River Flow 2010 - Dittrich, Koll, Aberle & Geisenhainer (eds) - © 2010 Bundesanstalt für Wasserbau ISBN 978-3-939230-00-7

Two Dimensional Modeling of Dam-Break Flows

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ABSTRACT: A two-dimensional flow model based on shallow water equations is developed. The spatial discretisation is obtained by the FVM cell centered type method. The numerical system is solved in explicit way. The flux modeling has been deployed by TVD WAF scheme with a second order accuracy both in space and time. The local Riemann problem is solved by the HLLC method. Verification of the model was carried out by comparison of model results and analytical and numerical solutions. Comparison of these two set of results present a reasonable degree of similarity. Finally, the results of numerical model were compared with available experimental data of dam- break test case.

Keywords: Numerical models, Dam failure, Shallow water equations, TVD WAF method

1 INTRODUCTION

In recent years free-surface flow models have been increasingly developed using explicit schemes. Because the result of numerical models behave better when used to simulate flows with sharp gradient free surfaces, such as dam-break flows (Namin et al. 2004). Several techniques have been published in the literature concerning the use of the finite volume method to solve the shallow water equations to model free surface flows. Shock capturing techniques in the framework of finite volume discretisation, especially Godunov type methods, have recently drawn more attentions. At least five approximate Riemann solvers i.e. Roe, FVS, Osher, HLL and HLLC can be found in the literature, all of which are based on the characteristics theory (Toro, 2001). Zoppou and Roberts (2003) examined explicit conservative schemes for the solution of the one dimensional homogeneous shallow water equations. In the conclusion, for the ease of implementation, efficiency and robustness, the HLLC and Osher schemes have been recommended for first-order schemes. Also in comprehensive study of Erduran et al. (2002), the performance of the approximate Riemann solvers has been evaluated according to five criteria including ease of implementation, accuracy, applicability, simulation time and stability. Finally, HLL and HLLC method were determined as the high-ranked schemes in terms of ease of implementation. It was highlighted that a first-order accurate solution algorithm using either Osher or HLLC schemes can be recommended for the simulation of all kinds of applications.

There have been a number of studies, aimed at developing numerical models to predict dam-break flows. Fraccarollo and Toro (1995) utilized a numerical model using a shock-capturing method of Godunov type. They used HLL method for modeling fluxes at the cell interface and TVD WAF method for achieving the second order accuracy. They compared the numerical model with their experiments of dam-break flow. Soares-Frazão and Zech (2002) used numerical simulations of the flow with three Roe-type finite volume schemes, including 1D, 2D and the hybrid approaches. They compared the results of numerical models with their experimental data for the dam break waves in a channel with a 90° bend. Valiani et al. (2002) simulated the flood wave using a Godunov-type method. In the model, the local Riemann problem is solved by the HLL method. They used the TVD MUSCL technique in order to achieve second order accuracy in space and also time. They utilized the model for simulating Malpasset dam-break. Yu-chuan and Dong (2007) simulated dam-break flows in curved boundaries by the finite-difference method, in a channel-fitted orthogonal curvilinear coordinate system.

In this paper, the HLLC approximate Riemann solver is selected for computing the fluxes in the interface of the cells. In order to achieve second-order accuracy, the Weighted Average Flux (WAF) method which was introduced by Toro (1989) is used. The following part consists of how to develop a two dimensional finite volume model. Then the model is verified by dam-break with a left dry bed and partial dam-break through a sluice gate. Then the model is applied for the dam-break in channel with 90° bend (CADAM test).

2 NUMERICAL MODEL

2.1 *Governing equations*

Under the assumption of hydrostatic pressure, and neglecting the diffusion terms and Coriolis and wind effects, the obtained two-dimensional model of depth averaged shallow water equations appears as:

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial H(U)}{\partial y}=S(U) \tag{1}$$

$$U=(h,\, hu,\, hv)^T$$

$$F(U)=(hu, hu^2+gh^2/2,\, huv)^T$$

$$H(U)=(hv, huv, hv^2+gh^2/2)^T$$

$$S(U)=(0, gh(S_{ox}-S_{fx}), gh(S_{oy}-S_{fy}))^T$$

where h = water depth, u, v = depth-averaged velocity components in the x and y directions, g = acceleration due to gravity, S_{ox}, S_{oy}= bed slopes in the x and y directions, S_{fx} , S_{fy} = friction terms in the x and y directions, respectively (Valiani et al., 2002).

