

# Development of supply-limited transport due to vertical sorting of a sand-gravel mixture

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**ABSTRACT:** If a partially mobile sediment is transported an immobile sediment layer can form below the bedforms. This immobile layer can cause a supply-limitation, i.e. the volume of mobile sediment on top of the layer available for transport becomes limited. This causes the bedforms, roughness and sediment transport to be reduced compared to alluvial conditions, i.e. the situation where all bed surface material consists of mobile sediment. We studied the development of the bed stratification in a series of flume experiments with different initial sand-gravel mixtures. In all experiments a thin immobile gravel layer developed with supply-limited bedforms on top; showing a strong similarity with the supply-limitation as observed in situations of sand transport over pre-installed flat immobile beds. Two phases were observed in the temporal development of the stratified bed: I) first a relatively quick development of dunes with gravel accumulating in the dune troughs, followed by II) a slower development of the level of the immobile gravel layer in the bed. In the final equilibrium situation the thickness and composition of the immobile layer appeared to be more or less independent of the initial sand-gravel mixture composition. However, the thickness of the mobile sediment layer (active layer) and the average dune height strongly reduced with increasing gravel concentration in the initial mixture.

*Keywords: Sediment transport, Flumes, Experimentation, River beds, Dunes, Sand,*

## 1 INTRODUCTION

Near initiation of motion, some of the coarsest size fractions in sand-gravel mixtures can become immobile (partial transport). Vertical sorting of size fractions can lead to the formation of an immobile coarse layer that prevents further entrainment of sediment from below this layer. The active layer – the layer of bed load material on top of the immobile layer that is being transported – is too thin for the bedforms to develop to the equilibrium dimensions that would be reached in fully mobile bed conditions (alluvial conditions). In this so-called supply-limited situation the limited sediment supply not only reduces the bedform dimensions, but also the bed roughness and the transport rate. Recent studies into bedform dimensions, roughness and sediment transport under supply-limited conditions have resulted in model concepts for the prediction of these variables (Tuijnder, 2010). The volume of available bedload sediment (per square meter) is a central parameter in these concepts. Accordingly, prediction of the vertical sorting and the associated volume of bed-

load sediment on top of the immobile layer is critical in this situation. However, prediction of vertical sorting is far from trivial (see e.g. Blom et al., 2006; Ribberink, 1987).

In order to be able to model the development of immobile layers and the supply-limited sediment transport over these layers, we would like to answer the following research questions I) How does the stratification and vertical sorting of a partially-mobile sand-gravel mixture evolve in time? II) Will a supply-limited condition develop that is comparable to the supply-limitation in case of sand transport over flat (pre-installed) immobile gravel beds? III) How does the initial gravel content affect a) the bedforms and active layer thickness, and b) the composition and thickness of the immobile gravel layer?

In order to gain insight into these questions and to obtain a data-set for model testing and development, we conducted a series of flume experiments at the TU-Braunschweig. These experiments are an extension to the experiments of Tuijnder et al. (2009). In these earlier experiments an immobile gravel layer was installed prior to the

experiments, while in the present experiments the bed layer system (immobile layer with an active layer on top) forms naturally. In the new experiments the initial bed is fully-mixed and thus no immobile layer and therefore no supply-limitation is initially present. Four experiments are presented in which the initial fraction of the gravel in the sediment is increased stepwise. In this way we want to study the effect of the gravel content on the vertical sorting situation that develops: the thickness and composition of the gravel layer, the thickness of the mobile sand layer above it (active layer) and the average dune height that develops. The measured bed stratification and vertical sorting is presented and interpreted considering the requirements of a morphological (layer) model that would be able to represent these processes. First, the set-up and procedure of the experiments are presented in Section 2. Next, the measured temporal development of the bed stratification is presented in Section 3. In Section 4, detailed gravel concentration measurements are presented. These measurements give further insight into the structure of the gravel layer that developed in the final equilibrium situation. Finally, in Section 5 the consequences of the observations for morphological modeling are discussed.

