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Diffusion coefficient of suspended sediment and kinematic eddy viscosity of flow containing suspended load

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ABSTRACT: Diffusion coefficient of suspended sediment (ε_s) and kinematic eddy viscosity (ν_t) of flow containing suspended load have not clearly been solved based on suspended sediment behavior. As for ε_s, the kinematic eddy viscosity or the turbulent diffusion coefficient of momentum of water for clear water, ν_{t0}, is often employed, or it is sometimes empirically modified. When a suspended sediment particle is expressed by using the probability density of its existence height $f(y)$ as a result of random motion driven by turbulence, its transitional probability can be related to the characteristics of turbulent flow represented by a parameter ν_t. While, $f(y)$ must be similar to the concentration distribution of suspended sediment $C(y)$ governed by a diffusion equation with a parameter ε_s. It has suggested us an estimation of the turbulent Schmidt number (ν_t/ε_s). As for an interaction between suspended particles and turbulent flow, the fluctuations of drag force and turbulence have been focused on in a k-ε turbulence model, the correlation of which appears in transport equations of k and ε. This approach has suggested how ν_t increases from ν_{t0} with depth averaged concentration of suspended load.

Keywords: Suspended sediment, Diffusion coefficient, Kinematic eddy viscosity, Diffusion model, Stochastic model, Turbulent model

1 INTRODUCTION

The mechanics of sediment transport with open channel turbulent is one of the most fundamental topics of river hydraulics, but the details have not systematically understood. The behavior of suspended sediment is subjected to the turbulent flow structure, while it is influenced by suspended sediment.

The suspended sediment concentration profile under equilibrium is governed by the following equation,

$$w_0 C(y) = \varepsilon_s \frac{dC(y)}{dy} \tag{1}$$

where $C(y)$=concentration distribution of suspended sediment, w_0=settling velocity of sand, ε_s=turbulent diffusion coefficient of suspended sediment, and y=height from the bed, respectively. Rouse (1937) obtained the concentration distribution by assuming that

$$\varepsilon_s = \beta \nu_t \tag{2}$$

and that the eddy kinematic viscosity, ν_t, is obtained by the log law for open channel turbulent flow as follows:

$$\nu_t = \kappa u_* h \frac{y}{h}\left(1 - \frac{y}{h}\right) \tag{3}$$

where κ=Karman's constant, u_*=shear velocity, and h=flow depth, respectively. Lane & Kalinske (1940) employed a constant value for ν_t which is for example given as the depth average value of Eq.(3) ($\nu_t = \kappa u_* h/6$).

Figure 1 shows the vertical distribution of diffusion coefficient of suspended sediment obtained by Coleman (1970). From the depth average, it demonstrates that the value of β, the ratio of ε_s to ν_t, And the reciprocal of which is termed "turbulent Schmidt number," changes with the ratio of w_0 and u_*. Furthermore, its vertical change has a different deviation from the form of Eq.(3) in the outer layer.

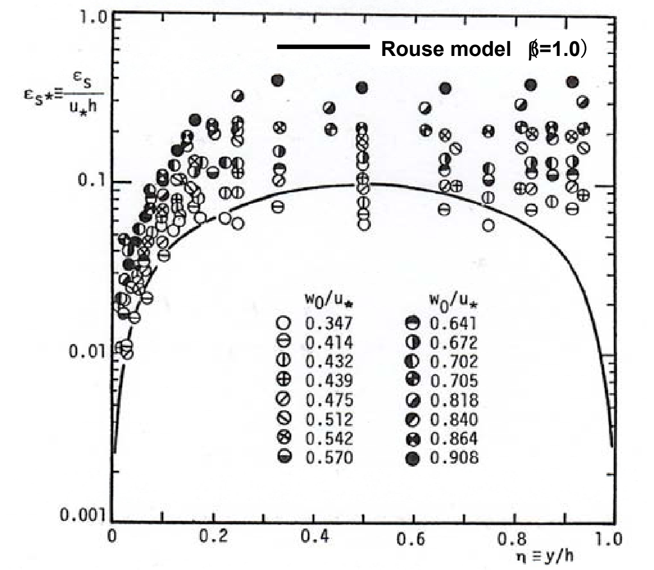

Figure 1. Vertical distribution of diffusion coefficient of suspended sediment.