$$S_{oy}=-\frac{\partial z}{\partial y}, S_{ox}=-\frac{\partial z}{\partial x} \tag{2}$$

$$S_{fy}=-\frac{n^2 v\sqrt{u^2+v^2}}{h^{4/3}}, S_{fx}=-\frac{n^2 u\sqrt{u^2+v^2}}{h^{4/3}} \tag{3}$$

where n = Manning's roughness coefficient.

2.2 *Discretisation*

The computational domain is divided into a set of quadrilateral finite volumes. Then the governing equations are integrated over each control volume, which yields by application of the divergence theorem:

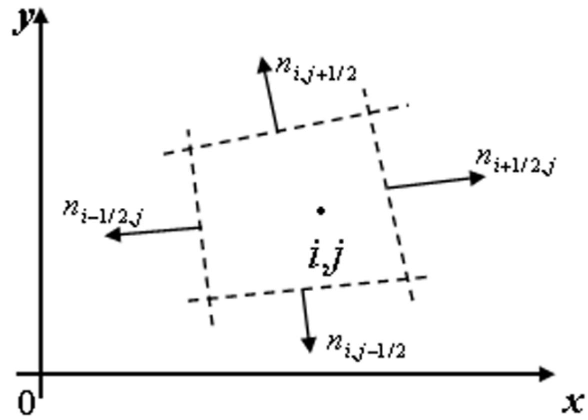


Figure 1. Generic control volume and notations

$$\int_v \frac{\partial U}{\partial t}\mathrm{d}V+\int_s G(U)n_i \mathrm{d}s=\int_v S(U)\mathrm{d}V \tag{4}$$

where $G(U)=(F,H)^T$, n_i =the unit normal vector outward from control volume that is shown in Figure 1. Equation (4) can be rewritten as:

$$\Delta V\frac{\mathrm{d}U}{\mathrm{d}t}=-\sum_{j=1}^{m} G(U)\, n_{ij} ds_{ij}+S\ (U)\Delta V \tag{5}$$

where ΔV= the area of each control volume, m= the number of cell sides, ds_{ij} =the length of side j. The Rotational Invariance Property is utilized between F and H over each side (Toro, 1992):

$$\Delta V\frac{\mathrm{d}U_i}{\mathrm{d}t}=-\sum_{j=1}^{m} T_{nij}^{-1}F(T_{nij}U_i, T_{nij}U_j)ds_{ij}+S_i(U_i)\Delta V \tag{6}$$

Consequently, in order to compute the fluxes in the cell interface, the local one-dimensional Riemann problems are utilized in the normal directions of the cell sides. Which $F(T_{nij}U_i,\ T_{nij}U_j)$ is resolved via an HLLC method. The numerical flux of HLLC, $F_{i+1/2}$, evaluated as follows (Toro et al., 1994):

$$F_{i+1/2}=\begin{cases} F_L & if\ \ S_L\geq 0 \\ F_{*L} & if\ \ S_L\leq 0\leq S_* \\ F_{*R} & if\ \ S_*\leq 0\leq S_R \\ F_R & if\ \ S_R\leq 0 \end{cases} \tag{7}$$

where S_L, S_* and S_R are the speeds of the left, contact and right waves, respectively that is shown in Figure 2, and $F_L=F(U_L)$, $F_R=F(U_R)$, U_L and U_R are the left and right Riemann states of a local cell interface, respectively, F_{*L} and F_{*R} are the numerical fluxes in the left and right sides of the star region of the Riemann solution which is divided by a contact wave (Toro et al., 1994).