## 2 EXPERIMENTS

### 2.1 Set-up

The experiments were conducted in a 30 m long and 1 m wide sediment recirculating flume at the Leichtweiss-Institute in Braunschweig. The set-up is schematically shown in Figure 1. A constant head tank above the flume supplied the flume with water. The discharge in the supply pipe was measured with an IDM, an Inductive Discharge Measurement device. The accuracy of this device is approximately 1% of the measured discharge according to its specifications. A funnel caught the sediment discharge at the end of the flume. The sediment was pumped back to the upstream end with a small water discharge ( $\pm 20$  l/s) using a recirculation pump where the sediment-water mixture was distributed over the width of the flume. In the return pipe another IDM measured the discharge.

Over a length of 17.45 m the bed and water level were measured continuously using ultrasound (echo) sensors that were mounted on a measurement carriage. The measurements were taken every 3 minutes from  $x = 6.3$  m to  $x = 23.75$  m, where  $x = 0$  is located at the upstream end of the flume, at the location of the pivoting point.

The carriage measured the bed level at three parallel transects at  $y = 0.165$ ,  $0.500$  and  $0.835$  m, where  $y = 0$  is the left sidewall when looking downstream. The flume bottom is  $z = 0$ . The water level was measured approximately in the centre of the flow.

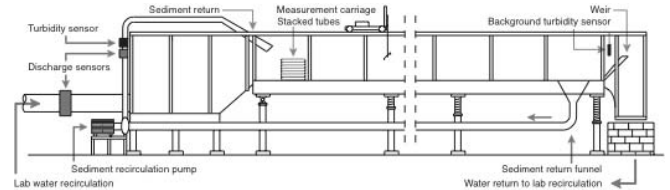


Figure 1: Side view of the used flume setup. The supply of water came from a constant-head tank ca 5 m above the level of the flume. The flume is supported by jacks that allow the flume to be tilted to realize an equilibrium flow.

### 2.2 Conditions

Between the experiments, the gravel content of the sand-gravel mixture was varied. A uniform gravel and sand fraction were mixed in varying proportions (see the sediment characteristics in Table 1).

We chose the experimental conditions in such a way that these experiments form an extension of earlier experimental work (Tuijnder et al., 2009). The conditions of these new experiments are the same as the conditions in Series 1 of Tuijnder et al. (2009). The water depth was 0.2 m and the flow velocity was 0.52 m/s. The bed slope was adjusted during the initial adjustment phase to maintain uniform conditions for a varying roughness.

Table 1. Sediment characteristics of the sand, gravel and the mixtures used in the experiments.

	$f_g$ (%)	$D_{10}$ (mm)	$D_{50}$ (mm)	$D_{90}$ (mm)	$\sigma_g$ (-)
Sand	0%	0.6	0.8	1.2	1.3
Gravel	100%	7.8	10.9	15.4	1.3
Exp 1	5%	0.6	0.9	1.3	1.9
Exp 2	10%	0.6	0.9	4.1	2.3
Exp 3	15%	0.6	0.9	9.7	2.6
Exp 6	20%	0.6	0.9	10.9	2.9

### 2.3 Procedure

Before the experiments, the sand and gravel were thoroughly mixed in the flume and distributed evenly over the length of the flume. The layer had a constant thickness and was screeded flat. An initial bed slope for equilibrium flow was set using the flume tilting mechanism and the flume was slowly filled with water. Once the required water level was reached, the required discharge was set and the measurements were started.

Every three minutes a measurement profile was made showing the water level, bed level, depth

and dune height. With this data, uniform flow was maintained by adjusting the flume slope and downstream weir. Once equilibrium conditions were established, the setup could continue measuring automatically without further adjustments.

The duration of each experiment was two days, apart from Exp 4, which was stopped after 30 hours. After the experiments, the bed was photographed, an additional bed level measurement was taken and the gravel concentration was measured at several locations along the flume.

#### 2.4 Gravel concentration measurements

We measured the vertical level of each gravel particle within a 15 x 15 cm square metal frame. This frame was placed on the bed on different locations along the centre line of the flume. The sand was carefully excavated and the level of the top of each gravel particle was measured with a point gauge before removing the gravel particle. The frame was gradually lowered to make sure that a constant sampling surface was maintained. This procedure was continued until the bed was excavated until 8 – 10 cm below the bed surface.

