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Comparison among different entrainment/deposition functions in the simulation of a 1D dam-break

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ABSTRACT: The evaluation of the net flux of sediments entering/leaving the flow over an erodible bed (entrainment) plays a key role in the description of solid transport. Furthermore, the expression of the entrainment is required as one of the closure relations of several recent morphological models suited for the analysis of fast erosional transients, as dam break over a loose bed. However, the choice among the formulations proposed in the literature for erosion/deposition (entrainment) evaluation is far from being straightforward. In order to investigate the effect of the choice of different entrainment/deposition formulas, in the present paper a comparison among some of the existing formulations, either deduced from experimental investigations or based on theoretical analysis, has been performed. As a benchmark, a well documented laboratory experiment of dam break over an erodible bed has been selected from the literature, while a two-phase morphodynamical model has been employed to simulate the process. The comparison between numerical simulations and experimental data highlights the different performances and limitations of the considered formulations.

Keywords: Entrainment/deposition, Non-equilibrium transport, Dam-break, Adaptation length.

1 INTRODUCTION

Modelling accurately geomorphic changes associated with dam-break flows is important for emergency planning, risk management and damage assessment. The propagation of a wave over a mobile bed after the sudden collapse of a dam causes important modifications, due to the complex interaction between the flow and the river bed particles constituting the bed. Through friction, inertial effects and momentum exchanges with the fluid phase, erosion of bed material may in turn significantly affect the development of the flood. Damages resulting from sediment erosion, transport and deposition may even be larger than the damages resulting from the flooding itself.

Since the propagation of a dam-break wave is a strongly unsteady flow, its prediction is still affected by the lack of knowledge about solid transport in unsteady conditions. As far as the early stages of the phenomenon are considered, the role of solid transport dynamics has been widely recognised in the literature of the last decade (Fraccarollo & Capart 2002, Cao et al., 2004), and several models aimed to describe unsteady sediment transport have been proposed so far. Apart from the conceptual framework under which momentum and mass balance equations are obtained, a feature common to most of these morphodynamic models is the presence of an entrainment/deposition source term, expressing the net flux of sediments entering/leaving the flow as the consequence of erosion and deposition processes.

However, the evaluation of particle fluxes exchanged between the flow and the mobile bed still constitutes an open challenge. Despite several formulations have been proposed, their use is far from be straightforward. A widely accepted and accurate formula describing the entrainment/deposition process is not available, and existing formulas may often exhibit significant prediction errors when applied to datasets different from the one used for their derivation (Pontillo et al., 2010)

With these premises, in the present paper several relations for the entrainment are compared in the reproduction of a laboratory dam-break test on a mobile bed, accurately documented in the literature. Their influence on the simulated results is furthermore discussed.

2 ENTRAINMENT MODELING

2.1 *Theoretical models*

The simplest approach for modeling river morphology is based on computing bed load transport by an equilibrium flow formula. This approach may not work well in highly unsteady flows, such as the propagation of dam-break wave in alluvial channel.

To overcome these limitations, an equation describing the delay in the adaptation of solid transport to equilibrium conditions has been introduced by Armanini & Di Silvio (1988), which is analogous to a "reaction" equation of chemistry. Following this approach, the bottom depth variation in time, *dZ/dt,* in the following named vertical bottom velocity, can be written as:

$$\frac{dZ}{dt} = \frac{Q_{s.cap} - Q_s}{L_*(1-\lambda)} \qquad (1)$$

where $Q_{s,cap}$ represents the transport capacity of the flow, commonly evaluated as the sediment discharge computed by a uniform flow formula, Q_s represents the instantaneous sediment discharge, L_* is a "characteristic length" which in general depends on the flow and sediment features and λ is the bed porosity.