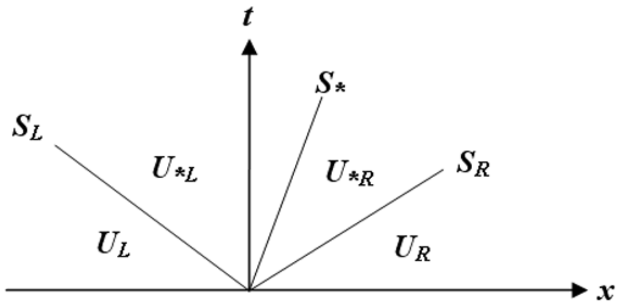


Figure2 Wave structure of HLLC Riemann solver

In order to achieve second-order accuracy, the WAF method was selected. The WAF scheme, as being second-order accurate in space and time, produces spurious oscillations near steep gradients, and so needs TVD stabilization. According to TVD WAF scheme the flux at the cell interface is calculated by the relation (Toro, 1992):

$$F_{i+1/2}^{TVD\ \ WAF} = 1/2(F_i + F_{i+1}) - 1/2\sum_{k=1}^{3} sign(C_k) A_{i+1/2}^{(k)} \Delta F_{i+1/2}^{(k)} \quad (8)$$

$$F_i = F(U),\ \ \Delta F_{i+1/2}^{(k)} = F_{i+1/2}^{(k+1)} - F_{i+1/2}^{(k)}, C_k = (\Delta t / \Delta x) S_k \quad (9)$$

where $A_{i+1/2}^{(k)}$ is a WAF flux limiter function. There are various choices for computing the limiter function in the present model such as Superbee, Van Leer, Van Albada and Minbee limiter functions (Toro, 2001).

In modeling the bottom slope source terms, the numerical pointwise treatment of S_o is not so difficult if the bottom slope in the x and y directions can be easily determined. However, generally the four vertices of each cell do not lie on the same plane; therefore the slope of the cell is not trivially computable. To avoid this problem, the technique has been developed by Valiani et al. (2002) is used.

3 MODEL TESTING

3.1 *Dam break problem with left dry bed*

The test has been designed by Toro (2001). The computational domain is a channel 50 m long, unit width, frictionless and horizontal bed. The downstream section of the dam is dry in this test case. Let h_L=0, h_R=1 m, u_L=0 and u_R=0, depth and velocity in the left and right side of x_0=30 m which is the position of initial discontinuity. In Figure 2, the model results and analytical water surface profiles are shown at t= 4 s. The solution consists of a single right rarefaction wave, with the wet/dry front attached to the tail of it. The propagation of wet/dry front at the correct speed is one major difficulty of numerical methods. In a real application in which such fronts are to be propagated by several kilometers, the propagation speed and thus the predicted wave arrival time will suffer from considerable errors (Toro, 2001). The model result is shown in Figure 2 by the TVD WAF scheme with Superbee limiter function. The model has a good agreement with analytical solution especially in the beginning of the rarefaction part that is indicated by a circle in Figure 2. Most of the models with the second order accuracy are more diffusive in this part.

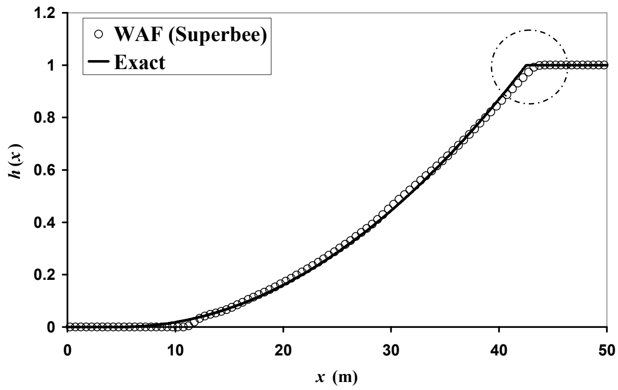


Figure 3. Dam-break problem with left dry bed

3.2 *Partial Dam-break through a sluice-gate*

The aim of this test case is to study the capacity of the present model to simulate the front wave propagation over a dry bed. The spatial domain is a 200×200 m^2 flat region (Figure 4).

