River Flow 2010 - Dittrich, Koll, Aberle & Geisenhainer (eds) - © 2010 Bundesanstalt für Wasserbau ISBN 978-3-939230-00-7

The effect of river dunes on the morphodynamic response to overloading

Astrid Blom
Environmental Fluid Mechanics section, Delft University of Technology, Delft, Netherlands

ABSTRACT: A new two-layer model for conservation of mixed sediment for bedform-dominated conditions is proposed. There is a need for such a new bedlayer-type sediment conservation model, as (i) the commonly applied Hirano (1971) model does not account for the effects of bedform stochastics and vertical sorting on bed level changes, (ii) the Ribberink two layer model is not sufficiently generic, and (iii) the stochastic Blom et al. (2008) sorting evolution model is cumbersome and requires a very small numerical time step. The new two-layer model is a combination of two sediment conservation models for mixed sediment: the Ribberink (1987) two layer model and the stochastic Blom et al. (2006) equilibrium sorting model. It incorporates the effects of both stochastics of bedform geometry and vertical sorting in the prediction of bed level changes in a parameterized form. After validating the new model against an aggradational flume experiment, we study the morphodynamic response to an excessive sediment supply (overloading) computed by the new two layer sediment conservation model and the commonly applied Hirano model in elementary numerical computations.

Keywords: Bedforms, Morphodynamics, Predictions, Sediment conservation

1 INTRODUCTION

The bed of the Dutch part of the Rhine river is eroding by about 2.5 cm per year, causing numerous problems such as (i) nonerodible parts of the bed forming sills, which cause problems for navigation, and (ii) loss of stability of groins, bridges and banks. In 2011 the Dutch Ministry of Transport, Public Works and Water Management will conduct a unique large-scale nourishment field experiment near Lobith to study if nourishment can counteract the Rhine river's bed erosion.

Such a field experiment is needed for providing more insight in the morphodynamic response to nourishment, as currently available mathematical models fail to predict the unsteady morphodynamic response under bedform-dominated conditions. Numerical analyses by Blom (2008) and Ravenstijn (2009) have demonstrated an incorrect mathematical description of the interaction between bedform dynamics, sorting, and morphodynamic effects. This also hinders the prediction of other unsteady morphodynamic behavior caused by flood events and dredging measures.

Sediment conservation models for mixed sediment are crucial in the modeling of the interaction among bedform dynamics, grainsize-selective sediment fluxes, sorting, and bed level changes. Hirano (1971) was the first to develop a sediment conservation model for mixed sediment. Its active layer represents the bed material that interacts with the flow and is available for entrainment by the flow. Blom & Parker (2004) provide an overview of the various types of sediment conservation models. For unsteady plane-bed conditions, Viparelli et al. (in press, a,b) recently conducted an extensive set of flume experiments, and applied the Hirano (1971) sediment conservation model to reproduce the observed trends.

Crickmore & Lean (1962) and Ribberink (1987) were the first to stress the importance of deep bedform troughs with respect to the time scales of sorting and morphodynamics. Ribberink (1987) developed a two-layer model to incorporate how deep troughs cause lower bed elevations to be reworked over a larger time scale. Following Ribberink's work, the author developed a new type of sediment conservation model that is deterministic in the computation of the morphodynamic response of the river bed, and stochastic in terms of the riverbed surface due to the presence of bedforms (Blom et al, 2006, 2008).

However, both the Ribberink (1987) and the stochastic Blom et al. (2008) model suffer from shortcomings. The Ribberink two-layer model is not sufficiently generic as (1) its vertical sediment exchange term was calibrated on the flume experiment in question; (2) the sediment exchange term is yet suitable for mixtures consisting of two size fractions only; and (3) under some conditions the sediment exchange term does not conserve mass. Application of the Blom et al. (2008) model is cumbersome as the model is complex and requires a small numerical time step. This paper presents a new two-layer sediment conservation model for mixed sediment under bedform-dominated conditions. It is a combination of the Ribberink (1987) two-layer model and the stochastic Blom et al. (2006) sediment conservation model and can be applied to unsteady conditions in the field such as due to sediment nourishment.

2 PROPOSED BED LAYER MODEL

2.1 *The active part of the bed*

Figure 1 shows the schematization of the active part of the bed in the new two-layer model for sediment conservation. The boundaries of the active part of the bed, i.e. elevations A and C, are determined from the probability distribution, P_s, of bed surface elevations ($\tilde{z}$) relative to the mean bed level. The probability distribution P_s is determined using submodels

- for the mean bedform height, e.g., a reduced version of the model developed by Shimizu et al. (2009) or Nabi et al. (2009);
- relating the mean relative trough elevation Δ_{ba} to the mean bedform height Δ_a by setting $\Delta_a = 2\,\Delta_{ba}$, where Δ_{ba} denotes the mean vertical distance between the mean bed level and the trough elevation (Figure 2);
- imposing a Weibull distribution for the probability distribution of relative trough elevations Δ_b (Van der Mark et al., 2008), relating the standard deviation of the relative trough elevation, σ_b, to its mean value, Δ_{ba}, by setting $\sigma_b = 0.63\,\Delta_{ba}$ (Van der Mark et al., 2008), and assuming individual bed forms to have a triangular shape (Blom et al, 2006).