An alternative expression of the entrainment/deposition term may be also obtained considering the bed surface as a shock surface, and applying the Rankine-Hugoniot conditions, which express the conservation of mass and momentum fluxes across the bottom surface (Fraccarollo & Capart 2002). This allows to relate the velocity of the bottom surface to the shear stresses exerted on its two sides. The specification of the stress tensor across the bed is required to achieve an amenable relation. In the present analysis, a two-phase mixture is considered and continuity of normal stresses is assumed. Furthermore, shear stress on the upper facet of the bottom is assumed as the sum of turbulent shear stresses exeterd by the flow, and of a frictional collisional shear due to the sediment phase. Finally, on the lower facet, Mohr-Coulomb friction due to the mixture augmented by the critical Shields shear stress is considered. The following relation is thus obtained:

$$\frac{dZ}{dt} = -\frac{\frac{U_w^2}{ghCh^2} + (\Delta+1)\alpha U_s^2 - g\Delta\left[\delta(\tan\varphi - \tan\varphi') + \vartheta_{*,cr}\left(1 - \frac{\tan\vartheta}{\tan\varphi}\right)\right]}{\eta U_w \lambda + (\Delta+1)U_s(1-\lambda)} \qquad (2)$$

in which U_w and U_p denotes the average water and sediment velocities, respectively, Ch is the non-dimensional Chezy coefficient, h the water depth, δ the solid phase volume for unit base area, α the Bagnold coefficient, φ the sediment repose angle, φ' the dynamic sediment repose angle, θ the bed slope. Finally, $\Delta = (\rho_s - \rho)/\rho$, with ρ and ρ_s water and sediment densities, respectively.

2.2 *Empirical formulations*

Numerous empirical relations, obtained by best fitting of experimental data, for describing entrainment exist (Pontillo et al., 2010). It is important to remark the scarcity of experimental data, due to the difficulty of measuring the vertical flux of sediment. They refer often to extremely simplified conditions, as uniform flow. Moreover, the definition of the entrainement term is itself not always clear. Most of existing formulas computes the rate in volume per unit area per unit time of bed sediment E [kg/(m²s)], but depending on the features of the experimental setting considered, they may implicitly refer either to difference between erosion and deposition fluxes or to the mere entrainment.

Usually E is expressed as function of flow parameters and sediment characteristics. In particular, it is mainly related to the bed shear stresses and the associated shear velocity u^* as stated by Van Rijn (1984 a) in his first relation:

$$E = 0.0033 \cdot \rho_s (\Delta g d)^{0.5} D_*^{0.3} T^{1.5} \qquad (3)$$

where ρ_s is the sediment density, g the gravity, d the grain mean diameter. In (3) the dimensionless particle (D_*) and a transport-stage (T) parameters are defined by

$$D_* = d_{50}\left[\frac{\Delta g}{\nu^2}\right]^{\frac{1}{3}} \qquad (4)$$

where ν is the cinematic viscosity;

$$T = \frac{(u_*^2) - (u_{*,cr}^2)}{(u_{*,cr}^2)} \qquad (5)$$

where $u_{*,cr}$ denotes critical bed shear velocity according to Shields'criterion.The function has been yield as "best-fitted" of experimental data obtained by the author himself at University of Delft.

Nino & Garcia (1994 a, b) proposed a formula with a similar structure :

$$E = 0.028 \cdot \rho_s (\Delta g d)^{0.5} (\vartheta_* - \vartheta_{*,cr})^{\frac{3}{2}} \tag{6}$$

with ϑ_* the non-dimensional Shields shear stress.

Another relation was obtained by Garcia & Parker (1991) based on dimensional analysis:

$$E = w_s \rho_s \frac{A \cdot Z_u}{1 + \frac{A}{0.3} Z_u} \tag{7}$$

with w_s the fall velocity of the sediment in quiescent fluid, the coefficient $A=1.3 \cdot 10^{-7}$ and Z_u function of the shear velocity expressed as

$$Z_u = \frac{u_* \cdot \left[\frac{(\Delta g d)^{0.5} d}{\nu} \right]^{0.6}}{w_s} \tag{8}$$

Van Rijn (1984b) proposed also a second formulation, based on the hypothesis of dynamic equilibrium characterized by the equality of the entrained and deposited material:

$$E = 0.015 \cdot w_s \rho_s \frac{d}{a} \frac{T^{1.5}}{D_*^{0.3}} \tag{9}$$

being a the elevation above the bed at which the equilibrium suspended sediment concentration is computed.

