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Numerical modeling of sediment transport in the Danube River: uniform vs. non-uniform formulation

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ABSTRACT: This paper presents the application of iSed, an integrated numerical sediment transport and morphology model, to a section of the Danube East of Vienna. The model is coupled with external 2-D or 3-D hydrodynamic codes to obtain the flow field and bed shear stress patterns driving sediment transport processes. It solves a convection-diffusion equation to account for suspended load; various empirical formulae have been included to calculate bed load transport, the Meyer-Peter/Müller equation being employed in the case of the Danube River. All equations are evaluated for an unlimited number of sediment size fractions, allowing the consideration of sorting processes. The numerical model was calibrated for the Danube River based on mean values of suspended load measurements using a US-P-61 suspended sediment sampler and bed load measurements using a basket sampler. Numerical modeling was performed employing both a uniform and a non-uniform formulation of the Meyer-Peter/Müller equation including hiding-exposure corrections. It was found that the uniform equation largely underestimates transport rates for discharges lower than mean flow as it predicts a later onset of bed load transport, while the nonuniform equation yields results in consistence with the measurements. For higher discharges, the uniform equation was found to overestimate the actual bed load transport.

Keywords: Sediment transport, Numerical models, Bed load movement, Empirical equations, Danube River

1 INTRODUCTION

Various numerical models for the prediction of sediment transport in rivers have been developed in the past, characterized by different dimensionality and degree of sophistication. For a large number of engineering problems, the use of one- and two-dimensional models proved successful (Zeng et al., 2008), while sediment transport processes under the influence of secondary currents could only be modeled correctly when parameterizing the vertical flow structure (Minh Duc et al., 2004) or employing a 3-D model (Olsen, 2003). However, practically all numerical models for sediment transport have in common that they also compute the flow field required as basis for transport processes, thus effectively restricting the applicability to the dimension and inherent limitations of the underlying hydrodynamics. In this study the sediment transport model iSed (Tritthart et al., 2009) is employed, which is coupled with external 2-D or 3-D hydrodynamic codes capable of starting from a previous solution (hot-start), and is therefore not subject to limitations imposed by a specific implementation of hydrodynamics.

While many of the transport formulae implemented in numerical models were originally derived for uniform sediment, in recent years a trend towards consideration of actual grain-size distributions using a non-uniform approach is notable. Often this is achieved by introducing a correction factor for hiding and exposure in bed load transport equations (Wu et al., 2000a) or by the solution of the governing equations for every size fraction of suspended sediment (Guo & Jin, 2002). Sun & Donahue (2000) presented a statistically derived bed load formula for non-uniform sediment; a stochastic approach towards considering hiding and exposure effects was taken in Wu et al. (2000b). Non-uniform sediment transport formulations were found to yield results in better agreement with measurements of bed elevation changes both using laboratory data (Fischer-Antze et al., 2009) and field data (Hung et al., 2009). However, a direct comparison of measured and predicted transport rates was rarely conducted so far, particularly not in the context of comparing uniform and non-uniform sediment transport formulae.

In this paper both the uniform Meyer-Peter/Müller bed load transport equation, as well as a modification intended to account for hiding and exposure effects (ATV-DVWK, 2003) and applied to several grain size fractions (Tritthart et al., 2009) are employed to a stretch of the Danube River East of Vienna. The results of the different formulations are compared to each other and to bed load transport measurements.

2 INTEGRATED SEDIMENT TRANSPORT MODEL (ISED)

Sediment transport computations are usually based on sediment properties as well as hydrodynamic properties, such as bed shear stress patterns, flow velocities and water depths. The iSed sediment transport model is coupled with an external hydrodynamic code in order to obtain these properties, which are then available for every time step (Tritthart et al., 2009). The transport model acknowledges various different mesh types and can therefore be coupled with either 2-D or 3-D hydrodynamic models. In the case study presented here, the sediment transport model is coupled with the RSim-2D hydrodynamic code, which provides a Finite Element solution to the shallow water equations on a triangular mesh, embedded into the RSim modeling framework (Tritthart, 2005).