The above procedure is explained in detail by Blom et al. (2006).

The active part of the bed consists of two active layers, layers AB and BC, which are exposed to the flow to a different extent. Elevations A, B, and C are derived from the probability distribution P_s at the specific time. Although the upper elevation of layer AB equals the mean bed level, η_a, layer AB reflects the sediment above elevation B up to the elevation where $P_s(\eta_A) = 0.01$ (Figure 1). Bed layer AB represents the bed material of which the vertical sorting profile can be assumed to have reached a steady-state at each point in time.

Bed layer BC represents bed elevations reached by deep bed form troughs (Crickmore & Lean, 1962, Ribberink, 1987, Di Silvio, 1992, Blom, 2008) and is defined such that, on the time scale of interest, its grain size distribution (GSD) cannot be assumed to reach a steady-state at each point in time. In other words, the GSD of bed layer BC adjusts to changing flow conditions much more slowly than bed layer AB. Elevation B, η_B, separates layer AB from layer BC and, for now, is the elevation where P_s equals 0.95 (Ribberink, 1987). In future work, elevation B, η_B, will be studied in further detail.

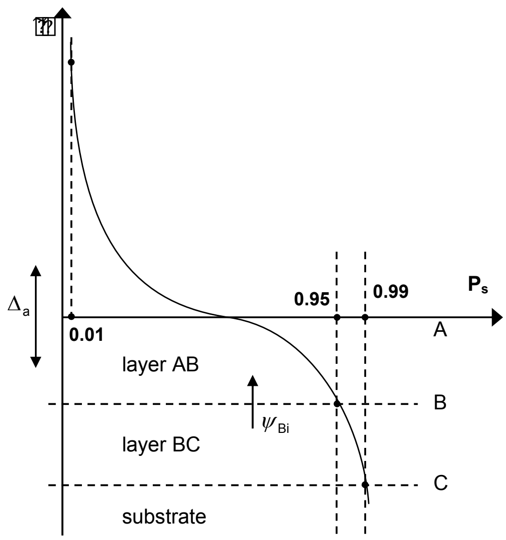

Figure 1. The new two-layer model for sediment conservation. The schematization of the active part of the bed is based on the probability distribution, P_s, of relative bed surface elevations, $\tilde{z}$.

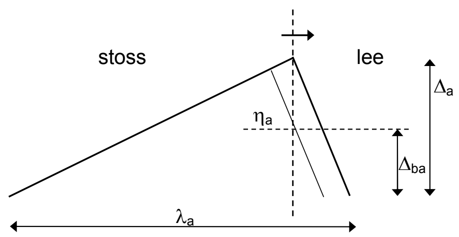

Figure 2. Geometric dune parameters.

2.2 *Governing equations of the new model*

Figure 3 shows a scheme of the morphodynamic model system for nonuniform sediment to which the new two-layer model for sediment continuity is applied. Like in the sediment conservation models analyzed by Blom (2008), we distinguish three types of vertical sediment fluxes affecting the vertical sorting profile:

I. vertical sediment fluxes through the migration of (irregular) dunes;
II. vertical sediment fluxes through a change in time of the probability distribution of relative bed surface elevations, P_s;
III. vertical sediment fluxes through net aggradation or degradation.

These sediment fluxes are assumed to act independently. Mass conservation in bed layer AB yields

$$c_b \frac{\partial F_{ABi}\delta_{AB}}{\partial t} + \underbrace{c_b F_{Bi}\frac{\partial \eta_B}{\partial t}}_{I,III} = \underbrace{\psi_{Bi}}_{II} - \underbrace{\alpha_{AB}\frac{\partial q_{ai}}{\partial x}}_{III} \qquad (1)$$

where c_b = 1-porosity, F_{ABi} = mean volume fraction content of size fraction i in layer AB, F_{Bi} = volume fraction content of size fraction i at interface B, η_B = elevation of interface B, ψ_{Bi} = grain-size-selective sediment flux from layer BC to layer AB (which will be explained in Section 2.4), α_{AB} = proportion of the aggradation/degradation flux that is attributed to layer AB (explained later in this section), and q_{ai} = the sediment transport rate of size fraction i. Mass conservation in bed layer BC yields