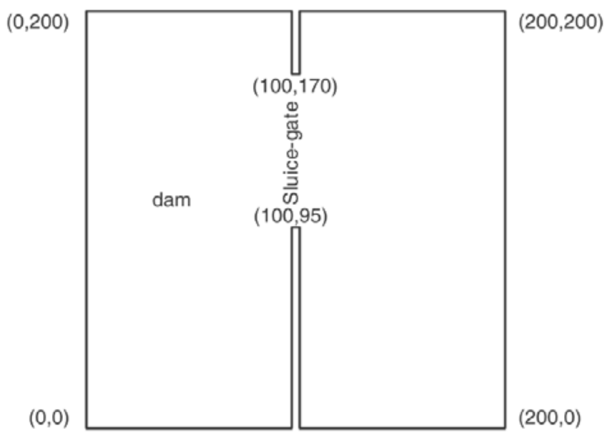


Figure 4. Partial dam-break layout (Loukili and Soulaïmani, 2007)

The bottom is frictionless. The computational domain is a 40×40 square mesh. The upstream discharge is zero. The initial water levels are 10 m upstream and 0 m in downstream. The asymmetric breach is 75 m wide. There is no analytical reference solution for this test case, but in the literature numerical results of various authors are available e.g. Mingham and Causon (1998), Loukili and Soulaïmani (2007). Figures 5 and 6 show the results of present model for water surface and water depth contours. The duration of the simulation is 7.2 s. Figure 7 shows the water depth contour of Loukili and Soulaïmani (2007). The present model result shows a good agreement with the published results.

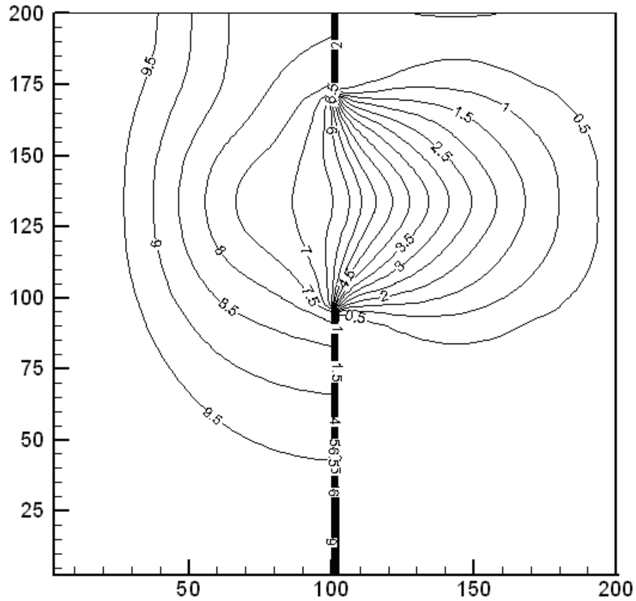


Figure 5. Results of present model for water depth contour

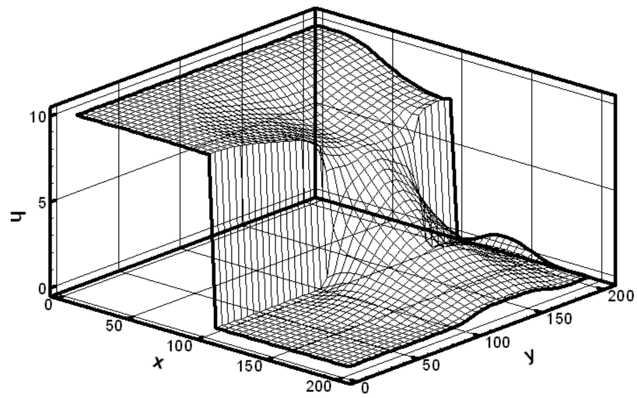


Figure 6. The results of present model for water surface

3.3 *Dam-break in channel with 90° bend*

This test problem is the experimental case designed by Soares-Frazão et al. (1998) for verifying the capability of numerical methods to simulate dam-break flows. The flow domain consists of a square reservoir and L-shaped channel as shown in Figure 8. The bottom level of the channel is 0.33 m higher than the bottom level of the reservoir, which means that there is a step at the entrance of the channel. Initially, the water depth was 0.53 m in the reservoir which is separated by a gate from the channel and then the gate is suddenly opened to produce a dam-break situation. The water depth in the channel was set to 0.01 m. Taking into account the effect of the bottom and the chute walls the average friction coefficient in the calculation is taken equal to 0.011 $s.m^{-1/3}$ (Prokof'ev, 2002).