The following relation, proposed by Elhakeem & Imran (2007), is based on similar hypotheses:

$$E = 0.0024 \cdot \rho_s (\Delta g d)^{0.5} (\vartheta_* - 0.0033)^{1.263} \tag{10}$$

It is worth of note that in the above formula stress excess is raised to an exponent smaller than the common value 1.5 of the previous ones.

3 BENCHMARK EXPERIMENTAL TEST

Spinewine & Zech (2007) published a considerable set of experimental data on small scale dam break over erodible bed. Distribution of these data in electronic format allows accurate comparisons with the results from numerical models.

Experiments were performed in a flume with on overall length of 6 m, i.e. 3 m on both sides of a central gate simulating an idealised dam. The considered test's configuration is characterized by flat erodible bed and a water height upstream the dam of 35 cm.

The sand has the following mechanical properties: particle sizes ranging from 1.2 to 2.4 mm, with mean diameter d_{50} = 1.82 mm, density ρ_s = 2683 kg/m^3, friction angle φ = 30° and negligible cohesion. Bottom solid packing concentration is $1-\lambda = 53\%$.

The instantaneous water and bed profile is recorded at times: 0.25, 0.5, 0.75, 1.00, 1.25, 1.50 seconds.

This test is reproduced using the dynamical model proposed by Greco et al. (2008) using the different entrainment relations reported in the previous section.

The use of a model requires the calibration of numerical parameters. Despite all parameters of the considered model have a clear physical meaning, their direct measurement is almost impossible. So, the parameters have been chosen using the relation obtained by application of Rankine-Hugoniot condition (2) after a sensitivity analysis and a best-fitting of experimental data. The adopted values are reported in Table 1.

Table 1. Parameters value

α	C_D	Ch	θ_c
0.0025	0.06	18	0.047

Figure 1 compares experimental and numerical results obtained with the above values of the parameters. Both free surface, Z_{fs}, and bottom, Z, profiles are reported in dimensionless variables, being h_0 the water depth in the reservoir before the dam break. Symbols denote experimental results, while solid, dashed and dotted lines the simulated ones.