$$c_b \frac{\partial F_{BCi}\delta_{BC}}{\partial t} + \underbrace{c_b F_{Ci}\frac{\partial \eta_C}{\partial t}}_{I,III} - \underbrace{c_b F_{Bi}\frac{\partial \eta_B}{\partial t}}_{I,III} = \underbrace{-\psi_{Bi}}_{II} - \underbrace{\alpha_{BC}\frac{\partial q_{ai}}{\partial x}}_{III} \qquad (2)$$

where F_{BCi} = mean volume fraction content of size fraction i in layer BC, F_{Ci} = volume fraction content of size fraction i at interface C, η_C = elevation of interface C, and α_{BC} = proportion of the aggradation/degradation flux that is attributed to layer BC. The volume fraction content of size fraction i at interface B, F_{Bi}, is given by

$$F_{Bi} = \begin{matrix} F_{ABzi}(\eta_{95}) & if\ \partial\eta_B/\partial t > 0 \\ F_{BCi} & if\ \partial\eta_B/\partial t < 0 \end{matrix} \qquad (3)$$

In case of an increase in interface elevation B, the lower (relatively coarse) material from layer AB ($F_{ABzi}(\eta_{95})$) is transferred to layer BC. The computation of $F_{ABzi}(\eta_{95})$ will be explained in Section 2.4. Likewise, the volume fraction content of size fraction i at interface C, F_{Ci}, is given by

$$F_{Ci} = \begin{matrix} F_{BCi} & if\ \partial\eta_C/\partial t > 0 \\ F_{oi} & if\ \partial\eta_C/\partial t < 0 \end{matrix} \qquad (4)$$

where F_{oi} = the volume fraction content of size fraction i just below layer BC. The constants α_{AB} and α_{BC} describe how vertical sediment fluxes through net aggradation and degradation (sediment fluxes of type III) are distributed between the two active layers of the bed. We distribute the amount of aggradation and degradation according to the bed layers' exposure to the flow (Figure 1):

$$\alpha_{AB} = \int_{AB} p_e dz = 0.95 \qquad (5)$$

$$\alpha_{BC} = \int_{BC} p_e dz = 0.05 \qquad (6)$$

where p_e = the probability density function of bed elevations. By definition, the values for α_{AB} and α_{BC} need to fulfill the constraint $\alpha_{AB} + \alpha_{BC} = 1$.

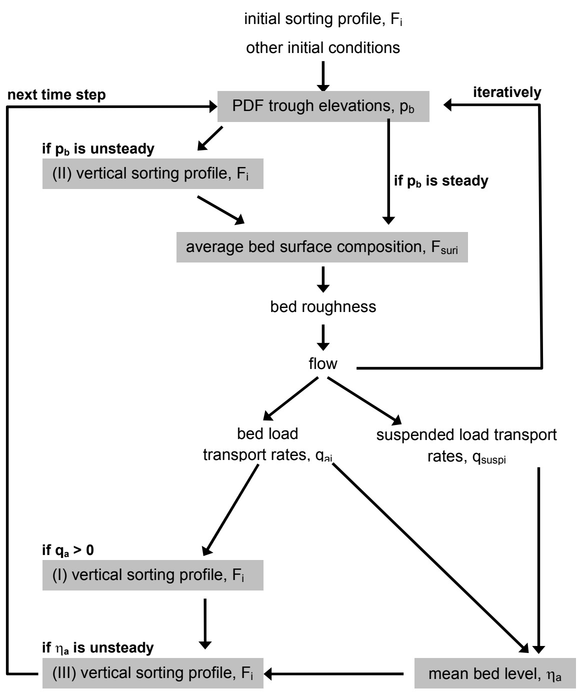

Figure 3. Scheme of the morphodynamic model system for nonuniform sediment to which the new two-layer model for sediment conservation is applied. Gray boxes represent sub-models that are part of the sediment conservation model. Evolution of the vertical sorting profile occurs through vertical sediment fluxes accompanying (I) dune migration, (II) a change in time of the PDF of relative trough elevations, and (III) net aggradation or degradation.

2.3 *Mean composition of the bed surface*

The mean composition of the bed surface needs to be known for computing skin friction, bedform height, and the grainsize-specific sediment transport rates. In the new two-layer model, the GSD of the bed surface is determined by weighting over the GSD of two active layers by their exposure to the flow, expressed by β_{AB} and β_{BC}. The mean volume fraction content of size fraction i at the bed surface, F_{suri}, then equals

$$F_{suri} = \beta_{AB} F_{ABi} + \beta_{BC} F_{BCi} \quad (7)$$

where

$$\beta_{AB} = \int_{AB} p_e dz = 0.95 \quad (8)$$

$$\beta_{BC} = \int_{BC} p_e dz = 0.05 \quad (9)$$

Note that the following constraint always needs to be fulfilled: $\beta_{AB} + \beta_{BC} = 1$.