The sampling volume ( $V_s$ ) and the number of gravel particles in a sample ( $n$ ) are known and therefore the gravel concentration ( $c$ ), expressed as a volume fraction of the total volume can be calculated as:

$$c = \frac{nV_g}{V_s} \quad (1)$$

Herein  $V_g$  is the average volume of a gravel particle (measured under water by water volume increase). The porosity of the sediment needs to be taken into account in order to calculate the gravel fraction  $f_g$ , i.e. the gravel volume relative to the total sediment volume.

The porosity of a sediment mixture with two size fractions varies with a varying composition. Yu and Standish (1987) developed a model to predict this porosity variation (Frings, 2008). We calibrated the Yu-Standish model, using a series of porosity measurements for different sand-gravel mixtures with the sediments used in our flume experiments. The calibrated porosity model that is needed to convert the gravel concentration ( $c$ ) into a gravel fraction ( $f_g$ ) reads as:

$$V = \frac{1}{1-\varepsilon} = \begin{cases} -1.70f_s + 1.75 & \text{if } f_s < 0.3 \\ 0.42f_s + 1.11 & \text{if } f_s \geq 0.3 \end{cases} \quad (2)$$

In this formula  $V$  is the specific volume,  $\varepsilon$  is the porosity and  $f_s$  is the sand fraction. With this porosity the gravel fraction was calculated using:

$$f_g = \frac{c}{(1-\varepsilon)} \quad (3)$$

### 3 IMMOBILE LAYER DEVELOPMENT

Under the chosen experimental conditions, the sand fraction is mobile and the gravel fraction is immobile. However, the critical shear stress for initiation of motion for a grain size fraction depends on the total composition of the sediment mixture. Hiding and exposure effects can decrease the critical shear stress for the gravel and increase it for the sand. At the start of the experiments, the gravel particles were mainly supported by the surrounding sand. This situation is called a matrix supported situation. If the sand is eroded, the gravel becomes exposed and is transported, or sinks to a lower level in place. The gravel particles are deposited in the troughs of the dunes that start to develop from the start of the experiment. On the dune crests the bed shear stress is high and the gravel particles do not find a stable resting place. However, in the troughs the probability of deposition is much larger, because of the smaller local bed shear stress, the coverage by sand-avalanches from the lee-side of the dune and the presence of more gravel particles. With dune dimensions growing in time, the concentration of gravel particles in the dune troughs increases. At the same time the threshold of motion of this gravel layer increases slowly because the gravel concentration increases. As a result of this vertical sorting process the transport rate of the gravel gradually decreases to zero.

The gravel layer protects the underlying sand against erosion and reduces the rate of entrainment of new sand into the layer above the gravel layer. The gravel layer in the dune trough can develop until it is strong and dense enough to prevent further entrainment of sand. The volume of mobile sediment on top of the gravel layer may still be smaller than required for alluvial conditions after the vertical exchange process. This total volume above the gravel layer is expressed per square meter by the parameter  $d$ . The conditions are supply-limited if  $d$  is smaller than needed for alluvial conditions ( $d < d_0$ ). In that case and the dune height, bed roughness and sediment transport are reduced compared to alluvial conditions.

#### 3.1 Determining the gravel layer level

We want to know the level of the gravel layer in order to know the volume of sediment available for bedform formation and sediment transport. The level of the immobile layer has been deter-

mined with the excavation method, but this only gives the level at a few locations in the final situation. We also determined the level of the gravel layer from the bed level measurements. We assume – and show later on in this paper – that the gravel layer lies at the level of the deepest dune trough that has occurred at each location. Because of the high measurement frequency relative to the development and migration speed of the dunes we have measurements from which we can select the deepest trough at each location. The advantage of this method is that it gives insight into the development in time towards the equilibrium sorting profile and it gives additional spatial information of the immobile layer level. Below, we will first present the results of this ‘deepest trough’ method since this can be used to study the temporal development towards the equilibrium conditions. Subsequently, we will present the equilibrium sorting profiles and compare the results of the excavation method and the deepest troughs method.

