

A General Analytical Model for Lateral Velocity Distributions in Vegetated Channels

X. Tang

School of Civil Engineering, The University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, 610065

M. Sterling & D.W. Knight

School of Civil Engineering, The University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

ABSTRACT: This paper presents a method for predicting the distribution of depth-averaged velocity in channels with either emergent or submerged vegetation. A general analytical solution to the depth-integrated Reynolds-Averaged Navier-Stokes (RANS) equation is given, where the drag force due to vegetation is modelled as an additional momentum ‘sink’ term. The method includes the effects of bed friction, drag force, lateral turbulence and secondary flows, via four coefficients f , C_D , λ & Γ respectively. The analytical solution gives good predictions of lateral velocity distribution when compared with two sets of experimental data of vegetated channels with submerged vegetation and two sets of data with emergent vegetation. The lateral distribution of bed shear stress can also be obtained through the Darcy-Weisbach friction factor. The predicted velocity and bed shear stress distributions can provide a basis for modelling flood conveyance and sediment transport in channels with vegetation.

Keywords: Vegetation, Open channel, Bed shear, Velocity

1 INTRODUCTION

Many natural rivers have significant vegetation, which introduces additional hydraulic resistance to the flow and reduces the local flow velocity. A strong shear layer often exists between the fast flowing water in the central region of a channel and the slower flowing water in the vegetated boundaries. In these circumstances, the exchange of mass and momentum between vegetated and non-vegetated regions will affect the channel conveyance as well as the velocity and boundary shear stress distributions. A method for predicting these is therefore clearly required when designing flood alleviation schemes, as well as for studies on bank protection and sediment transport.

There have been several studies on the flow in composite channels with vegetation, based either on a simplified one-dimensional (1-D) approach with an empirical Darcy-Weisbach friction factor, or on a simple eddy viscosity model for the turbulence (e.g. Darby 1999; Lopez & Garcia, 2001; Helmio, 2004). These one-dimensional models do not describe lateral distribution of velocity. More recently White & Nepf (2008) proposed a vortex-based model for predicting the lateral distribution of velocity and shear stress in a partially

vegetated channel. This model showed good agreement with the experimental data, despite the fact that the influence of secondary flow being ignored.

The present paper proposes a new general analytical solution for predicting the lateral distributions of velocity and bed shear stress for both submerged and non-submerged vegetation. The modelling is based on a depth-averaged form of the streamwise Reynolds Averaged Navier-Stokes (RANS) equation with an additional momentum term to deal with the drag force arising from the presence of the vegetation. The method relies on four hydraulic parameters related to the bed friction factor (f), lateral eddy viscosity (λ) and depth-averaged secondary flow (Γ), and a special parameter for the vegetation, namely a spatially averaged drag coefficient (C_D).

The predicted results, based on this analytical solution, are shown to agree well with the experimental depth-averaged velocity data from vegetated channels. The experimental data for flow with emergent vegetation are taken from White & Nepf (2008) and Pasche (1984), and that for submerged vegetation from Shimizu & Tsujimoto (1993) and Yan (2008). The particular case of

flow through emergent vegetation is also discussed elsewhere by Tang *et al.* (2010).

2 FLOW IN A VEGETATED CHANNEL

2.1 Governing equation of depth-averaged flow

For steady flow in a prismatic open channel, the equation for the streamwise momentum may be combined with the continuity equation to give:

$$\rho \left[\frac{\partial}{\partial y}(UV) + \frac{\partial}{\partial z}(UW) \right] = \rho g S_o + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \quad (1)$$

where $\{UVW\}$ = velocity components in the $\{xyz\}$ directions, x -streamwise parallel to the channel bed, y -lateral and z -normal to the bed, ρ = fluid density, g = gravitational acceleration, S_o = channel bed slope, and $\{\tau_{yx}, \tau_{zx}\}$ = Reynolds stresses on planes perpendicular to the y and z directions respectively, as illustrated in Figure 1. For flow in a vegetated channel, the drag force due to vegetation (plants) may be introduced into Eq. (1) (time-averaged RANS) by an additional momentum sink term to give (Tang *et al.* 2010):