2.4 *Fluxes through dune migration*

The GSD of the sediment flux due to dune migration between layers AB and BC, i.e. the vertical sediment flux ψ_{Bi} is computed from

$$\psi_{Bi} = \frac{1}{T_F}\left(F_{BCi} - F_{ABzi}(\eta_{97.5})\right) \quad (11)$$

where $F_{ABzi}(\eta_{97.5})$ = the volume fraction content of size fraction i at the mean elevation of layer BC, $\eta_{97.5}$, where $\eta_{97.5}$ denotes the elevation above which 97.5% of the bed surface elevations occur (i.e. $P_s(\eta_{97.5})$ = 0.975). The computation of $F_{ABzi}(\eta_{97.5})$ is explained later in this section. The time scale T_F of sediment flux ψ_{Bi} is given by Ribberink (1987):

$$T_F = \frac{c_b}{0.06} \frac{\lambda_a}{q_a} \quad (12)$$

where λ_a = mean bedform length, and q_a = mean bed load transport rate.

As bed layer AB is defined such that the sorting profile can be assumed to have reached a steady state, we can apply the equilibrium sorting model developed by Blom et al. (2006) to bed layer AB. We assume that volume fraction content of size fraction i in the sediment transported over the crest of each single dune equals the mean volume fraction content of size fraction i in the transported sediment, F_{ai}. The equilibrium sorting model by Blom et al. (2006) then provides a tool to compute the mean volume fraction content of size fraction i at elevation z within layer AB, F_{ABzi}, from

$$F_{ABzi}(z) = F_{ai} \frac{\int \frac{J(z)}{\lambda\Delta} \omega_i(z) p_b d\eta_b}{\int \frac{J(z)}{\lambda\Delta} p_b d\eta_b} \quad (10)$$

where F_{ai} = the mean volume fraction content of size fraction i in the bedload transport, ω_i = the lee sorting function, which is explained by Blom et al. (2006), η_b = relative trough elevation, p_b = Weibull probability density function of relative trough elevations, $J(z)$ = a Heaviside function which equals 1 when considering an elevation covered by bedform, λ = the bedform length, and Δ = the bedform height. We apply the formulation for lee face sorting parameter δ_i developed by Blom & Kleinhans (2006) to compute ω_i. The parameters $J(z)$, Δ, and λ are all dependent on the specific trough elevation, η_b.

3 VALIDATION AGAINST EXPERIMENTAL DATA

3.1 *The flume experiment*

The author applies the new sediment conservation model to reproducing a flume experiment in which (1) mixed sediment was used; (2) conditions with dunes prevailed; (3) net aggradation or degradation occurred; and (4) the vertical sorting profile was measured. As far as known to the author, Ribberink (1987) has been the only one who conducted such a flume experiment (i.e., experiment E8-E9). The length, width, and height of the flume's measurement section were 30 m, 0.3 m, and 0.5 m, respectively. The sediment mixture consisted of two sand fractions (grain sizes d_1 = 0.78 mm, d_2 = 1.29 mm) with very little overlap.

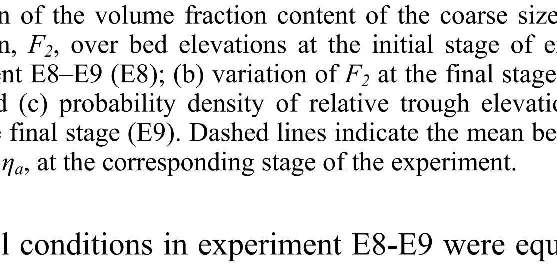

Figure 4. Data measured by Ribberink (1987) on (a) variation of the volume fraction content of the coarse size fraction, F_2, over bed elevations at the initial stage of experiment E8–E9 (E8); (b) variation of F_2 at the final stage (E9); and (c) probability density of relative trough elevations at the final stage (E9). Dashed lines indicate the mean bed level, η_a, at the corresponding stage of the experiment.

All conditions in experiment E8-E9 were equal to the ones of the equilibrium stage of the previous experiment, i.e., experiment E8, except for the grainsize-specific sediment feed rates. A downward coarsening trend characterizes the initial vertical sorting profile, which equals the equilibrium sorting profile of stage E8 (Figure 4). From the start of experiment E8-E9, the sediment recirculation system was changed to a sediment feed system. The initial feed rate was equal to the equilibrium sediment transport rate and GSD in experiment E8 (q_a= 5.64 10^{-6} m^2/s, F_{a1} = 0.5). Then, over a period of 30 h, Ribberink (1987) gradually reduced the volume fraction content of the fine size fraction in the sediment fed to the flume to zero, while the total feed rate was maintained steady. Because of technical problems, the total feed rate, q_a, decreased by about 5% over the first 30 h of the flume experiment. The duration of the experiment was 120 h.