2.1 *Bed load transport*

Bed load as the dominant form of transport in gravel bed rivers is calculated by the evaluation of empirical formulae in the iSed model. In the case study presented in this paper, the Meyer-Peter/Müller transport equation was used, both in its standard uniform formulation (Meyer-Peter & Müller, 1948) and including a hiding-exposure correction in order to account for non-uniform sediment transport (ATV-DVWK, 2003). The uniform formula is implemented as:

$$q_s = c_{MP}\sqrt{\frac{\rho_s-\rho}{\rho}gd_m^3\left[\frac{u^{*2}}{\frac{\rho_s-\rho}{\rho}gd_m}-\theta_c\right]^3} \quad (1)$$

where q_s denotes the overall bedload transport capacity, ρ_s the sediment density, ρ the density of water, and g the gravitational acceleration. u^* denotes the shear velocity and d_m the mean diameter of the sediment. While the original formulation of the Meyer-Peter/Müller equation is based on the mean diameter, there exist also formulations based on the median diameter d_{50} (ATV-DVWK, 2003). Substituting d_m by d_{50} has also shown to improve sediment transport calculations in the case of highly non-uniform sediment (Hauer et al., 2010). The critical mobility parameter θ_c can either take a fixed value (usually θ_c = 0.047) or is alternatively calculated from a parameterization of the Shields curve. Although originally determined as c_{MP} = 8, in practice the pre-factor c_{MP} requires calibration (Habersack & Laronne, 2002) and usually ranges between 2 and 8 (ATV-DVWK, 2003).

The non-uniform formulation of the Meyer-Peter/Müller equation, including a hiding-exposure correction and suitable for several size fractions, is implemented as:

$$q_{si} = p_i c_{MP}\sqrt{\frac{\rho_s-\rho}{\rho}gd_i^3\left[\left(\frac{d_i}{d_{ref}}\right)^{\alpha}\frac{u^{*2}}{\frac{\rho_s-\rho}{\rho}gd_i}-\theta_c\right]^3} \quad (2)$$

where p_i is the mass percentage occupied by grain size fraction i in the sediment mixture, d_i the mean diameter of the sediment fraction and d_{ref} a reference diameter, selected as d_{ref}=d_{50}. The influence of hiding and exposure is dominated by the exponent α which must be calibrated for a river reach and usually takes values in the range of 0 (no correction) to 1 (linear correction).

2.2 *Suspended sediment transport*

Transport of suspended sediment follows a convection-diffusion equation which incorporates an exchange term to account for interaction with the river bed:

$$\frac{\partial c}{\partial t}+\frac{\partial(u_1c)}{\partial x_1}+\frac{\partial(u_2c)}{\partial x_2}=\frac{\partial}{\partial x_1}\left(K_t\frac{\partial c}{\partial x_1}\right)+ \\ +\frac{\partial}{\partial x_2}\left(K_t\frac{\partial c}{\partial x_2}\right)+s_{dep}-s_{ero} \quad (3)$$

In Equation (3), c denotes the suspended sediment concentration, u_j the flow velocities in the corresponding coordinate directions x_j, K_t the depth-averaged diffusion coefficient, and t the time. s_{dep} is the deposition flux and s_{ero} the erosion flux with respect to the sediment layer at the river bed. The deposition flux is estimated from the depth-averaged suspended sediment concentration using the approximation of the concentration profile given by van Rijn (1984); the erosion flux is calculated according to Garcia and Parker (1991). Equation (3) is evaluated using the Finite Volume Method for every size fraction i smaller than 1 mm, yielding the suspended sediment concentration for every node of the computation mesh.

2.3 *Bed evolution*

The temporal evolution of the bed elevation at every computation node is calculated for every size fraction i following the Exner equation:

$$\left(1-n_p\right)\frac{\partial z_i}{\partial t}+\frac{\partial q_{si,x}}{\partial x}+\frac{\partial q_{si,y}}{\partial y}=s_{dep,i}-s_{ero,i} \qquad (4)$$

where z_i is the vertical change in the bed elevation due to sediment transport processes in the fraction i; $q_{si,x}$ and $q_{si,y}$ represent the bed load transport rate, split into coordinate directions x and y, according to the direction of the near-bed flow vector; n_P is the pore content of the sediment. The right-hand side of Equation (4) denotes the balance between deposition and erosion flux due to suspended sediment transport.

The equation is solved applying the Finite Volume Method on an arbitrarily shaped control volume surrounding the computation node, applying the numerical technique described in Tritthart & Gutknecht (2007), thus eventually yielding the vertical bed level changes for every grain size fraction.