### 3.2 Growth to equilibrium

Each of the experiments was started with a flat bed. During the experiment the dune height slowly increased in time and finally reached an equilibrium dune height. We determined the average dune height using a zero-crossing method (Van der Mark and Blom, 2007). Figure 2 shows the temporal development of the average dune height with time intervals of 1 hour for each of the experiments. This figure shows that the dune height increases during approximately 24 hours and then remains approximately constant. Additionally, it shows that the dunes grow to a larger equilibrium dune height if the gravel concentration in the initial sediment mixture is lower. An alluvial experiment (without gravel) with the same flow velocity and water depth resulted in a dune height of 0.08 m (Tuijnder et al., 2009). This shows that in all four experiments a supply-limitation has developed because of the gravel layer that was formed.

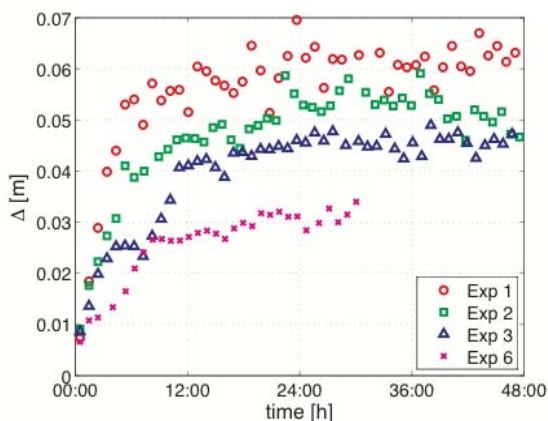


Figure 2. The increase of the dune height in time for the 4 experiments.

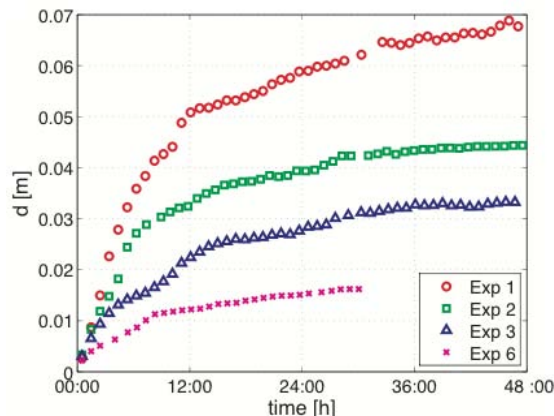


Figure 3. The increase of active layer thickness  $d$  in time for the 4 experiments.

In Tuijnder et al. (2009) it is shown that the average thickness of the sand layer on top of an immobile layer ( $d$ ) determines the relative dimensions to which the dunes can grow under supply-limited conditions. A ‘bedform dimension reduction’ (BDR-) function was presented, which describes the reduction in the dune height because of supply limitation relative to the alluvial dune height. This function reads as:

$$\frac{\Delta}{\Delta_0} = 1 - \exp\left(-\frac{d}{0.39\Delta_0}\right) \quad (2)$$

In this function  $\Delta_0$  is the dune height that occurs with the same water depth and flow velocity but without the presence of gravel. This function has been developed for equilibrium conditions.

In order to see whether this model concept is also applicable in the dune development stage we estimated  $d$  during the development stage. We applied the ‘deepest troughs’ method to determine to what depth the sediment had been active until then as a function of time. The average bed level minus this ‘deepest trough level’ provides an estimate of the average layer thickness  $d$ . Figure 3 shows that  $d$  initially increases quickly, simultaneously with growth of the dunes (compare with Figure 2). The initial rate of increase is smaller for the experiments with more gravel. After this quick initial adjustment process (in the first 6 – 12 hours), the rate of increase reduces. The adjustment process proceeds with a lower rate until the final equilibrium (until 48 hours or longer). The results indicate that the entrainment rate of new sediment from below the gravel layer decreases with an increasing gravel concentration in the gravel layer and increasing depth of the gravel layer. The layer thickness approaches a constant value towards the end of the experiments.

The hourly values of  $d$  and  $\Delta$  are compared to the BDR-function in Figure 4, which shows the relative dune height  $\Delta/\Delta_0$  against the relative sediment availability  $d/\Delta_0$ . Two dashed lines are shown; one shows the relation  $\Delta = 2d$ , the other

the BDR-function. The figure shows that development of the dune height follows the BDR-function until the gravel layer becomes sufficiently strong to stop further entrainment of sand from below the layer. The location of this final equilibrium point (with supply-limitation) on the the BDR-function is different for each experiment and has a clear relation with the initial gravel content. The more gravel was initially present, the sooner the development of  $d$  and  $\Delta$  stops.