Because of the imposed increase of coarse sediment fed to the flume, the active part of the bed started to coarsen at the upstream end of the flume. As a result, the sediment transport capacity decreased and a small degradation wave migrated in the downstream direction. As the total feed rate was steady, an aggradation wave succeeded the small degradation wave.

3.2 *Results of validation*

We now apply the following sediment conservation models in reproducing experiment E8-E9:

A. the Hirano (1971) active layer model;
B. the Ribberink (1987) two-layer model;
C. the stochastic Blom et al. (2008) sorting evolution model;
D. the Blom two-layer model presented in this paper.

Figure 5 shows that models B through D well predict the timescale of adaptation of the GSD of the transported sediment at the downstream end of the flume. It illustrates that including the stochastics in bedform geometry has a positive effect on the predicted timescale of the physical processes.

Figure 6 shows the computed time evolution of the geometric mean grain size of the active bed, d_{m95}, at various positions. Ribberink (1987) estimated the GSD of the active part of the bed by averaging over all bed material above elevation η_{95}, where η_{95} denotes the elevation above which 95% of the bed surface elevations occur (i.e. $P_s(\eta_{95}) = 0.95$). Figure 6 shows that due to the coarse sediment feeding the active part of the bed coarsened at the upstream end of the flume and a coarsening wave migrated in the downstream direction through the flume.

Although the Ribberink two-layer model shows good results, it has some shortcomings: (1) its vertical sediment exchange term was calibrated on the flume experiment in question; (2) the sediment exchange term is yet suitable for sediment mixtures composed of two size fractions only; (3) under some conditions the sediment exchange term does not conserve mass; and (4) the elliptic character of the set of equations is not eliminated completely (although the probability of becoming elliptic appears to be small). The two-layer model proposed in this paper (Model D) overcomes these problems and we can see that it suffices for this case study. For a more extensive assessment of the proposed model, the author foresees an extensive set of flume experiments under unsteady conditions. These experiments will be conducted in the Fluid Mechanics Laboratory of Delft University of Technology.

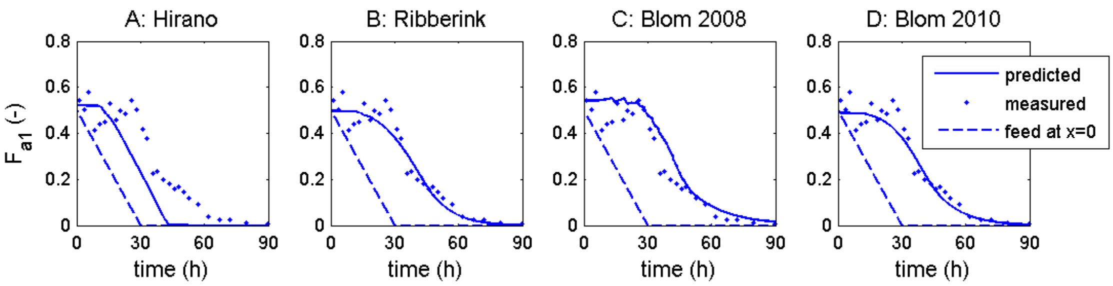

Figure 5. Time evolution of the volume fraction content of the fine size fraction in the transported sediment, F_{a1}, at $x = 28.5$ m. Predictions (lines) by (left to right) model A, the Hirano active layer model; model B, the Ribberink two-layer model; model C, the sorting evolution model; and model D, the Blom two-layer model presented in this paper. The dashed lines show the feed rate at the upstream end of the flume. Measured data (dots) originate from Ribberink (1987).

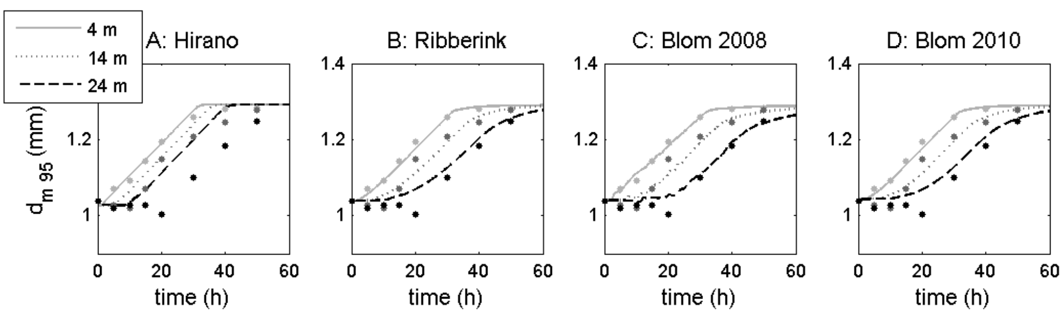

Figure 6. Time evolution of the geometric mean grain size of the active part of the bed, $d_{m\,95}$, at various positions ($x = 4$ m, $x = 14$ m, and $x = 24$ m). Predictions by (left to right) models A-D. Measured data (dots) originate from Ribberink (1987).