$$\rho \left[\frac{\partial}{\partial y}(UV) + \frac{\partial}{\partial z}(UW) \right] = \rho g S_o + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} - F_v \quad (2)$$

where F_v is the drag force per unit fluid volume due to the vegetation, and represented by:

$$F_v = \frac{1}{2} \rho (C_D \beta A_v) U^2 \quad (3)$$

where C_D is the drag coefficient, β is a shape factor of vegetation, and A_v is the projected area of the vegetation in the streamwise direction per unit volume. It should be noted that Eq. (2) is a time averaged momentum equation for the streamwise flow in the x -direction. F_v corresponds to a drag force component in the x -direction, and as such is linked only to the primary flow (U). It is appropriated that an ‘‘additional dispersive’’ term due to correlation of spatial deviations of the mean velocity components (Finnigan, 2000) is negligible, hence not considered due to uniformity in the even distributed vegetated region.

By integrating Eqs (2) and (3) over the water depth, H , (provided $W = 0$ when $z = 0$ and H , as assumed by Shiono & Knight, 1991), the depth-averaged momentum equation becomes:

$$\begin{aligned} \frac{\partial [H(\rho UV)_d]}{\partial y} &= \rho g H S_o \\ &+ \frac{\partial H \bar{\tau}_{yx}}{\partial y} - \tau_b - \frac{1}{2} \rho (C_D \beta A_v) h U^2 \end{aligned} \quad (4)$$

where the overbar or the subscript d refers to a depth-averaged value, τ_b is the bed shear stress, and U_v and U_d are the depth-averaged velocity over the vegetated height (h) and the total flow depth (H), defined by $U_v = \frac{1}{h} \int_0^h U dz$ and

$$U_d = \frac{1}{H} \int_0^H U dz \text{ respectively.}$$

For the non-submerged vegetation, $U_v = U_d$ due to $h = H$; otherwise $U_v < U_d$ for submerged vegetation, as illustrated in Figure 2.

By taking into account the porosity, $\delta (= 1 - \phi)$, for the blockage effects of vegetation on the flow, where ϕ is the volumetric vegetation density, defined as the ratio of the volume of vegetation to the flow, it follows that $\phi = h^* (\pi/4) D^2 n_v$, where D represents a characteristic diameter of the vegetation, n_v is the number of plants per unit bottom area, and h^* represents the ratio between the height of vegetation (h) and the flow depth (H), defined by $\min [H, h]/H$. Therefore $h^* < 1$ if the vegetation is submerged whereas $h^* = 1$ for emergent vegetation. The projected area of the vegetation in the streamwise direction, $A_v = 4\phi/(\pi D)$. It is worth noting that for submerged vegetation, U_v in Eq. (4) is replaced by U_d , where their relationship can be determined by Stone & Shen’s (2002) equation as follows:

$$U_v^2 = k_v U_d^2 \left(\frac{h}{H} \right) \quad (5)$$

where k_v is a coefficient defined by $[(1 - D n_v^{1/2}) / (1 - D h^* n_v^{1/2})]^2$, which is about 1.0 for most practical cases.

Eq. (4) may be rewritten in the form of effective water volume as:

$$\begin{aligned} \rho \delta \frac{\partial H (UV)_d}{\partial y} &= \rho \delta g H S_o + \delta \frac{\partial H \bar{\tau}_{yx}}{\partial y} \\ &- \delta \tau_b - \frac{1}{2} \rho (C_D \beta A_v) (k_v h^*) H U_d^2 \end{aligned} \quad (6)$$

