{50} increases with θ_b as well (see Eqs. 4-5) and hence the bed roughness should grow with the concentration of solids transported in flow above the bed.

At θ_b values round the lower limit of the upper plane bed regime in the database, the solids discharge was very low and so was the value of the roughness ratio for the bed, $k_s/d_{50} \approx k_s/d_{84}$ (typically round the value of 1). Values of two orders of magnitude higher could be reached for k_s/d_{50} during pressurized-pipe tests and the increase in k_s/d_{50} was associated with a significant increase in solids flow rate in a pipe. Figure 4 shows a relationship between k_s/d_{50} and the delivered volumetric concentration, C_{vd}, of grains in the flow of mixture above the bed measured during the medium-sand test in the 150-mm pipe.

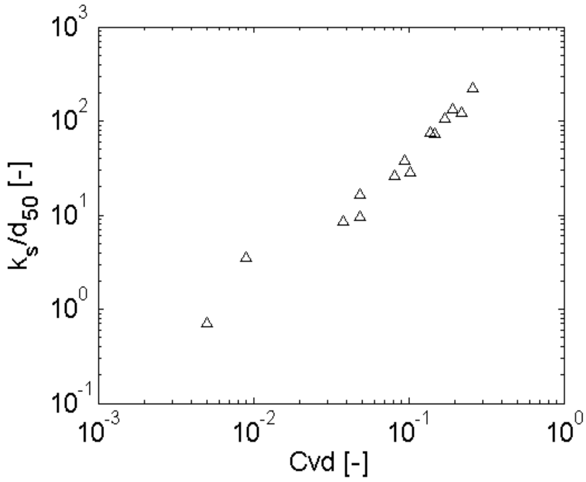

Figure 4. Relation between k_s/d_{50} and delivered concentration of solids in medium-sand flow in circular pipe of inner diameter of 150 mm.

4.2 *Interpretation of laboratory results to gravel-bed streams*

In gravel-bed streams the relation between the bed roughness and the solids discharge may be more complex than in laboratory pipes but the basic trend should hold. In the literature, values higher than 1 (typically between 1.6 and 5.1) are suggested for $\alpha = k_s/d_{84}$ in gravel-bed streams at bed shear conditions typical for usual flow discharges. Even higher values of α can occur for flood flow discharges as our field tests suggest. At least a certain portion of the increased roughness of the bed should be attributed to the interaction of the transported grains with the top of the stream bed. Perhaps, the upper plane bed regime was not reached in the two observed streams (θ_b was too low) and the roughness was predominantly affected by solids transport through bed forms (although no signs of significant bed forms were observed in the reaches after the flood event).

The upper plane bed is more likely to occur in mountain streams with longitudinal slopes larger than say 0.02. For instance, a flood event on the north-Bohemian mountain torrent Dubská Bystřice in August 2002 produced the following conditions. The observed reach had the longitudinal slope of 0.05. A typical value of the water depth at the peak discharge was 1.4 meter and the hydraulic radius $R_b \approx 0.78$ meter. Hence, the maximum bed shear stress reached the value of about 400 Pa. The grain size distribution in the bed was $d_{16} \approx 1{,}3$ cm, $d_{50} \approx 5$ cm, $d_{84} \approx 18$ cm a $d_{90} \approx 22$ cm. Hence, $\theta_b \approx 0.11$ for d_{90}, $\theta_b \approx 0.47$ for d_{50}, and $\theta_b > 1.8$ for grains smaller than d_{16}. During the flood condition the critical value of θ_b for initial motion ($\theta_{b,cr} \approx 0.05$ according to the Shields diagram) was exceeded for a great majority of bed grains. The bed grains of sizes up to 0.5 m were in motion at the peak discharge. Grains smaller than say d_{16} were transported as suspended load, the coarser grains were transported as bed load. Presumably, intense transport of sediments considerably affected bed resistance and hence the relation between the flow discharge and the water stage. No gauge station was available anywhere nearby the reach and the flow rate was unknown. Therefore the bed roughness could not be determined from the field post-flood measurements. However, a rough estimation of a flow rate suggested that Manning's n should reach a value of about 0.06 during the event.