4 MORPHODYNAMIC RESPONSE TO OVERLOADING

Although the proposed model awaits a more extensive validation based on a new set of unsteady flume experiments under bedform-dominated conditions, we now apply the commonly applied Hirano (1971) and the new Blom two-layer model to numerical elementary overloading experiments with uniform and mixed sediment:

N1. Hirano active layer model, uniform sediment;
N2. Blom two-layer model, uniform sediment;
N3. Hirano active layer model, mixed sediment;
N4. Blom two-layer model, mixed sediment.

We simply apply the same conditions as in experiment E8-E9 (for details, see previous section and Blom, 2008), except for the initial bed material and the sediment feed rate. In the mixed sediment experiments, the sediment consists of 2 sand fractions (d_1 = 0.5 mm, and d_2 = 1 mm). The initial bed material is homogeneous and consists of 50% of each of the sand fractions. The uniform sediment experiments naturally consist of a single size fraction with the same geometric mean grain size as the mixed sediment experiments (d_m = 0.707 mm). The value for d_m is determined from:

$$d_m = d_{ref} 2^{-\phi_m} \tag{13}$$

where

$$\phi_m = \overline{F_1}\phi_1 + \overline{F_2}\phi_2 \tag{14}$$

$$\phi_i = -\log_2\left(\frac{d_i}{d_{ref}}\right) \tag{15}$$

where $\overline{F_i}$ = the mean volume fraction content of size fraction i in the bed, d_i = the grain size of size fraction i, ϕ_i = the arithmetic grain size of size fraction i, ϕ_m = the arithmetic mean grain size, and d_{ref} = the reference grain size (d_{ref} = 1 mm).

The overloading factor is set equal to 2, which means that the sediment feed rate, q_{feed}, is equal to two times the *initial* sediment transport capacity (q_{feed} = 1.57 10^{-5} m^2/s). Note that by definition the *final* sediment transport capacity equals the sediment feed rate, q_{feed}. The volume fraction content of the fine size fraction in the sediment fed to the flume is set equal to the initial volume fraction content in the transported sediment ($F_{feed,1}$ = 0.55).

Figure 7 illustrates that for the uniform sediment numerical experiments (models N1-N2) the aggradation occurs more quickly and the final amount of aggradation before reaching a new steady-state and the final slope are larger than for the mixed sediment experiments (N3-N4). For the uniform sediment experiments a larger slope is required to be able to transport the sediment fed to the flume.

Although the Hirano active layer model (model N3) shows similar results with respect to the rate and final amount of aggradation as the Blom two-layer model (model N4), the results of the Hirano model differ significantly from the ones of the Blom two-layer model with respect to the mean grain size of the bed surface (Figure 8). The time scale of changes in the GSD of the bed surface is much longer for the Blom two-layer model, which is due to the latter model incorporating of the effect of the stochastics of bedform geometry. These larger time scales are due to the fact that stochastics of bedform geometry cause sediment to be (temporarily) stored at elevations reached by relatively deep bedform troughs. This sediment only becomes available for entrainment again when a new deep troughs migrates over the area. In contrast to the Hirano model, the Blom two-layer model includes this delaying effect of bedform stochastics. Figure 9 shows the geometric mean grain size d_m of the bed material at various times. It illustrates how the development of the vertical sorting within the bed shows a very different behavior due to the inclusion of vertical sorting and bedform stochastics in the proposed Blom two-layer model.

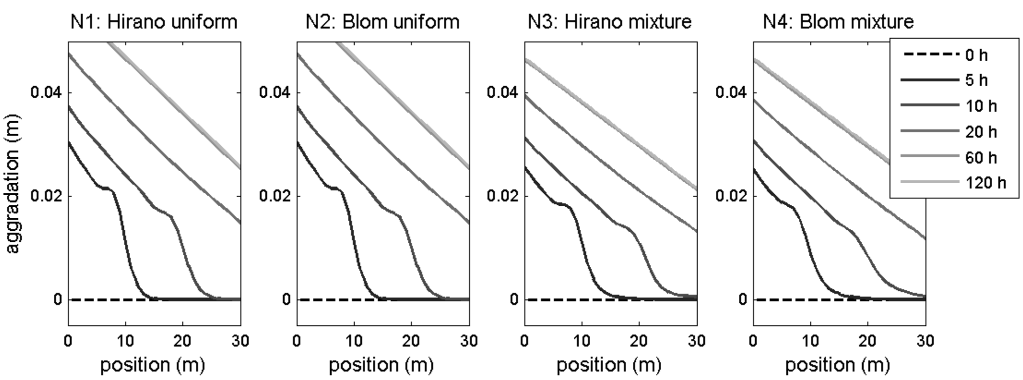

Figure 7. Variation of aggradation over space, at various times. Predictions by (left to right) models N1-N4.