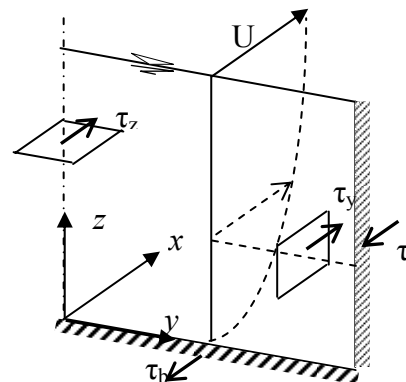


Figure 1. Bed and wall shear

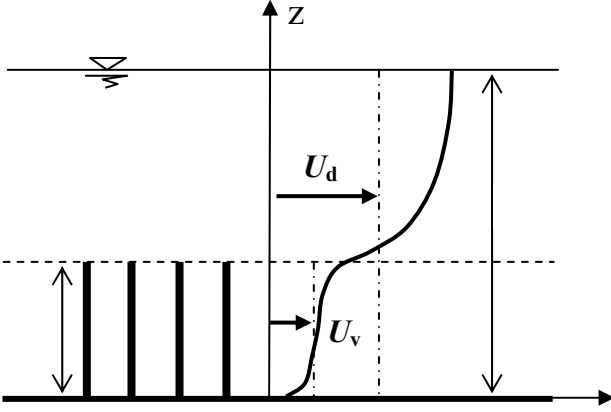


Figure 2. Sketch of open channel flow with submerged vegetation

Therefore Eq. (6) is a general depth-averaged momentum equation in the streamwise direction for shallow-water flow with vegetation. It should be noted that $h^* = 1$ and $k_v = 1$ for non-submerged vegetation. For the non-vegetated channel, i.e. $A_v = 0$ and $\delta = 1$, Eq. (6) then takes the same form as that given by Shiono & Knight (1991). By making the following assumptions:

$$\tau_b = \left(\frac{f}{8}\right)\rho U_d^2; \quad \bar{\tau}_{yx} = \rho \bar{\varepsilon}_{yx} \frac{\partial U_d}{\partial y};$$

$$\bar{\varepsilon}_{yx} = \lambda U_* H; \quad \Gamma = \frac{\partial}{\partial y} [H(\rho UV)_d] \quad (7)$$

By using the notation $A_v' = A_v k_v h^{*3}$, Eq. (6) becomes

$$\rho g H S_0 - \rho \frac{f}{8} U_d^2 - \frac{1}{2\delta} \rho (C_D \beta A_v') H U_d^2$$

$$+ \frac{\partial}{\partial y} \left\{ \rho \lambda H^2 \left(\frac{f}{8}\right)^{1/2} U_d \frac{\partial U_d}{\partial y} \right\} = \Gamma \quad (8)$$

These parameters are discussed more fully by Shiono & Knight (1990, 1991), Knight & Shiono (1996), Tominaga & Knight (2004), Abril & Knight (2004), and Knight *et al.* (2010) for non-vegetated channels and by Rameshwaran & Shiono (2007), and Tang *et al.* (2010) for vegetated channels.

2.2 Analytical solution of depth-averaged flow

For a vegetated channel, where the drag coefficient C_D , density of vegetation (ϕ), local friction factor (f), eddy viscosity (λ) and secondary flow term (Γ) are given, an analytical solution to Eq. (8) for U_d can be obtained as follows:

$$U_d = [A_1 e^{\gamma y} + A_2 e^{-\gamma y} + k]^{1/2} \quad (9)$$

$$k = \frac{g S_0 H - \Gamma / \rho}{f / 8 + H / (2\delta) C_D \beta A_v'} \quad (10a)$$

$$\gamma = \sqrt{\frac{2}{\lambda} \left(\frac{8}{f}\right)^{1/4}} \frac{1}{H} \sqrt{\frac{f}{8} + \left(\frac{H}{2\delta}\right) C_D \beta A_v'} \quad (10b)$$

For the non-vegetated channel, i.e. $A_v = 0$ and $\delta = 1$, Eqs (10a&b) become:

$$k = \frac{8(g S_0 H - \Gamma / \rho)}{f}; \quad \gamma = \sqrt{\frac{2}{\lambda} \left(\frac{f}{8}\right)^{1/4}} \frac{1}{H} \quad (11)$$

It can be seen that Eq. (11) is the same as that given by Shiono & Knight (1991).