Initially, the dunes develop along the relation  $\Delta = 2d$ . Considering that dunes are generally approximately triangular this suggests that all available sand above the gravel layer is effectively used for dune formation and regularly entrained. As the layer thickness  $d$  increases, an intermediate sand layer starts to develop on top of the gravel layer that is entrained only occasionally. Figure 4 shows that this occurs if  $d/\Delta_0$  is between 0.2 and 0.4. We will call this layer the exchange layer (after Ribberink, 1987). From this point on the relation  $\Delta = 2d$  and the BDR-function start to deviate since the sand in the exchange layer does not contribute to the dune height very effectively ( $\Delta < 2d$ ).

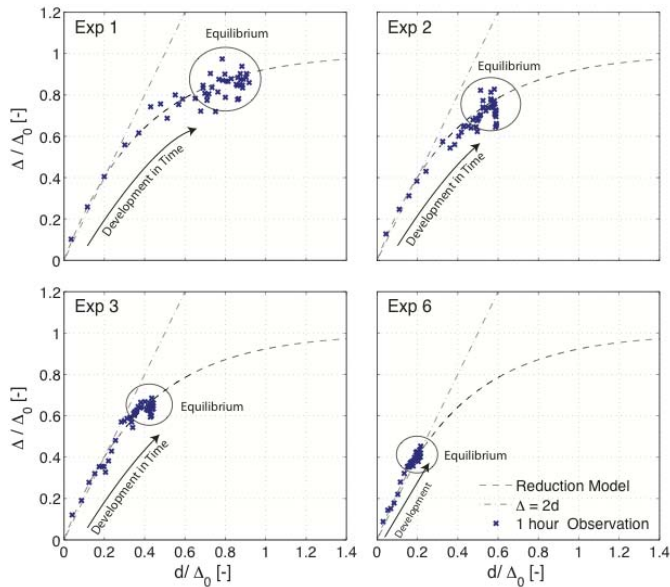


Figure 4. The development of the relative dune height as a function of the relative layer thickness.

#### 4 FINAL BED STRATIFICATION

In the final equilibrium conditions, the average dune height remains constant and the immobile layer has fully developed. We studied the vertical sorting profile that developed in the end situation with the excavation method and the deepest trough method. The results of these methods are compared in Figure 6. This figure shows a longitudinal bed level profile of the end situation of the experiments. The level of the immobile layer, as

obtained from the deepest trough method, is shown in a (spatially) filtered and in an unfiltered way. The echo sensor does not pick up the tops of the gravel particles, but the stronger sound reflections from a few millimeter beneath it where the bed porosity is smaller. Tests showed that the echo sensor measures approximately a half gravel diameter ( $D_{gr}$ ) below the top of the immobile layer. The fat solid line shows the filtered level of gravel, the thin gray lines parallel to it are  $\frac{1}{2}D_{gr}$  above and below it. The thin irregular line through it shows the unfiltered level measurements.

The gravel content, as measured in the excavations, is shown in the overlay graphs at the location in the flume where they were measured. The vertical axis of each overlay graph indicates the sampling level. On the x-axis the gravel fraction  $f_g$  is shown. The dashed vertical line in this graph shows the initial gravel content of the fully-mixed bed.

Figure 6 shows that the peak of the gravel content is generally observed at the deepest-trough level. Furthermore it can be seen that the dune troughs in Exp 1 only occasionally reach the gravel layer and that the gravel layer has indeed developed at the deepest trough level that occurred at that location until then. Comparison to the experiments 2, 3 and 6 reveals that the immobile layer develops at a higher level within the bed if more gravel is present initially. At the same time a smaller volume of sand is present above the gravel layer and the supply-limitation is stronger.

The increase in  $d$  and  $\Delta$  with decreasing initial gravel fraction in the sediment mixture is shown in Figure 5. It can be seen that  $d$  increases more rapidly than  $\Delta$ . This is caused by the formation of the exchange layer as discussed above. The available sand ( $d$ ) is used less efficiently for dune formation.