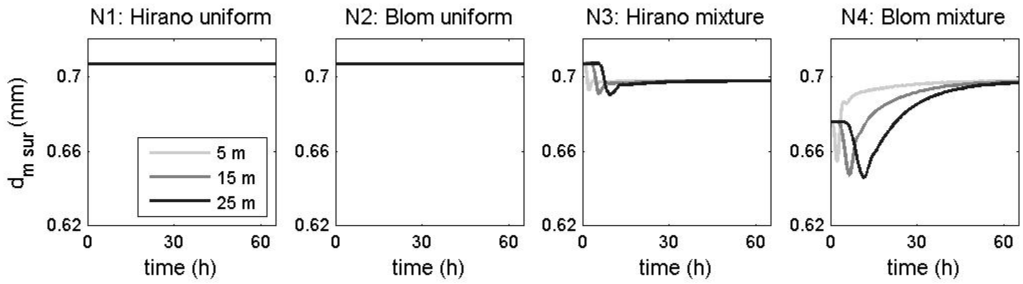

Figure 8. Variation of the geometric mean grain size of the bed surface over time, at various locations. Predictions by (left to right) models N1-N4.

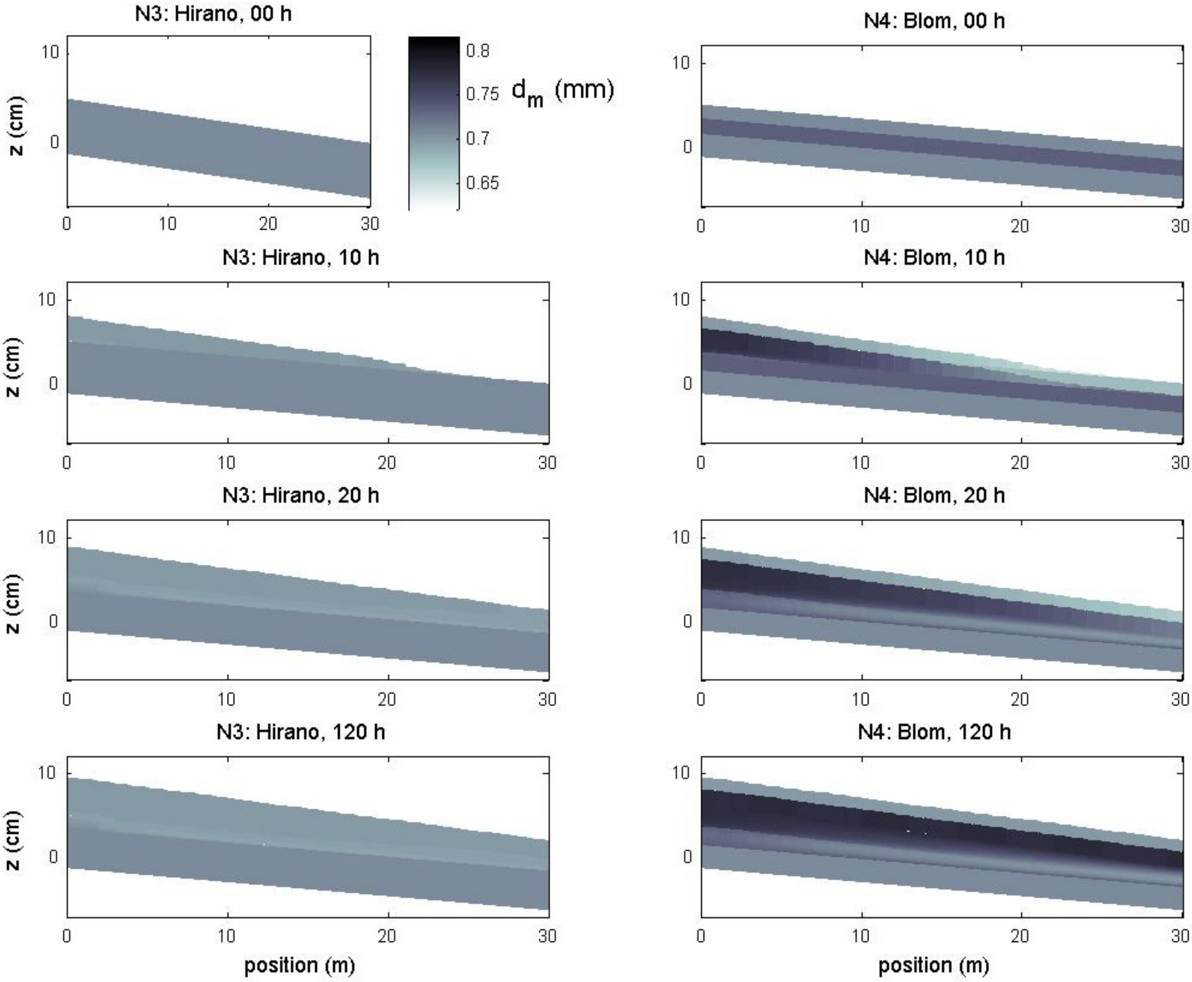

Figure 9. Predicted geometric mean grain size d_m of the bed material, at various times. Predictions by (left to right) models N3 and N4.