2.4 *Grain sorting*

Grain sorting follows the theoretical concept of applying an exchange layer at the interface between river bed and water column where all mixing processes take place. The width of this layer, which is usually in the range of d_{90} to 4 x d_{90} of the bed material, is a model calibration parameter and influences the coarsening and fining of the bed sediment due to erosion and deposition processes, respectively (Tritthart et al, 2009). In case of erosion, sediment from lower subsurface layers is added to the mixing layer; if deposition is occurring, the deposited material is added while the layer always retains the same thickness. Thus, a new grain size distribution is obtained for every time step.

3 CASE STUDY AT THE DANUBE EAST OF VIENNA

3.1 *Integrated River Engineering Project*

The Austrian Danube River East of Vienna (Figure 1b) is characterized by several conflicting interests: (i) ecology, as most of the river reach is part of a national park; (ii) navigation, as the Danube river corridor is part of an international waterway; (iii) flood protection and river engineering.

Due to the retention of sediments in upstream reservoirs and the prevention of side erosion by stabilized banks, a significant sediment deficit is present in the river reach, consequently leading to ongoing river bed degradation. Observations (Donauconsult, 2006) demonstrate an average bed degradation rate of 2-3.5 cm per year, leading to a strong down-cutting of the river into its alluvium, eventually causing a river bed break-through into fine, marine deposits (Habersack et al., 2007).

Furthermore, the river section is characterized by ecological deficits, as side arms have been disconnected and morphodynamic processes restricted since a major river restoration scheme in the late 19th century. Concerning navigation, the minimum water depth required for an international waterway is frequently not met during low flow periods, particularly in areas of fords. Hence, yearly ford dredging is performed to ensure ship passage, which is not a sustainable measure (Habersack et al., 2007).

In order to combine all the interests and to solve the problems for all stakeholders, the "Integrated River Engineering Project on the Danube East of Vienna" was initiated. Several measures will be undertaken, such as the removal of bank protection structures, the reconnection of side arms and the modification of groin structures in shape and crest elevation. The problem of ongoing river bed erosion will be tackled by a granulometric bed improvement: by adding larger gravel sizes within the natural grain size spectrum it is anticipated to reduce sediment transport to about 10 to 15% of its current value. The added material will have a dimension of 40/70 mm which is larger than the average size but smaller than the largest grain of the natural sediment of the Danube River in the project reach. Initially, a layer of 25 cm thickness will be superimposed on defined areas of the river bed, which is expected to mix with the normal load in the following years. Thus a paved river bed with undesirable effects for benthic life will be prevented (Liedermann et al., 2006).

Measures will be first implemented in a test reach of 3 km length (Figure 1a), comprising a nature-scale experiment. In order to obtain insights into the processes taking place and to ensure success of the measures planned, a comprehensive monitoring campaign has been conducted. Lessons learned from the monitoring and evaluation of these measures will be considered for the entire river reach throughout the future course of the project (Tritthart et al., 2008).

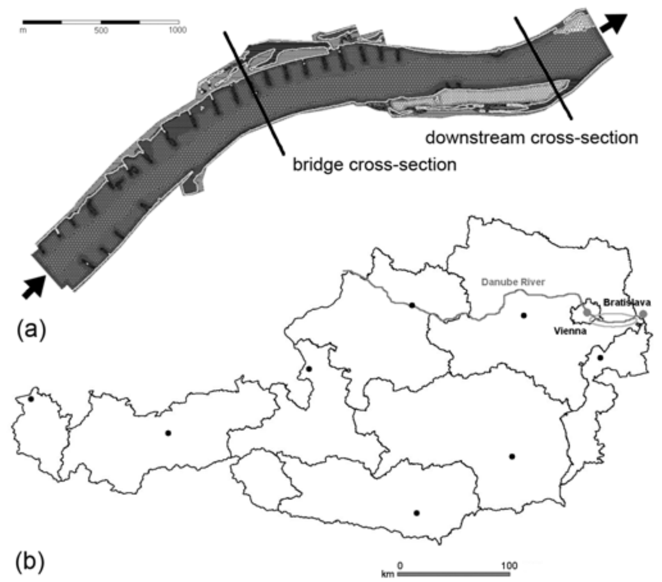

Figure 1. (a) Computational domain of the 3 km test reach at the Danube River East of Vienna; (b) location of the project reach within Austria.

3.2 *Set-up, calibration and validation of the numerical sediment transport model*

In the scope of the integrated river engineering project, a numerical sediment transport model was set up to assess and predict sediment transport and morphodynamics in the river section. For this purpose, the iSed model was coupled with the two-dimensional shallow water hydrodynamics code RSim-2D. The hydrodynamic model consisted of 72,500 triangular mesh cells; surface roughness was calibrated using numerous measurements of flow velocity and water surface elevations.