The unknown constants, A_1 to A_2 in Eq. (9), can be obtained through applying appropriate boundary conditions to the cross-section shown in Figure 3. For each panel the two unknown constants, A_1 to A_2 can be eliminated using the following boundary conditions (Knight *et al.* 2004, Tang & Knight, 2008):

- The no-slip condition, i.e. $U_d = 0$ at remote boundaries or $U_d =$ given values (U_1 or U_2) as shown in Figure 4;
- The continuity of the velocity U_d at each domain junction, i.e. $U_d^{(i)} = U_d^{(i+1)}$;
- The continuity of unit force ($H \bar{\tau}_{yx}$) at each domain junction, i.e. $[H \bar{\tau}_{yx}]^{(i)} = [H \bar{\tau}_{yx}]^{(i+1)}$

It follows for a continuous depth domain that the continuity of unit force implies

$$\left(\mu \frac{\partial U_d}{\partial y}\right)^{(i)} = \left(\mu \frac{\partial U_d}{\partial y}\right)^{(i+1)} \quad \text{with } \mu = \lambda \sqrt{f} \quad (12)$$

where the superscript (i) indicates the number of an individual panel. Knight *et al.* (2004) point out that the continuity of $\partial U_d / \partial y$, previously used by most researchers [Shiono & Knight, 1990; Ervine *et al.* 2000], is only appropriate for certain cases, notably where f and λ are the same in the two adjoining domains.

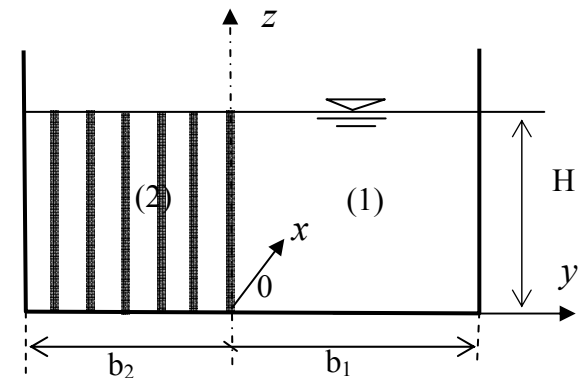


Figure 3. Cross-section of a vegetated channel with notation

For a rectangular channel, the lateral depth-mean velocity for half the channel becomes:

$$U_d = \sqrt{\frac{-k}{\cosh(\gamma b)} \cosh(\gamma y) + k} \quad (13)$$

where b is the half channel width, and k with γ are given by Eqs (10a) and (10b) respectively.

Therefore Eq. (9) gives the lateral distribution of depth-mean velocity and boundary shear stress [via Eq. (7)] in a vegetated channel with either emergent or submerged vegetation. It should be noted that the solutions (9) - (10) for U_d are also suitable for non-vegetated channels, in which case $A_v = 0$ and $\phi = 0$.

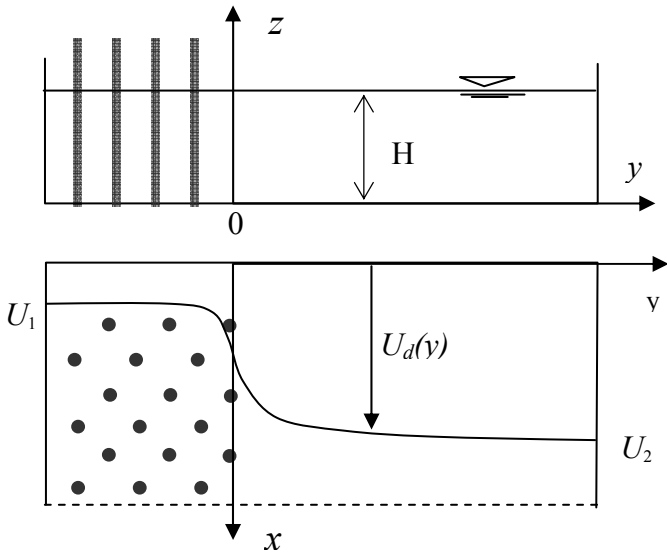


Figure 4. Sketch of partially vegetated laboratory channel by White & Nepf (2008)