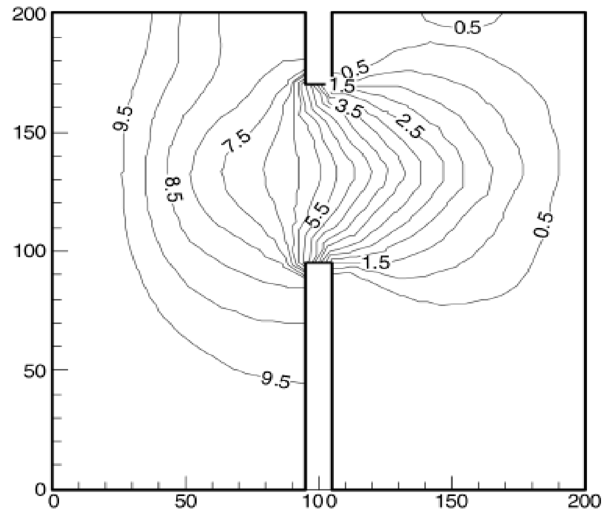


Figure 7. Results of Loukili and Soulaïmani for water depth contour (Loukili and Soulaïmani, 2007)

The variation of water surface elevation with time is compared with the experimental data at the different gauge positions as shown in Figure 9. The comparison of results for gauge P1 is satisfactory as it shown in Figure 9 (a). It shows that the numerical model computes the right discharge coming in to the channel. Figure 9(b) shows the arrival of the first shock traveling downstream of the reservoir and the arrival of the second shock reflected from the channel bend. The agreement for the second shock is very good. However, there are differences at P3 and P4 for the arrival of second shock. At most gauge positions, there is good agreement between the present model results and the experimental data. The deviation between the model results and experimental data may be due to the local head loss caused by sudden change in flow geometry at the entrance to the channel and due to eddy losses, which are not taken into account in the numerical computations.

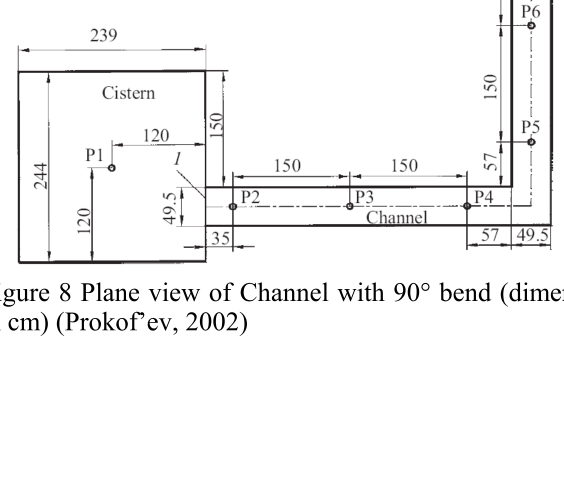
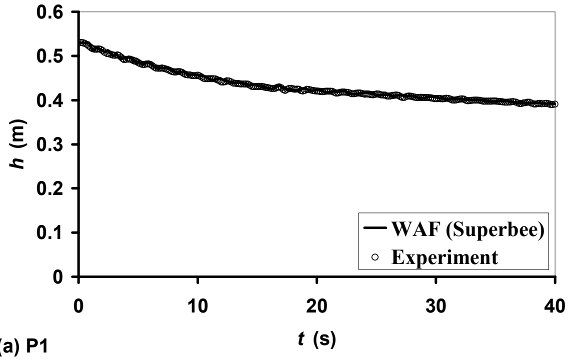
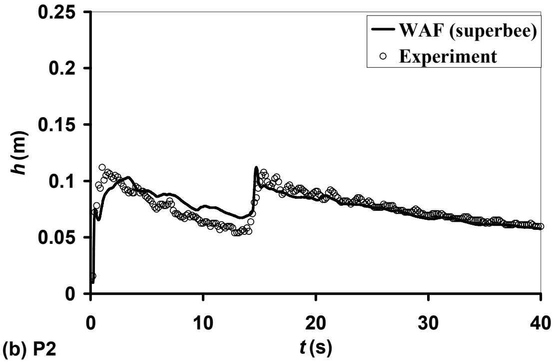