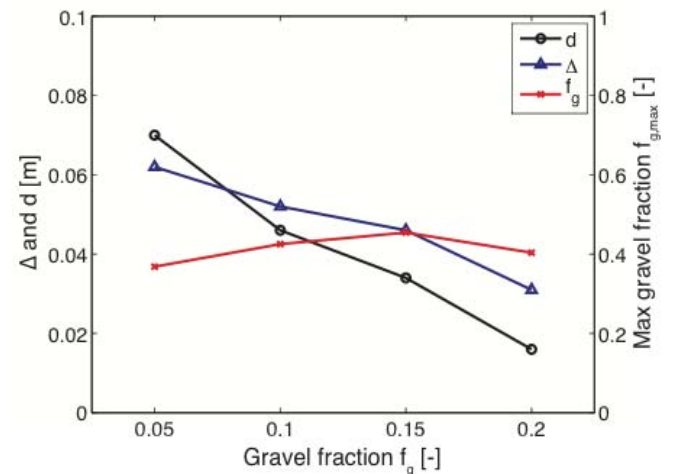


Figure 5. Variation in the parameters  $d$ ,  $\Delta$  and  $f_{g,max}$  with varying initial gravel content.



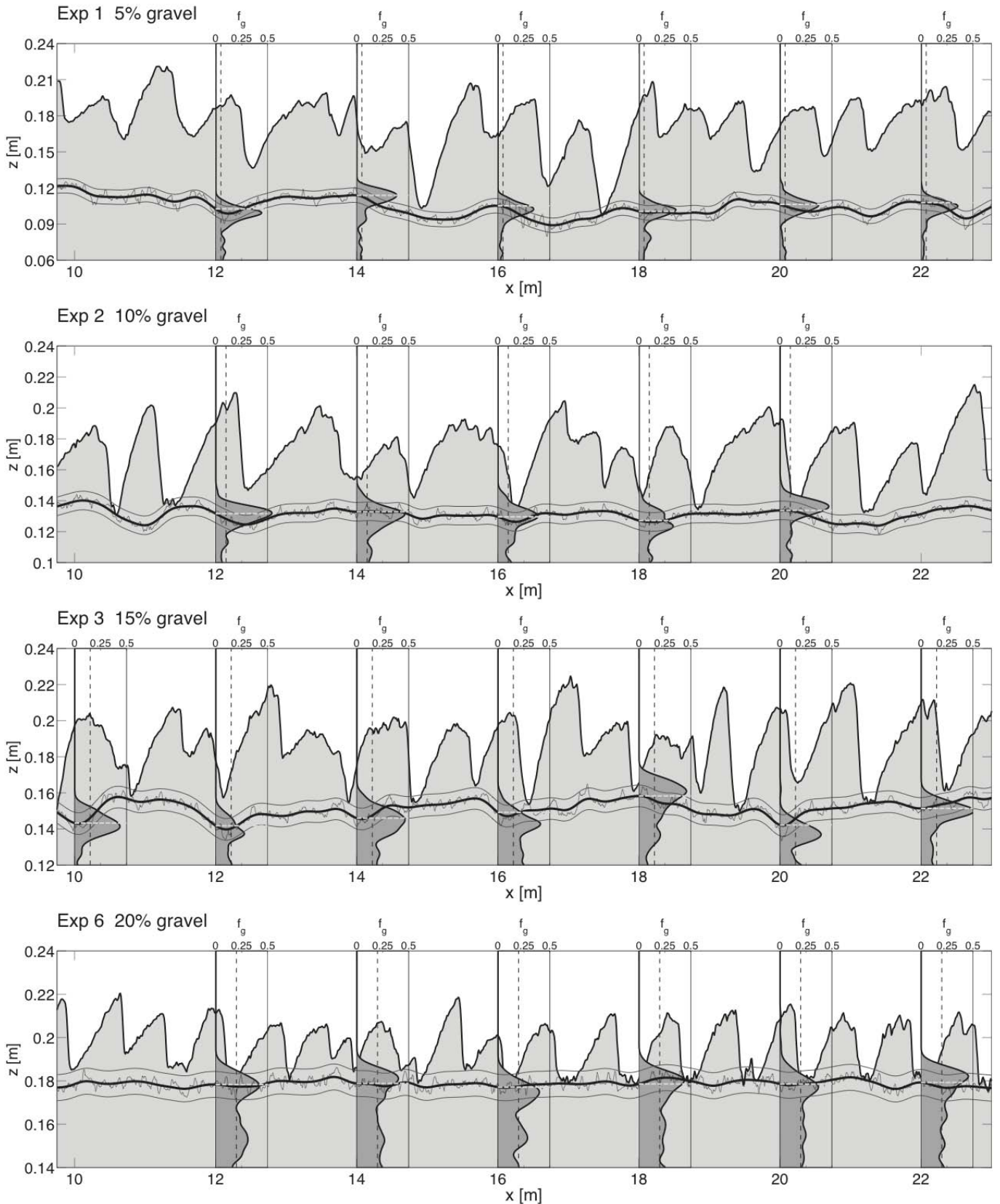


Figure 6. Bed level profile showing the position of the gravel layer and the measured gravel fraction profile in the overlaying graphs. The left side of the overlaying graphs indicates the x-coordinate of the measurement in the