3 APPLICATION OF ANALYTICAL SOLUTION TO FLOW IN A CHANNEL WITH SUBMERGED VEGETATION

3.1 Introduction to the experiments

Two sets of experiments in rectangular channels with submerged vegetation were used in this study. One was with the channel partially vegetated, and the other was with vegetation placed across the whole width of the channel.

In the first, Shimizu and Tsujimoto (1993) conducted an experiment in a 0.4 m wide, 12.5 m long tilting flume with a bed slope of 1/2000. Detailed turbulence measurements were carried out with a two-colour fibre-optic Laser Doppler Anemometer (LDA) in a flow with a depth of 0.128 m and half the channel filled with a 20 x 20

mm array of 1.8 mm diameter circular cylinders. The height of these wooden rods was 0.060 m, thus representing a model for submerged vegetation, as illustrated in Figure 4. The corresponding volume density of model plants (ϕ) was about 0.003. As the channel was configured by Vinyl Chloride plates, it was assumed to be a hydraulically smooth channel.

The second set of experiments were undertaken by Yan (2008) in a 0.42 m wide, 12 m long tilting flume. In the series F1, where the bed slope of channel was set to be 0.72%, detailed turbulence measurements were carried out with a LDA for a flow with a depth of 0.120 m and with the whole channel filled with an array of 6 mm diameter circular rods, in a 20 x 50 mm pattern. The height of the aluminum rods was 0.060m, thus also representing a model for submerged vegetation, as illustrated in Figure 2. The volume density of model plants (ϕ) in this case was about 0.0142. The Manning coefficient (n) of the channel was reported to be about 0.010.

3.2 Modelling results of lateral velocity distribution

In order to apply Eq. (9) with (10) to predict the depth-averaged velocity, the drag coefficient (C_D), local friction factor (f), eddy viscosity (λ) and secondary flow term (Γ) are required, as well as the shape factor β . Each of these parameters is now addressed in detail below.

The friction factor (f) is obtained using the standard Colebrook-White equation, given by:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{3.02\nu}{Re\sqrt{f}} + \frac{k_s}{12.3H} \right) \quad (14)$$

where Re is the local Reynolds number, defined by $4U_dH/\nu$, and the equivalent sand roughness height, k_s , was assumed to be 0.16 mm, and was obtained through the relationship $n = k_s^{1/6}/(8.25 g^{1/2})$, with n corresponding to a hydraulically smooth channel surface (i.e. $n = 0.009$) (Ackers, 1993). Thus the local friction factors (f) were estimated to be 0.0173 and 0.0166 for the above two experimental cases respectively, based on (14).

The shape factor β was set equal to 1.0, as cylindrical rods were used in the experiment to simulate the vegetation. Tanino & Nepf (2008) showed that the drag coefficient (C_D) decreases as the Reynolds number based on the rod diameter, ($Re_D = U_d D/\nu$) increases. However, values are typically in the range 1.0-1.05 for $\phi < 0.09$ for Re_D up to $O(10^3)$. Therefore it is assumed that C_D can be taken as 1.0, which it is considered appropriate for the studied case ($\phi = 0.003$), whereas for the

high density case ($\phi = 0.0142$), C_D was assumed to be 1.10.