In order to set up and calibrate the numerical sediment transport model, several field samples and measurements were taken: (i) bed load, using a basket sampler at various cross-sections during discharges spanning the entire range from low flow to a 15-year flood; (ii) suspended load, using a US P-61-A point integrating suspended sediment sampler at various cross-sections during several different runoffs; (iii) grain size distribution of bed layers, by analysis of around 150 volumetric samples and 50 freeze core probes, divided into 10 layers of 0.1 m thickness. The 50 freeze core probes were arranged in three points per cross-section every 200 m along the entire length of the test reach. The grading curves obtained after sieving were characterized on average by a mean diameter d_m = 25.4 mm, a median diameter d_{50} = 21.5 mm, a diameter d_{90} of 53.5 mm and a standard deviation $\sigma = (d_{84}/d_{16})^{0.5} = 3.0$.

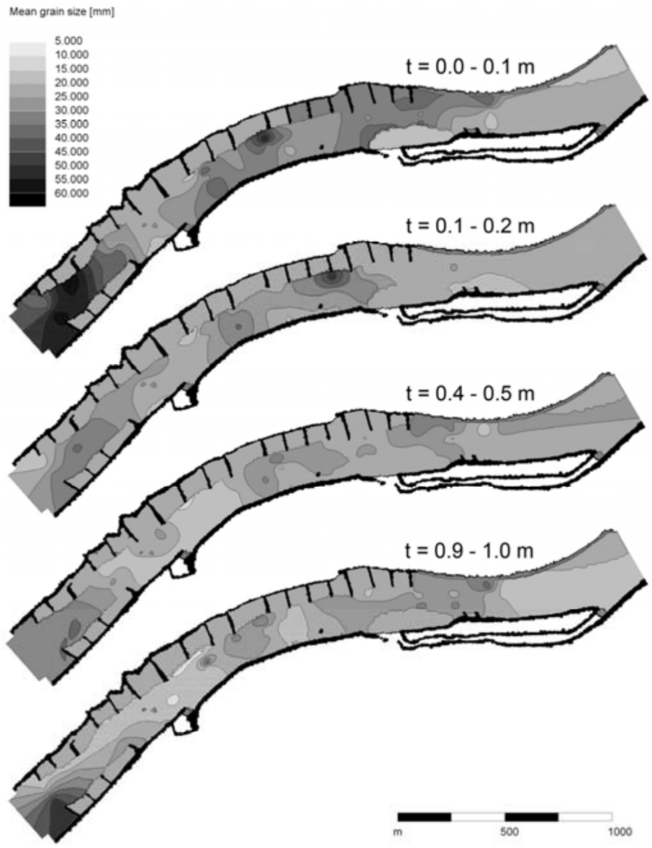

Figure 2. Mean grain sizes of bed material in four selected layers (t = depth below the river bed).

Using the Kriging methodology, grain size distributions between the sampling points of all 10 bed layers were interpolated to obtain the entire subsurface sediment composition, which was entered into the model; hydraulically or sedimentologically differentiated regions (i.e. groin fields) were considered separately. Sediments were found to be characterized by a high spatial variability as visible from the mean grain size distribution (Figure 2).

The non-uniform formulation of the Meyer-Peter/Müller bed load transport formula (Equation (2)) was calibrated on the mean sediment transport rates obtained from basket sampler measurements. This way, the pre-factor was calibrated as c_{MP}=5.0, which is also used in a modification of the formula by Hunziker (1995). Concerning the critical mobility parameter θ_c, both the usage of a Shields parameterization and a fixed value were compared during the calibration process. It was found that the application of a Shields parameterization led to a fining of the transported sediment compared to measurements; therefore a constant value was used instead (θ_c =0.047). The general importance of a calibration of the critical mobility parameter was also outlined in Habersack & Laronne (2002).

The computed bed load transport capacity was found to exhibit high sensitivity to the hiding-exposure correction exponent α (Tritthart et al., 2009). The best fit with measured transport rates was obtained by α=0.25.