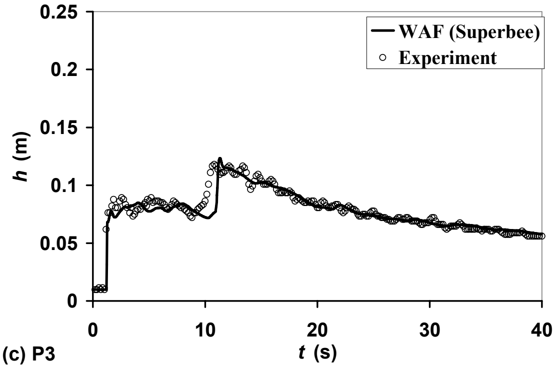
Figure 8 Plane view of Channel with 90° bend (dimensions in cm) (Prokof'ev, 2002)



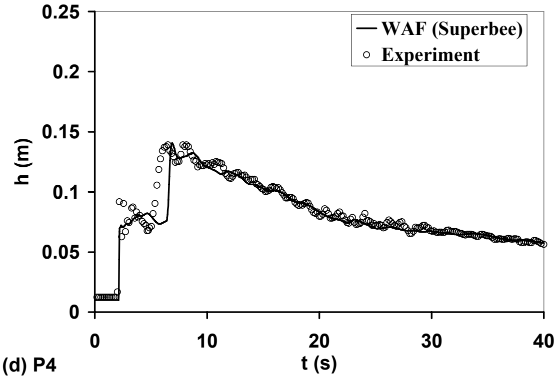
(a) P1



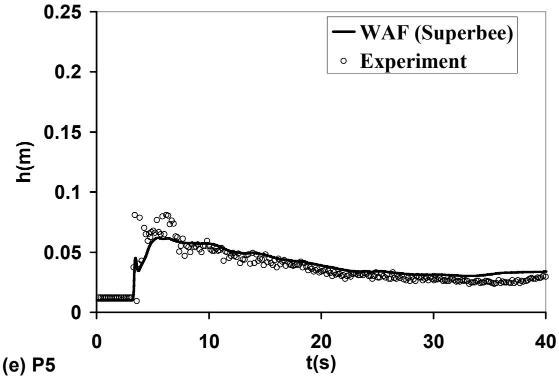
(b) P2



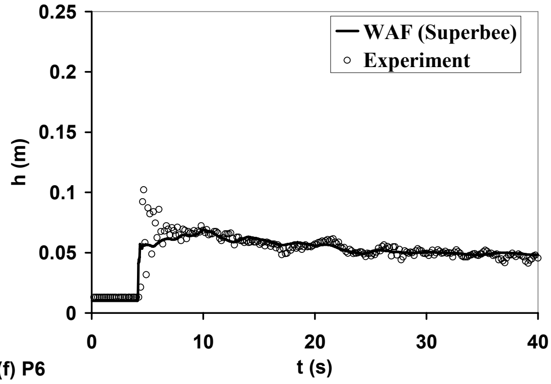
(c) P3



(d) P4



(e) P5



(f) P6

Figure 9. Dam-break flows on wet bed at different gauges

It is reported some discrepancy between the results of numerical models and the experimental data at the gauge stations P2 and P5 by previous researchers such as Prokof'ev (2002). However, the results of the present model at the mentioned stations are good agreement with experimental data which can be caused by good accuracy of TVD WAF method as mentioned in section 3.1.

4 CONCLUSIONS

A two dimensional finite volume model has been developed based on shallow water equations. In the current study the TVD WAF method with second order accuracy both in space and time has been used for modeling fluxes at the interface of the cells. Model verification has been made by comparison of the model results with analytical and numerical solutions. Comparison of the two set of results represent a reasonable degree of similarity. Finally, the model was applied to one of CADAM test and the results of numerical model compared with measurements. The results of present model indicate that:

- The model is applicable when the bed is dry.
- The model has a good agreement in the beginning of the rarefaction wave in 1-D dam break problem.
- Comparison of the model results and experimental data in different gauge stations in channel with 90° bend shows that the model operates well in predicting the wave caused by dam-break.

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