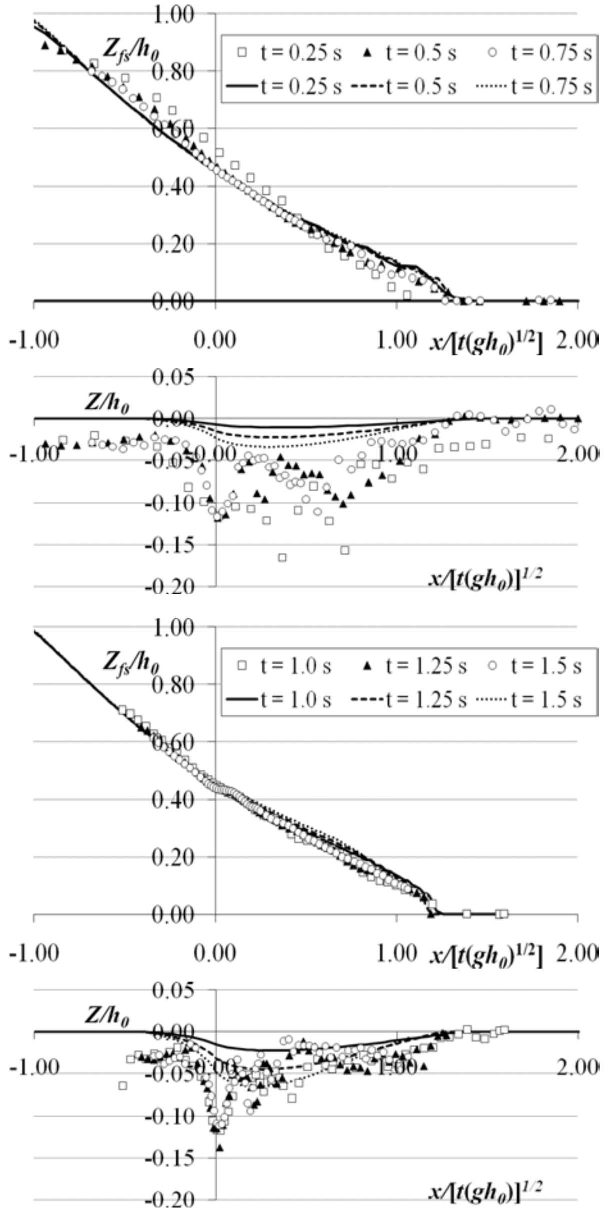

Figure 1. Measured and simulated free surface and bottom profiles. Top panel: t = 0.25, 0.5 and 0.75 seconds. Bottom panel: t = 1.0, 1.25 and 1. 5 seconds.

The agreement is reasonably good, and the used formulation reproduces qualitatively the erosion process, capturing also the magnitude of the scour depth. It is worth of note that the excavation close to the original dam location is not reproduced. It is probably due to the discrepancy in the theoretical assumption of instantaneous fluid release and the experimental procedure used for dam removal. In fact, the dam break is reproduced by a quick downward movement of the thin rigid wall: in this movement the sand tends to fill the space in the bottom previously occupied by the dam.

The same experiment has been then reproduced by the two-phase model, using the adaptation formula (1) for the entrainment, and evaluating the characteristic length L_* accordingly to Armanini & Di Silvio (1988). The graphical representation of the results is given in Figure 2.

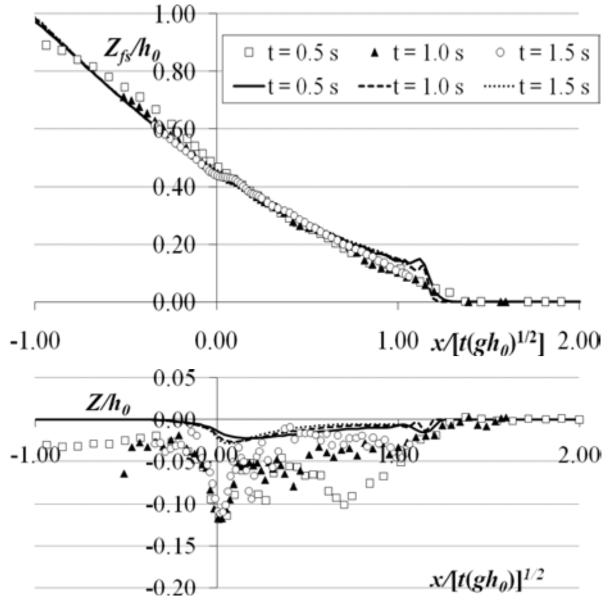

Figure 2. Measured and simulated free surface and bottom profiles at times t = 0.5, 1.0 and 1.5 seconds.

Although the hydrodynamic is well reproduced, the bed erosion is not. The overall entrainment of sediments from the bottom is significantly smaller than the experimental evidence.

The characteristic length was then changed to the water depth value, but still the reproduced entrainment has been found to be smaller than the experimental one. Therefore, apart from the order of magnitude of the characteristic length L_* this kind of approach seems to be not suitable for simulating short-time dam-break waves.

Also the empirical formulations reported in paragraph 2.2 have been tested in order to understand their applicability to the simulation of this kind of process. The results obtained by applying the first formulation proposed by Van Rijn are shown in Figure 3.

As in the latter case, the bottom erosion computed by the Van Rijn relation (3) is less than the observed one.

So far the chosen entrainment relation does not affect significantly the simulated water profile. The hydrodynamic is quite well reproduced in all cases, and the magnitude of the entrainment varies in a range so that the celerity of the downstream water wave still agrees with the experiments.