As for the flow containing suspended sediment, the change in the turbulent structure was investigated, and previously the decrease in Karman's constant was focused (Einstein & Chien, 1958; Hino, 1963; Karim & Kenedy, 1983). Current interpretation is that Karman constant is universal but the wake effect which is superimposed on the log law in the outer layer changes the turbulent structure (Coleman, 1981). Then, the velocity profile is written as

$$\frac{U}{u_*}=\frac{1}{\kappa}\ln\frac{y}{k_s}+B_s+\frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi y}{2h}\right) \tag{4}$$

where Π=wake strength parameter, k_s=equivalent sand roughness, B_s=a function of $Re_*=u_*k_s/\nu$ given by Nikuradse's experiment, and ν=kinematic viscosity. According to Coleman, Π value increases with depth-averaged concentration of suspended sediment, and it corresponds to larger value of ν_t in the outer region and thus larger ε_s.

In this paper, it is focused on how the turbulent diffusion coefficient of suspended sediment differs from the kinematic eddy viscosity which is turbulent diffusion coefficient of momentum of water and how the turbulent structure deviates from the clear water flow by containing suspended sediment, based on minute description of behaviors of flow and sediment.

2 TURBULENT DIFFUSION COEFFICIENT OF SUSPENDED SEDIMENT

Yalin & Krishnappan (1973) conducted an analysis of concentration profile of suspended sediment, where the probability density function of the existing height of a single particle, $f(y)$, is regarded to be similar to $C(y)$.

When the existence of the bottom and the water surface are neglected and it is assumed that the probability of displacement of a particle is independent of y for simplicity, the governing equation of the probability density that a particle exists at the height y at the time t, $f(y;\,t)$, is written as follows:

$$f(y;t+\Delta t)=\int_{-\infty}^{\infty}f(y-\eta;t)g(\eta)d\eta \tag{5}$$

where Δt=time step for stochastic process; and $g(\eta)$=probability density of $\{\eta\}$, the particle displacement during Δt, respectively. The expected value and standard variation of $\{\eta\}$, $E[\eta]$ and σ_η, are given as follows:

$$E[\eta]=-w_0\Delta t,\quad \sigma_\eta=k_\eta v'_{rms}\,\Delta t \tag{6}$$

where k_η=the ratio of vertical component of particle-speed fluctuation to v'_{rms} ($k_\eta\cong1.0$). Taylor's expansion of $f(y-\eta;\,t)$ around $f(y;\,t)$ and manipulating the right hand term of Eq.(5) gives the following expression.

$$f(y;t+\Delta t)=f(y;t)-E[\eta]\frac{\partial f}{\partial y}+\frac{1}{2}E\left[\eta^2\right]\frac{\partial^2 f}{\partial y^2}+\cdots\cdots \tag{7}$$

where $E[\cdot]$=expected value operator.

Under equilibrium, $f(y,\,t+\Delta t)=f(y,t)$, and thus Eq.(7) yields the following equation with respect to $f(y)$ after neglecting the higher-order terms.

$$E[\eta]f=\frac{1}{2}E\left[\eta^2\right]\frac{\partial f}{\partial y} \tag{8}$$

This form is similar to Eq.(1) with respect to $C(y)$, and $f(y)$ and $C(y)$ are regarded proportional to each other. Comparing Eq.(8) with Eq.(1), we obtain the following equation (Tsujimoto, 1984).

$$\varepsilon_s=\frac{E\left[\eta^2\right]}{2\Delta t}=\frac{u_*^2\Delta t}{2}\left[k_\eta^2\left(\frac{v'_{rms}}{u_*}\right)^2+\left(\frac{w_0}{u_*}\right)^2\right] \tag{9}$$

When $(w_0/u_*)\rightarrow0$, particle motion responds to the turbulence so well to regard ε_s as ν_t, and then, Δt should be decided as follows, which is important on conducting stochastic simulation.

$$\Pi_T\equiv\frac{u_*\Delta t}{h}=\frac{2\nu_{t*}}{\left(k_\eta\phi_v\right)^2} \tag{10}$$

where $\nu_{t*}\equiv\nu_t/(u_*h)$; $\phi_v\equiv v'_{rms}/u_*$; and these depth-averaged values are $\kappa/6$ and 0.8, respectively; then Π_T is around 0.21, which corresponds to Lagrangian time-scale of turbulence.