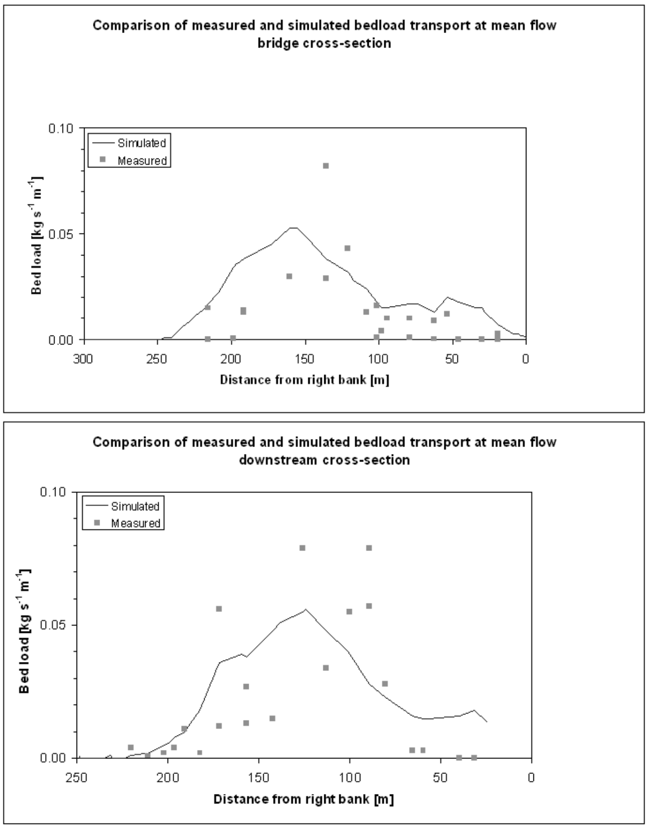

Figure 3. Comparison of measured and simulated (non-uniform calculations) bed load transport profiles at mean flow; top: bridge cross-section; bottom: downstream cross-section.

In order to validate the modeling results, measured and simulated bed load transport profiles for discharges near mean flow (MQ ± 10%) were compared at two cross-sections (Figure 3; the location of the bridge cross-section is indicated in Figure 1a, the downstream cross-section is located near the downstream end of the test reach). It was found that the computed profiles fit very well into the range of measured data, which generally showed a significant spatio-temporal variability, though in consistence with literature (Habersack et al., 2008).

3.3 *Results and discussion*

Using the calibrated and validated numerical sediment transport model, bed load transport capacities were computed for various discharges, comparing the uniform and non-uniform formulations of the Meyer-Peter/Müller equation (Equations (1) and (2)). Figure 4 shows the calculated transport capacities for regulated low flow (RNQ, 915 m^3s^{-1}), mean flow (MQ, 1930 m^3s^{-1}) and highest navigable flow (HSQ, 5060 m^3s^{-1}) after three consecutive days of the respective discharge. This three-day approach phase was selected in analogy to the spatial approach section required by numerical models, in order to allow the model to achieve equilibrium state not only concerning hydrodynamics but also regarding sediment transport due to mixing of bed load and bed sediment material.

The non-uniform formula predicts bed load transport even for low flow (RNQ), which is in consistence with bed load measurements. Higher transport rates are calculated particularly in the first kilometer of the test reach and in a section narrowed by the existence of a gravel bank near the right bank of the river, taking values of over 0.1 $kgs^{-1}m^{-1}$ (Figure 4, a-1). In contrast, the uniform formula predicts zero bed load transport in almost the entire river reach with the exception of a small patch close to the gravel bank, where the calculated bed shear stress values exceed the critical shear stress for the mean grain size (Figure 4, a-2).

When comparing the transport calculations for mean flow (MQ), a similar pattern can be seen. The non-uniform formulation of the Meyer-Peter/Müller equation yields non-zero bed load transport capacities in the entire river bed, exhibiting maxima in the navigation channel of the river – a section of 120 m width near the center line of the river – (Figure 4, b-1). In general, the bed load transport predicted is slightly higher than during low flow. In contrast, the calculations using the uniform formula again show practically no transport in the entire river with the exception of the outflow cross-section, where the river is narrowed by a gravel bank near the left bank, and several smaller patches distributed over the entire bed (Figure 4, b-2).

For the highest navigable flow (HSQ), which is close to annual flood level, flow is distributed through the main channel as well as through several side arms. The non-uniform formula predicts bed load transport rates of around 0.3 $kgs^{-1}m^{-1}$ in the entire main channel, with transport rates in some confined patches exceeding 0.5 $kgs^{-1}m^{-1}$ (Figure 4, c-1). Though there is virtually no bed load transport in the side arm connected at the right bank, the general flow situation in the river bend leads to significant flow and bed load transport in the side arms connected at the left bank.