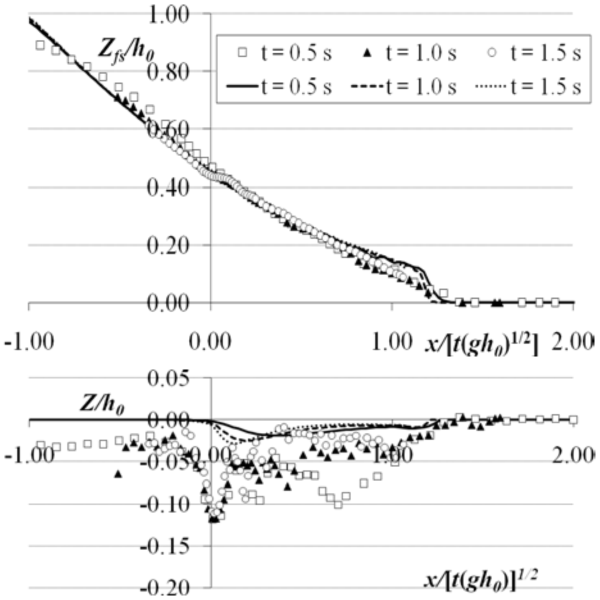

Figure 3. Measured and simulated free surface and bottom profiles at times t = 0.5, 1.0 and 1.5 seconds.

Differently, the second relation for the entrainment proposed by Van Rijn, Eq.(9), simulates an erosion process close than the experimental one, but predicts that the downstream front of the water wave propagates with a smaller celerity than measured one (see Figure 4).

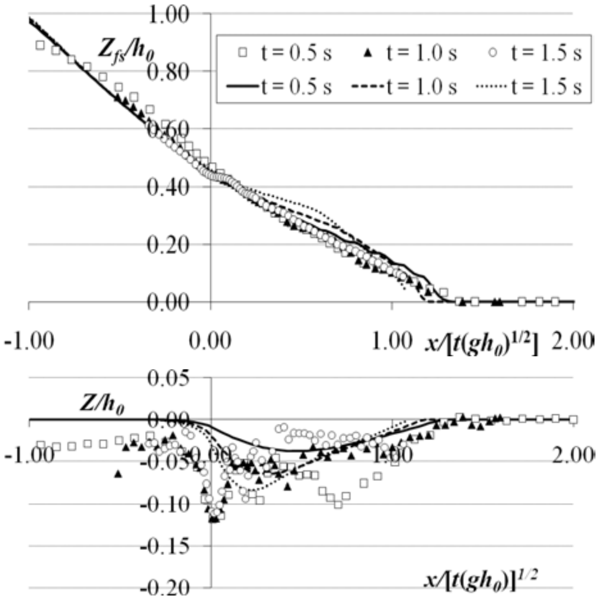

Figure 4. Measured and simulated free surface and bottom profiles at times t = 0.5, 1.0 and 1.5 seconds.

A re-calibration of model parameters has been carried out to investigate the effect of the entrainment formulation chosen for the initial model calibration. The water profile (Figure 5) is quite similar to the observed one and also the bed erosion seems to be reproduced quite well.