The ratio of the turbulent diffusion coefficient of suspended sediment to the kinematic eddy viscosity (the inverse of Schmidt number) is estimated as a function of (w_0/u_*) as follows.

$$\beta \equiv \frac{\varepsilon_s}{\nu_t} = 1 + \frac{1}{\left(k_\eta \phi_v\right)^2}\left(\frac{w_0}{u_*}\right)^2 \tag{11}$$

Kerssens *et al.* (1979) obtained the following empirical relation for β by regression of the data from the Enoree River (Coleman 1970), where the depth-averaged value of β was shown.

$$\beta \equiv \frac{\varepsilon_s}{\nu_t} = 1 + \alpha_1\left(\frac{w_0}{u_*}\right)^{\alpha_2} \tag{12}$$

where α_1=1.54 and α_2=2.12. The theoretically deduced Eq.(11) is consistent to Eq.(12) where α_1=1.56 and α_2=2.0 and it is extremely close to the above empirical regression.

Figure 2 shows the comparison between the data by Coleman and Eq.(11). In addition, Figure 3 shows the fitting parameter in Rouse profile of suspended sediment concentration, $Z_1=w_0/(\beta\kappa\, u_*)$ is well explained by the present analysis.

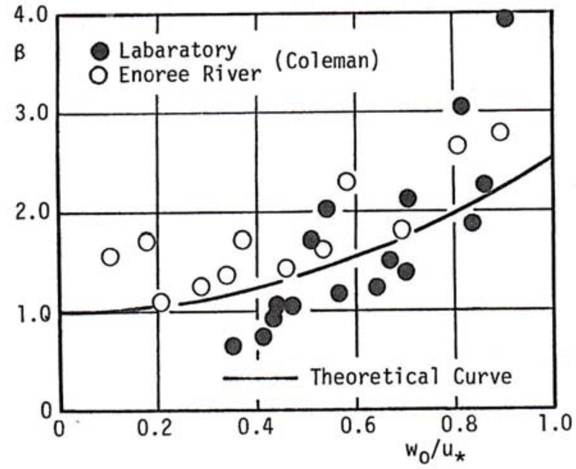

Figure 2. Comparison between the present theory and Coleman's data in Schmidt number.

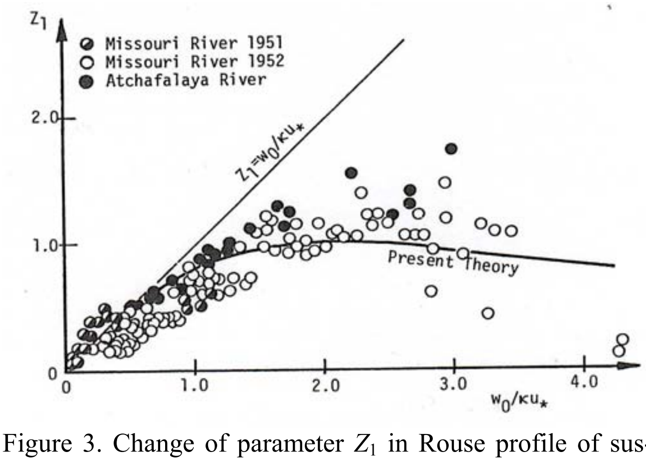

Figure 3. Change of parameter Z_1 in Rouse profile of suspended sediment concentration with (w_0/u_*).

3 OPEN CHANNEL TURBULENT FLOW CONTAINING SUSPENDED LOAD

Though suspended sediment particles follow turbulent motion from the time average view point, their instantaneous behaviors never respond to the turbulence instantaneously, and they show a statistical correlation. Hence, the drag force acting on a paricle is written as (F_{dx}+f_{dx}, f_{dy}), where F_d is time average and f_d is fluctuation of drag force, respectively. The subscripts x, y imply the longitudinal and vertical components, respectively. Then the governing equations for turbulent flow with suspended sediment are given as follows including the equations with respect to the turbulent energy and its dissipation.

$$gI - \frac{\sum F_{dx}}{\rho} + \frac{d}{dy}\left[\left(\nu + \nu_T\right)\left(\frac{du}{dy}\right)\right] = 0 \tag{13}$$

$$\frac{d}{dy}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{dk}{dy}\right] + P_k - \varepsilon + G - S_E = 0 \tag{14}$$

$$\frac{d}{dy}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{d\varepsilon}{dy}\right] + \frac{\varepsilon}{k}\left\{C_{1\varepsilon}\left[P_k + \left(1 - C_{3\varepsilon}\right)G\right] - C_2\varepsilon - C_{4\varepsilon}S_E\right\} = 0 \tag{15}$$

$$G \equiv \beta_S \nu_t \left(\sigma/\rho - 1\right) g \frac{dC}{dy} \tag{16}$$

$$S_E \equiv \frac{\sum\left(\overline{u' f_{dx} + v' f_{dy}}\right)}{\rho} \tag{17}$$