In order to calculate the secondary flow term, Γ , recourse was made to the work of Shiono & Knight (1991), who, based on the compound channel data of the UK-FCF flume, suggested $\Gamma / (\rho g H S_o) = 0.15$ for the non-vegetated main channel. It is worth noting that the relationship is valid for a compound channel, but it is appropriate to be used for the channel with partially vegetated channel, which has a similar role as a floodplain, as demonstrated by Shimizu & Tsujimoto (1993). However, the work of Ghisalberti & Nepf (2004) and Yang *et al.* (2007) have demonstrated that not only is the flow around vegetation elements complex but there is evidence of weak secondary current cells within a vegetated floodplain. The effect of the secondary flow on the vegetated floodplain was ignored, i.e. $\Gamma = 0$. This can also be supported by the velocity contour plot shown by Shimizu & Tsujimoto (1993).

The final parameter value to be addressed relates to the eddy viscosity, λ . The value of λ was taken as 0.07, which is close to the standard value 0.067 ($= \kappa/6$, where κ is the von Karman constant). However, previous work has illustrated that the value of λ can change significantly.

The results of the simulation for above two cases are shown in Figures 5 and 6, where Figure 6 shows only the simulation for half the channel due to the symmetry of the channel. Figure 5 shows that a strong shear layer exists between the slow flowing vegetated region and the fast flowing non-vegetated region. The simulated results of Figures 5 and 6 agree well with the experimental data. Figures 5 and 6 illustrate the ability of the analytical solution to provide reasonably good results for submerged cases.

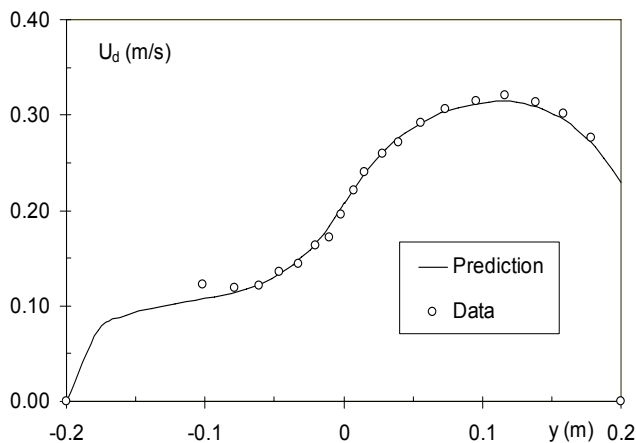


Figure 5. Comparison of predicted U_d distributions with experimental data (Shimizu & Tsujimoto, 1993)

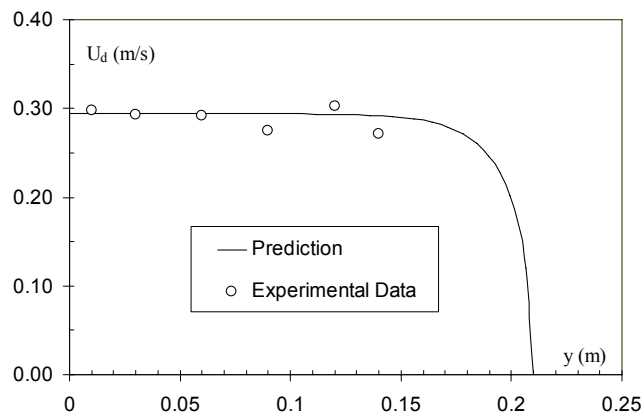


Figure 6. Comparison of predicted U_d distributions with experimental data (Yan, 2008)

4 APPLICATION OF ANALYTICAL SOLUTION TO FLOW IN A CHANNEL WITH PARTIALLY EMERGENT VEGETATION

White & Nepf (2008) carried out detailed two-dimensional velocity measurements with a Laser Doppler Velocimetry (LDV) in a 1.2 m wide, 13 m long flume, partially filled with a 0.4 m wide array of 6.5 mm diameter wooden circular cylinders. The wooden rods pierced the water surface, thus representing a model for emergent vegetation, as illustrated in Figure 4. The flow depth varied from 55 mm to 150 mm, ensuring large aspect ratios (> 8) to minimize the impact of the side walls on the flow. Three volume densities of model plants ($\phi = 0.02, 0.045$ and 0.10) were used in their experiments. In this study, only one case X ($\phi = 0.10$) was used here, with the corresponding flow parameters: flow depth, $H = 0.78$ m, $S_o = 0.0715\%$, $f = 0.05$ and $\alpha C_D = 1.77$. Further examples of simulations for other experimental runs may be found in Tang *et al.* (2010).