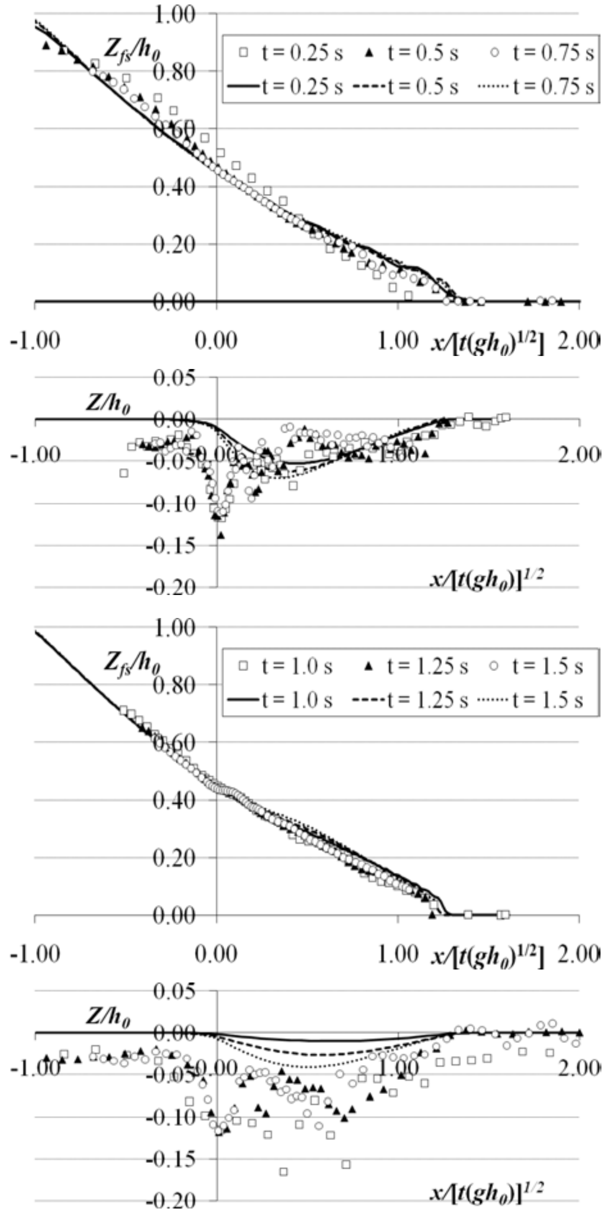

Figure 5. Measured and simulated free surface and bottom profiles. Top panel: t = 0.25, 0.5, and 0.75 seconds. Bottom panel: t = 1.0, 1.25 and 1. 5 seconds.

The results obtained with the other formulations, Eqs. (6), (8) and (10), are not reported herein for sake of brevity. However, applied to the considered case-study, they predict a negligible entrainment rate. This can be due to the fact the sand constituting the bed in the test is not fine and those formulas do not have a parameter depending on the particles diameter.

The relation proposed by Garcia and Parker, Eq. (7), accounts for the particles diameter in the parameter Z_u, but still it gives an almost null entrainment rate.

To have a quantitative assessment of the agreement between numerical and experimental results (Figure 6), the mean square error in the simulated bed profile has been evaluated for each of the times at which experimental data are available. In particular, the performance of Eqs. (2) and (9) are compared in

Figure 6. The error measure is made non-dimensional by dividing by the initial water depth.

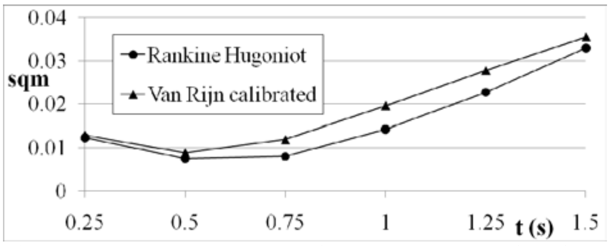

Figure 6. Square mean error between computed and estimated bottom elevation as a function of time.

A part of the error is given to the scour hole close to the original dam location, which is not reproduced in the simulated results. The entrainment formulation derived from the Rankine-Hugoniot conditions performs better than the Van- Rijin formula, provided that the same formula is used also for parameter calibration.

4 CONCLUSIONS

The large use of mathematical models in the analysis of rapid river morphodynamical processes implies the choice of a relation describing the entrainment and deposition of sediment particles constituting the river bottom.

In this paper the influence of the choice of the entrainment/deposition formulation has been investigated. To this aim, a well documented experiment regarding the propagation of a dam break wave over an erodible bed has been chosen to compare the performances of some existing formulations as closure relations of a recently proposed morphodynamic model.

The application of Rankine-Hugoniot conditions to the bed surface has lead to a relation expressing the entrainment rate as a function of the stresses acting on both side of the same surface. The use of this relation gives good results if compared to the experimental evidence. Good results are also obtained by using the second relation proposed by Van Rijn (1984 a), provided that the other parameters of the morphodynamical model are calibrated.

The other literature considered formulations produce a less satisfactory agreement with the experimental data. The erosion process simulated by the numerical model using those formulas is less than the observed one.

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