$$P_k \equiv \nu_T\left(\frac{du}{dy}\right)^2 \tag{18}$$

$$\nu_t \equiv \frac{C_\mu k^2}{\varepsilon} \tag{19}$$

where G=turbulent energy production due to buoyant force acting on suspended particles (Eq.16), S_E=turbulent energy dissipation due to suspended sediment motion in turbulent flow (Eq.17), and Σ implies the summation with respect to the particles to require the assumed depth-averaged concentration. The drag term for each particle is written as follows:

$$F_{di} + f_{di} = \frac{1}{2}\rho C_D \rho\left(u_{pi} - u_i\right)\left|\mathbf{\vec{u}_p} - \mathbf{\vec{u}}\right|\frac{A_2 C}{A_3 d} \tag{20}$$

Where $\vec{\mathbf{u}}_{\mathbf{p}} = (u_p,\ v_p)$=instantaneous velocity of suspended particle; $\vec{\mathbf{u}} = (u+u',\ v')$=instantaneous fluid velocity and they are expressed as vectors.

When the flow behavior is calculated, the numerical parameters for k-ε modeling, are set as the standard values except $C_{4\varepsilon}$, as follows (Rodi, 1984): C_μ=1.44, $C_{1\varepsilon}$ =1.44, $C_{2\varepsilon}$ =1.92, $C_{3\varepsilon}$ =0.8, σ_k=1.0, σ_ε=1.3 and β_S=1.0.

The particle behavior is governed by the following equation (in vector form) when the flow-field is given

$$\rho\left(\frac{\sigma}{\rho}+C_M\right)A_3 d^3\frac{d\mathrm{u}_\mathrm{p}}{dt}=-\frac{1}{2}\rho C_D \rho\left(\vec{\mathbf{u}}_\mathbf{p}-\vec{\mathbf{u}}\right)\left|\vec{\mathbf{u}}_\mathbf{p}-\vec{\mathbf{u}}\right| -\rho\frac{A_2 C}{A_3 d}\left(\frac{\sigma}{\rho}-1\right)\vec{g}A_3 d^3+\rho\left(1+C_M\right)A_3 d^3\frac{d\vec{\mathbf{u}}}{dt} \tag{21}$$

where $\vec{\mathbf{g}}$ =gravity acceleration vector.

When the uniform flow is considered, the particle behavior in the vertical direction is simulated by the following equations:

$$v'(t)=\varsigma\cdot v'_{rms}\ ;\quad v'^2_{rms}=0.34k \tag{22}$$

where $\{\zeta\}$=random number uniformly distributed in (0, 1). By Monte-Carlo simulation, one can obtain $\mathrm{u}_\mathrm{p}(t)$ and the existing height probability density $f(y)$ by solving Eq.(21). This procedure is employed to describe the correlation between turbulence and stochastic behavior of a suspended particle to apply to Eq.(17).

If the depth-averaged concentration C_m is given, the concentration distribution is written as $C(y)=C_m f(y)$. Using Eq.(20), the time series of the drag is obtained, and one can evaluate the interaction term (S_E) in Eqs.(14) and (15).

Figure 4 shows the calculated velocity profile of sediment laden flow for the experiment by Vanoni & Nomicos (1960), and the present analysis fairly well describes the deviation of velocity profile for sediment laden flow from the one for clear water flow. The calculation was conducted with the increase of the sediment concentration. As a result, Figure 5 demonstrates how the velocity profile deviates from the log law with the increase of C_m.

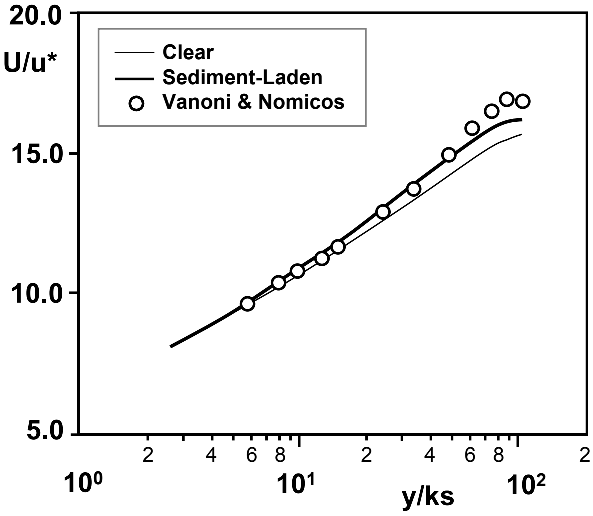

Figure 4. Comparison between the calculated velocity profile of sediment laden flow and measurement by Vanoni & Nomicos (1966).