flume. The fat solid line below the dunes shows the filtered level of gravel, the thin gray lines parallel to it are  $\frac{1}{2}D_{gr}$  above and below it. The irregular line through it shows the unfiltered bed level measurements.

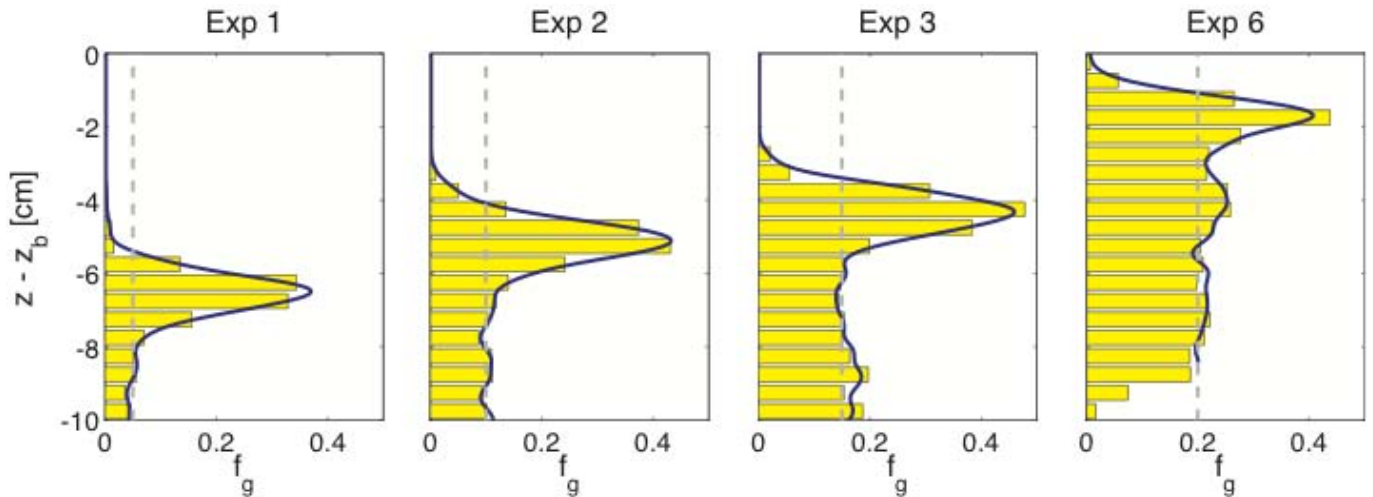


Figure 7. Histogram with smoothed trend line of the average gravel fraction in the bed as function of vertical distance to the average bed level.