The results of simulating emergent vegetation for the case X are shown in Figure 7, where the modelling parameters assumed in the analytical solution are outlined above in section 4.2. The agreement with the experimental data of White and Nepf (2008) is again seen to be quite good.

The second set of experimental data used to evaluate the analytical solution pertains to that of Pasche (1984) and Pasche & Rousve (1985). These data relate to laboratory experiments which were conducted in an asymmetric compound channel with a vegetated floodplain. The emergent vegetation was modelled using 12 mm diameter cylindrical wooden rods. The channel cross section had a bank full height (h) of 0.124 m, with the remaining dimensions shown in Figure 8. Manning's coefficient n was reported to be 0.010 for the smooth part of the channel for all

experiments. In what follows, data from two experiments (Case I: flow depth (H) = 0.2015 m, $S_o = 0.05\%$ and Case II: $H = 0.224$ m, $S_o = 0.1\%$) will be used. The corresponding volume densities (ϕ) were set at 0.63% and 2.54% respectively. In keeping with the above analysis, the shape factor β was set equal to 1.0, as cylindrical rods were again used in the experiment to simulate the vegetation. It is worth noting that the local friction factors for the main channel and floodplain were obtained as values of 0.015 and 0.031 respectively (Tang *et al.* 2010).

For flow over a linearly sloping bed without vegetation in the main channel, U_d is given by (Shiono and Knight, 1991)

$$U_d = [A_3 \xi^\alpha + A_4 \xi^{-(\alpha+1)} + \omega \xi + \eta]^{1/2} \quad (15)$$

where the constants α , ω and η are given by

$$\alpha = -\frac{1}{2} + \frac{1}{2} \left\{ 1 + \frac{s(1+s^2)^{1/2}}{\lambda} (8f)^{1/2} \right\} \quad (16)$$

$$\omega = \frac{gS_o}{\frac{(1+s^2)^{1/2}}{s} \left(\frac{f}{8} \right) - \frac{\lambda}{s^2} \left(\frac{f}{8} \right)^{1/2}} \quad (17)$$

$$\eta = \frac{-\Gamma}{\rho \left(\frac{f}{8} \right) \left(1 + \frac{1}{s^2} \right)^{1/2}} \quad (18)$$

where ξ is the local depth given by $H - (y - b)/s$ (for $y > 0$) and $H + (y+b)/s$ (for $y < 0$) as shown in Figure 8. Similar to the flat bed case, A_3 and A_4 are unknown constants for each panel, but are obtained by applying appropriate boundary conditions, as outline in section 2.2. Also see Knight *et al.* (2004 & 2007) for further details.

The eddy viscosity (λ) was based on the assumptions outlined below. In the main channel, λ was taken as 0.07. However, previous work has illustrated that on the floodplain the value of λ can change significantly. Hence, recourse was made to the work of Abril and Knight (2004), who stated that

$$\lambda_{fp} = (-0.2 + 1.20 \text{Dr}^{-1.44}) \lambda_{mc} \quad (19)$$

in which the subscripts mc and fp refers to the main channel and the floodplain respectively.

The results of the analysis are shown in Figure 9, which again illustrates the ability of the analytical solution to provide reasonably good results over a wide range of vegetation densities (the RMSSE varied between 0.0230 and 0.0270). Simulations for other experimental runs can be also found in Tang *et al.* (2010).