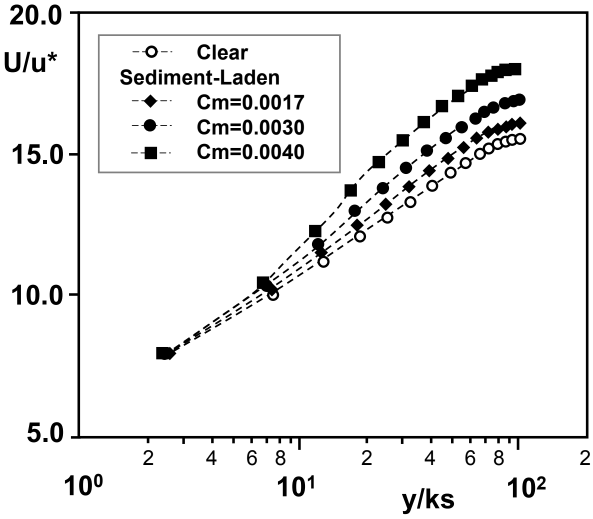

Figure 5. Deviation of velocity profile of sediment laden flow from clear water flow with increase of suspended sediment concentration.

The change of velocity profile is characterized by the increase of velocity gradient and previously it has been represented by the decrease of Karman's constant. Figure 6 is prepared from this view point, and the result of the present model is consistent to the previously collected data and Hino's theoretical results (1963), where κ_0=Karman's constant for clear water flow (=0.4). As mentioned also in Chapter 1, current interpretation is that the deviation of velocity profile appears in the outer layer, and the wake strength parameter Π appearing in Eq.4 should be discussed. In Figure 7, the wake strength parameters for the measured profiles by Coleman (1970) are plotted against the depth averaged concentration of suspended sediment, and the calculated results for Figure 5 are also plotted. This figure supports that the present analysis can describe the essential properties of flow containing suspended sediment.

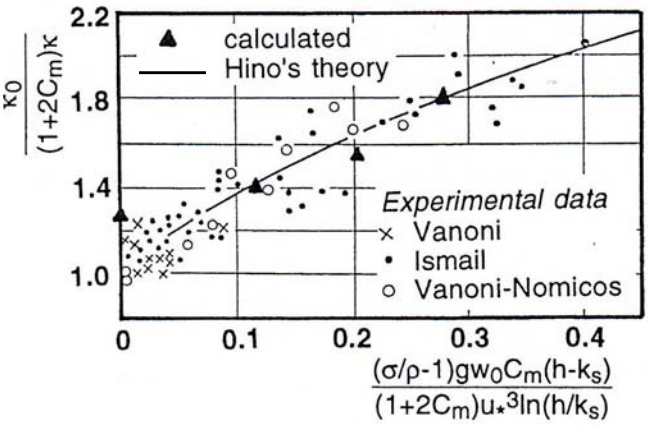

Figure 6. Change in apparent Karman's constant for sediment laden flow .

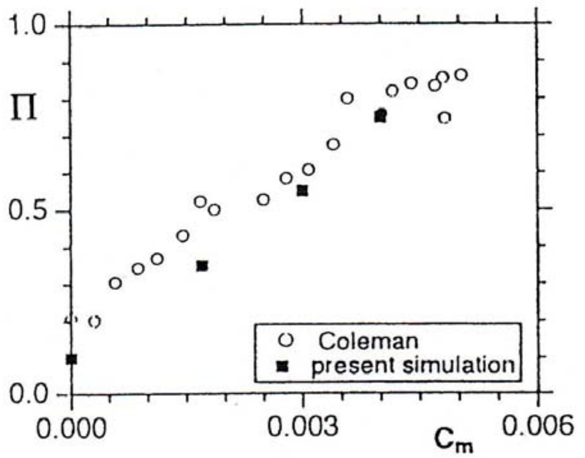

Figure 7. Relation between wake strength parameter and depth averaged concentration of suspended sediment.

On the other hand, Figure 8 shows the calculated distribution of Reynolds stress, $\tau_t(y)$, and it hardly deviates from the triangular profile for the clear water flow. It is consistent to the fact that the drag term of suspended sediment is zero in the time average and it is the most significant difference from bed-load sediment.