The friction conditions could be considered the upper plane bed regime conditions and hence the roughness formula derived from the laboratory tests may be applicable. However, a direct application is conflicted by e.g. a broad grain size distribution of the gravel bed. The broad distribution gives rise to the question which characteristic sizes should be used in particular dimensionless groups in Eq. 5. Furthermore, a broad grain size distribution affects the solids transport rate at a certain applied bed shear stress. Perhaps, the solids rate can be higher for broadly graded bed sediment than for a narrow graded bed fraction of the same d_{50} at the particular bed shear stress (400 Pa in the Dubská Bystřice reach)..

It would be interesting to simulate a wide grain distribution in laboratory pipe tests in order to evaluate the effect of the grain size distribution on the solids discharge and the bed roughness. A preparation of much laboratory tests is a work currently in progress.

4.3 *Application of bed-roughness formula*

Pressurized pipe: A formula for the mobile-bed roughness is an important part of predictive models for stratified flows through slurry pipes with both a sliding bed and a stationary bed (e.g. Matoušek and Krupička 2010).

Open channel: The roughness of a mobile bed considerably affects a relationship between the flow rate and the water stage in open channels. This relationship is of major practical importance for an estimation of water stages under flood conditions giving e.g. a prediction of the maximum water stage for a certain (flood) discharge in a channel. In practice, the relationship is also used for an estimation of the flood discharge from flood marks (the marks assigning the maximum water level) in a channel after a flood event.

5 CONCLUSIONS

Field measurements in two gravel-bed streams showed that during a flood event the bed roughness was much bigger than predicted using formulae in literature. It is suggested that the high value of the bed roughness at high bed shear is due primarily to transport of sediment at the flood discharge.

A significant effect of solids transport on the bed roughness was observed in slurry flows in upper plane bed regime in enclosed pressurized pipes. Tentative formulae were proposed for the Nikuradse's equivalent roughness of a bed composed of a narrow graded grain fraction and eroded by a flow of solids-liquid mixture.

Additional pipe tests are required to incorporate conditions typical for gravel-bed streams, i.e. a wide size distribution of grains in a bed and in transported load.

ACKNOWLEDGEMENT

The research was supported by the CIDEAS VS project 1M0579 of the Ministry of Education, Youth and Sports of the Czech Republic.

REFERENCES

García, M.H. et al. 2008. Sedimentation engineering: processes, management, modeling, and practice. Edited by M.H. García; American Society of Civil Engineers.

Limerinos, J.T. 1970. Determination of the Manning coefficient from measured bed roughness in natural channels. USGS Water-Supply paper 1898-B.

Krupička, J., Matoušek, V. 2010. Semi-empirical formulae for upper plane bed friction. Proc. Hydrotransport 18, BHRG, accepted.

Matoušek, V. 2009. Concentration profiles and solids transport above stationary deposit in enclosed conduit. J. Hydraul. Eng., ASCE, 135(12), 1101-1106.

Matoušek, V., Krupička, J. 2009. On equivalent roughness of mobile bed at high shear stress. Journal of Hydrology and Hydromechanics, 57(3), 191-199.

Matoušek, V., Krupička, J. 2010. Modeling of settling-slurry flow around deposition-limit velocity. Proc. Hydrotransport 18, BHRG, accepted.

Nnadi, F.N., 1992. Bed-load transport at high shear stress: with application to friction in rivers and sand waves. PhD Thesis, Queen's University at Kingston.

Nnadi, F.N., Wilson, K.C. 1992. Motion of contact-load particles at high shear stress. J. Hydraul. Eng., ASCE, 118(12), 1670-1684.

Pugh, F.J., 1995. Bed-load velocity and concentration profiles in high shear stress flows. PhD Thesis, Queen's University at Kingston.

Sumer, B.M., Kozakiewitz, A., Fredsøe, J., Deigaard, R., 1996. Velocity and concentration profiles in sheet-flow layer of movable bed. J. Hydraul. Eng., ASCE, 122(10), 549-558.

Wilson, K.C. 1989. Mobile-bed friction at high shear stress. J. Hydraul. Eng., ASCE, 115(6), 825-830.

Wilson, K.C. 2005. Rapid increase in suspended load at high bed shear. J. Hydraul. Eng., 131(1), 46-51.