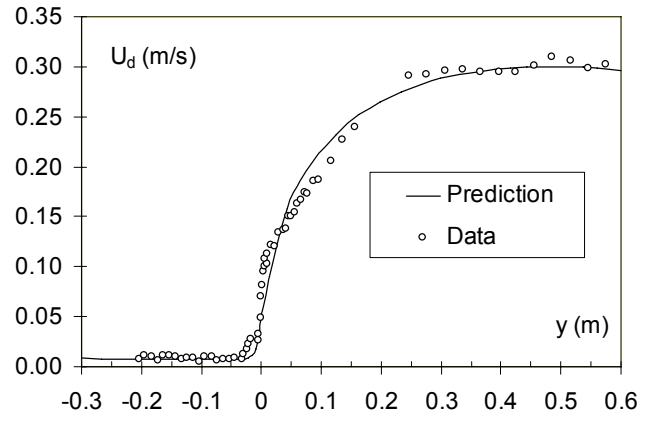


Figure 7. Comparison of predicted U_d distributions with experimental data (White & Nepf, 2008)

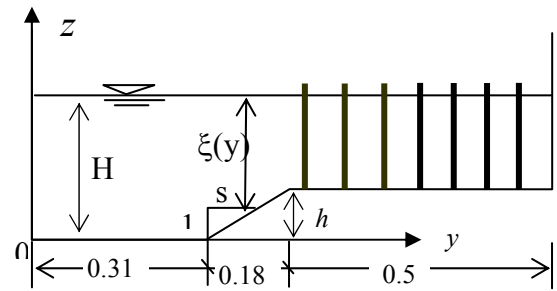


Figure 8. Cross-section of vegetated compound channel by Pasche (1984): Unit: m

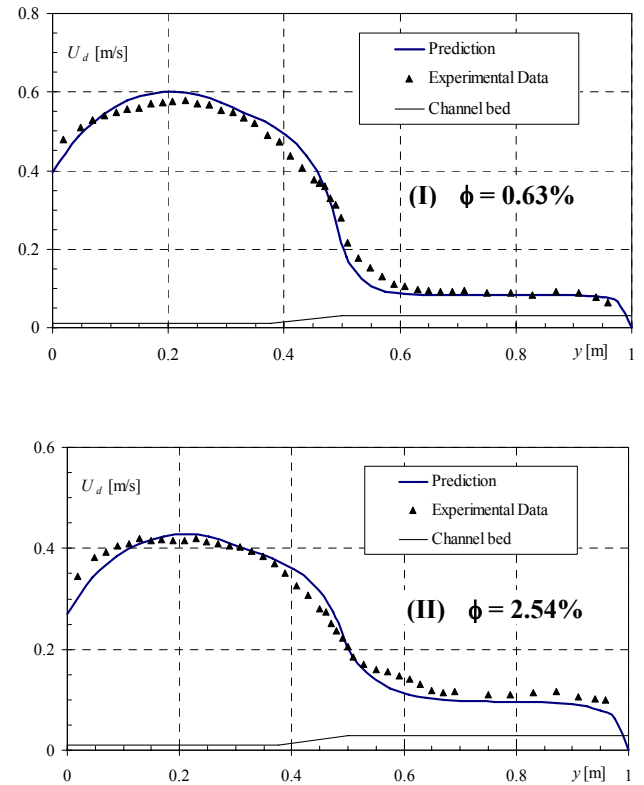


Figure 9. Comparison of modelled U_d distributions with Pasche's experimental data

5 CONCLUSIONS

The following conclusions may be drawn from this study:

- A general analytical solution (9) for depth-averaged velocity in a vegetated channel has been obtained. It is based on the depth-integrated form of the Reynolds-Averaged Navier-Stokes equation, (8), with the drag force due to vegetation being modelled as an additional momentum term.
- The analytical solution (9) with (10) simulates the lateral depth-averaged velocity distribution in vegetated channels (either emergent or submerged vegetation). It may also be used for non-vegetated channels, in which case $A_v = 0$ and $\delta = 1$.
- The predicted velocity distributions agree well with the experimental data for flow in a vegetated channel for both emergent and submerged vegetation.
- The proposed analytical solutions (9) can be used to predict boundary shear distributions through the Darcy-Weisbach friction factor.

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