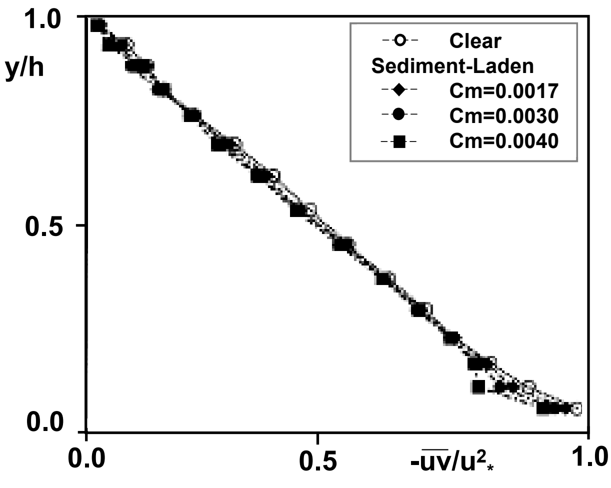

Figure 8. Reynolds stress distribution.

In the flow with bed-load, the Reynolds stress suppressed in the near bed layer and the velocity profile deviates from clear water flow (Tsujimoto *et al.*, 1995). The internal turbulent structure, represented by the kinematic eddy viscosity, for example, is hardly degenerated.

On the other hand, in the flow with suspended sediment, the Reynolds stress cannot be degenerated but the internal turbulent structure is degenerated. This is clearly seen in Figure 9, where the change of kinematic eddy viscosity over the depth is shown and it deviates from the clear water condition (represented by Eq.4) significantly in the outer layer.

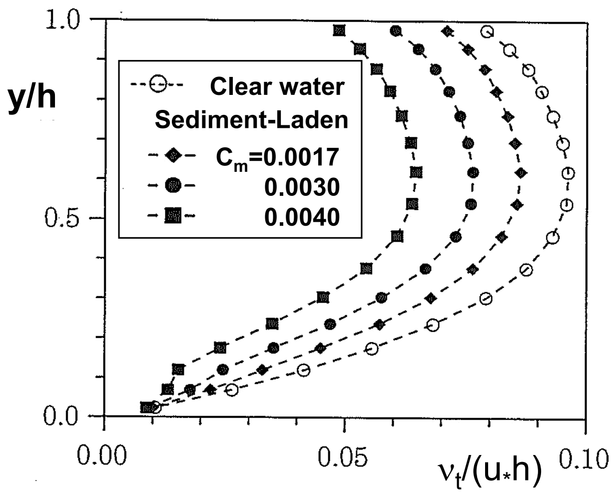

Figure 9. Change in distribution of kinematic eddy viscosity along depth with suspended sediment concentration..

4 CONCLUSION

In turbulent flow containing suspended load, the turbulent structure changes because of the correlation between turbulence and stochastic behavior of suspended particles (the cross correlation between turbulent velocity and fluctuation of drag force on a particle), and it promotes larger wake strength parameter. This effect is emphasized with increases of the depth averaged concentration of suspended sediment. By taking account of cross correlation term of turbulence and drag force, the change of the kinematic eddy viscosity (ν_t) from that in clear water flow (ν_{t0}) against the depth averaged sediment concentration (C_m) can be evaluated. Such a change of internal structure of turbulence brings larger gradient of velocity profile in outer layer, which has been recognized as a decrease of the apparent Karman constant and currently recognized as the increase of the wake strength parameter (Π).

On the other hand, the turbulent diffusion coefficient of suspended sediment ε_s, which governs the concentration profile of suspended sediment, is somewhat different from the kinematic eddy viscosity ν_t which is the turbulent diffusion coefficient of momentum of water, though they have been often identified or empirically related to each other. In this study, by comparing diffusion theory and stochastic model for a suspended particle, the ratio of ε_s to ν_t is investigated. Based on the fact that the governing equation for concentration profile $C(y)$ and that for probability density of existence height of a suspended sediment $f(y)$ are mathematically similar, the time scale for conducting stochastic simulation of suspended particle in turbulent flow and the ratio of ε_s to ν_t have been deduced simultaneously. The obtained relation for the ratio of ε_s to ν_t against the ratio of shear velocity and settling velocity of a suspended particle is quite consistent to the empirical regression relation.

Summarizing the former and latter parts of this paper, we are now able to evaluate how the turbulent diffusion coefficient deviates from the kinematic eddy viscosity for clear water flow or the diffusion coefficient distribution of suspended sediment from the diffusion coefficient of momentum of water, as shown in Figure 1.

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