#### 4.1 Structure of the gravel layer

Figure 6 shows the individual gravel fraction profiles. Each profile is different, even in the undisturbed still fully-mixed bed below the gravel layer, due to small imperfections in the mixing and sampling process. Below the gravel layer, the gravel fraction should equal the gravel fraction of the initial sediment mixture since this sediment never moved after installation. The measured values vary around the initial gravel content. In order to remove this variation, we compose an average profile by taking the average over all profiles of an experiment. Figure 6 also shows that the peaks of the individual profiles are not at the same level with respect to the mean bed level. The comparison with the deepest trough levels shows that this is caused by the wavy nature of the gravel layer. For the average gravel fraction profile we therefore shift the profiles up or down, matching the gravel fraction peaks at the average peak level. Figure 7 shows the result of the averaging of the gravel profiles. The zero-level is the average bed level over the reach 12-23 m.

Figure 7 shows that the gravel layer has a thickness of approximately 1.5 – 2 cm. Compared to the average grain size of the gravel layer of 12 mm this is surprisingly thin. The maximum gravel fraction is approximately constant, with a value of 0.4, and does not show a systematic increasing or decreasing trend with the initial gravel content (see Figure 5). The most compact packing of the sand-gravel mixture occurs if the gravel particles form a clast supported framework with the void space completely filled with sand. This is the situation that occurs if the sand fraction is approximately 0.3 according to the calibrated Yu-Standish model (Eq. 2). The sand fraction in the gravel layer is larger, which suggests that the

closest packing of the gravel in the gravel layer is not reached. These measurements indicate that the gravel layer is mainly a single row of loosely packed gravel particles, with some scattered gravel particles above and below it.

## 5 IMPLICATIONS FOR MORPHOLOGICAL MODELLING

The goal of these experiments was to gain insight into, and to obtain a set of quantitative data on: I) the bed stratification and vertical sorting process and II) the development of supply-limitation if a sand-gravel mixture is transported. A follow-up goal of the research is to apply these new insights for the development of a morphological layer model concept that would be able to represent these processes.

A number of implications of the results presented above for morphological modeling can be mentioned here:

1) Supply-limited bedforms which develop naturally by vertical sorting and bed stratification show a strong similarity with those observed above pre-installed flat immobile beds. Supply-limited sediment model concepts as developed for the latter situation (Tuijnder, 2010) may therefore also be applicable for the former situation.

2) Depending on the degree of supply-limitation three bed layers can be distinguished, an active layer, an exchange layer and an immobile layer, each having a different function in the vertical exchange process.

3) Possibly the 2-layer model concept as suggested by Ribberink (1987) or the diffusion model of Armanini (1995) can be extended for this purpose. See also Blom (2008) for a discussion on this issue, and Sloff and Ottevanger (2008) for some first experiences with a similar approach.

4) Additional sub-models will be needed for I) the vertical sediment fluxes between the layers, II) the (equilibrium) thickness and composition of the layers, and III) the mobility of size fractions in the layers (hiding / exposure effects). The data and insights obtained from the present experiments can contribute to the development / validation of these sub-models.

5) Apart from the modelling of layer development from fully-mixed sediment beds, a further challenge will be to model the transition from one layer system to the next, which should also include the possible breaking-up of immobile layers.

## 6 CONCLUSIONS

A set of new laboratory experiments with different sand-gravel mixtures showed that a small percentage of immobile gravel in the initial mixture is able to induce supply-limitation with strongly reduced bedform dimensions, bed roughness and sand transport.

The BDR-function derived for supply-limited situations in case of pre-installed immobile layers (Tuijnder et al., 2009) is also applicable for (thin) immobile layers as formed naturally by vertical sorting. This agreement was found during the development process of the dunes as well as in the final supply-limited equilibrium situation.

The new data show the following characteristics of the immobile layer in the final equilibrium situation: I) The immobile gravel layer has a thickness of approximately 1.5-2 gravel diameters and has a gravel content of approximately  $f_g = 0.4$ . These values are more or less constant and not influenced by the dunes on top of the immobile layer. The dunes differed considerably in size and shape between the different experiments. II) With increasing gravel content the level at which the immobile layer is formed is higher and the supply-limitation is stronger. If the gravel content is high (20%) the transport layer forms directly on top of the gravel layer.

These characteristics confirm the idea that the growth of dunes and of the active layer can only come to an end after a certain critical volume of immobile gravel - sufficient for the formation of a more or less continuous single gravel layer - has been exposed to the flow. For higher gravel contents of the initial mixture this critical volume is reached earlier.

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