5 CONCLUSIONS

A new two-layer model for conservation of mixed sediment under bedform-dominated conditions is proposed. The model consists of two active layers that are exposed to the flow to a different extent. The upper active layer represents the bed material that reaches a steady-state with respect to vertical sorting instantaneously. The lower active layer represents the bed elevations that are reached by relatively deep bedform troughs only. This layer shows a larger time scale for reaching a steady-state grain size distribution. The new two-layer model has successfully been validated through reproducing the unsteady aggradational flume experiment conducted by Ribberink (1987). We then applied the model in elementary numerical predictions of the morphodynamic response to overloading. The new model shows a significantly different time scale of the adaptation of the grain size distribution of the bed surface, as well as a distinct development of vertical sorting of the bed material, compared to the commonly applied Hirano model.

REFERENCES

Blom, A., Parker, G. (2004), Vertical sorting and the morphodynamics of bed form-dominated rivers: A modeling framework, J. Geophys. Res., 109, F02007, doi:10.1029/2003JF000069.

Blom, A., Kleinhans, M.G. (2006), Modelling sorting over the lee face of individual bed forms, in River Flow 2006: Proc. Int. Conf. on Fluvial Hydraulics, edited by R. M. L. Ferreira et al., Taylor and Francis, Leiden, Netherlands, pp. 807–816.

Blom, A., Parker, G., Ribberink, J.S., de Vriend, H.J. (2006), Vertical sorting and the morphodynamics of bed-form-dominated rivers: An equilibrium sorting model, J. Geophys. Res., 111, F01006, doi:10.1029/2004JF000175.

Blom, A., Ribberink, J.S., Parker, G. (2008), Vertical sorting and the morphodynamics of bed form-dominated rivers: A sorting evolution model, J. Geophys. Res., 113, F01019, doi: 10.1029/2006JF000618.

Blom, A. (2008), Different approaches to handling vertical and streamwise sorting in modeling river morphodynamics, Water Resour. Res., 44, W03415, doi:10.1029/2006WR005474.

Crickmore, M.J., Lean, G.H. (1962), The measurement of sand transport by means of radioactive tracers, Proc. R. Soc. London, Ser. A, 266, 402– 421.

Di Silvio, G. (1992), Sediment exchange between stream and bottom: A four layer model, paper presented at International Grain Sorting Seminar, IAHR, Ascona, Switzerland.

Hirano, M. (1971), River bed degradation with armouring, Trans. Jpn. Soc. Civ. Eng., 3(2), 194–195.

Nabi, M., de Vriend, H.J., Mosselman, E. Sloff, C.J., Shimizu, Y. (2009), Simulation of subaqueous dunes using detailed hydrodynamics. In Proc. 6th IAHR symposium on River, Coastal and Estuarine Morphodynamics, Santa Fe, Argentina.

Ravenstijn, E. (2009), Behaviour of nourishments in quasi 3-dimensional graded sediment models. MSc thesis, Delft University of Technology, Netherlands.

Ribberink, J.S. (1987), Mathematical modelling of one-dimensional morphological changes in rivers with non-uniform sediment, Ph.D. thesis, Delft Univ., Delft, Netherlands.

Shimizu, Y., Giri, S., Yamaguchi, S., Nelson, J. (2009), Numerical simulation of dune–flat bed transition and stage-discharge relationship with hysteresis effect, Water Resour. Res., 45, W04429, doi:10.1029/2008WR006830.

Van der Mark, C. F., Blom, A. , Hulscher, S.J.M.H. (2008), Quantification of variability in bedform geometry, J. Geophys. Res., 113, F03020, doi:10.1029/2007JF000940.

Viparelli, E., Haydel, R., Salvaro, M., Wilcock, P., Parker, G. (in press, a), Modeling of river morphodynamics with creation/consumption of grain size stratigraphy. Part 1: Laboratory experiments, J. Hydr. Res.

Viparelli, E., Sequeiros, O., Cantelli, A., Wilcock, P., Parker, G. (in press, b), Modeling of river morphodynamics with creation/consumption of grain size stratigraphy. Part 2: Numerical model, J. Hydr. Res.