While the overall pattern of sediment transport is consistent between uniform and non-uniform formulae at HSQ, however, the transport rates predicted by the uniform formula are significantly higher, ranging between 0.5 and 3 $kgs^{-1}m^{-1}$. As opposed to the results of the non-uniform calculations, most areas characterized by high bed load transport are predicted to lie directly adjacent to areas of zero bed load transport, without a gradual increase in transport rates (Figure 4, c-2). Since the difference in bed load transport capacity directly translates into morphodynamic activity according to Equation (4), a steep gradient in this parameter consequently leads to undesired local sedimentation and erosion phenomena.

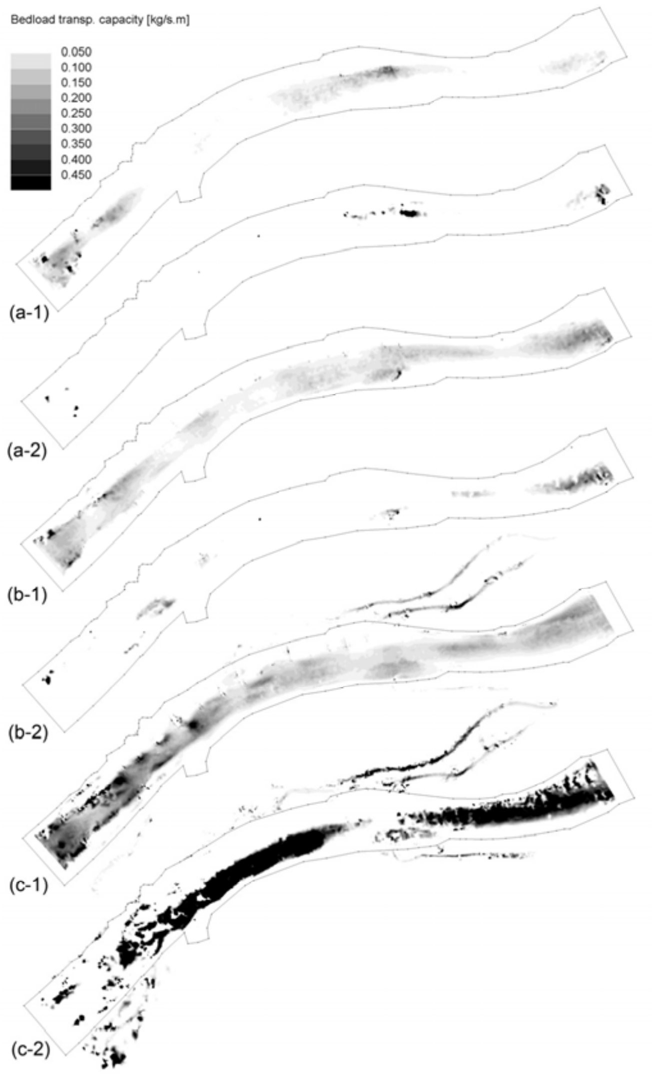

Figure 4. Plan view of calculated bed load transport capacities for uniform and non-uniform formulations of the Meyer-Peter/Müller transport equation; (a-1) 915 m^3s^{-1} (RNQ), non-uniform; (a-2) 915 m^3s^{-1} (RNQ), uniform; (b-1) 1930 m^3s^{-1} (MQ), non-uniform; (b-2) 1930 m^3s^{-1} (MQ), uniform; (c-1) 5060 m^3s^{-1} (HSQ), non-uniform; (c-2) 5060 m^3s^{-1} (HSQ), uniform.

In order to compare the computed bed load transport rates with measured data, transport through the bridge cross-section (Figure 1a) was calculated from the spatially distributed data both for simulation results and measurements; the result is presented in Table 1. For regulated low flow (RNQ), the average of all bed load transport measurements (4.4 kgs^{-1}) is very close to the non-uniform calculations (4.7 kgs^{-1}). The uniform calculations predict a bed load transport rate of 0.0 kgs^{-1} since the bed shear stress does not exceed the critical shear stress of the mean sediment diameter. A similar result is obtained for mean flow (MQ): the average value of all measurements is equal to the non-uniform calculations (6.0 kgs^{-1}). Again, the uniform calculations yield zero bed load transport since the critical shear stress is not exceeded.

Table 1. Comparison of measured and computed bed load transport through the bridge cross-section for various discharges, using uniform and non-uniform formulations of the Meyer-Peter/Müller sediment transport equation.

Discharge	**RNQ**	**MQ**	**HSQ**
Q [m^3s^{-1}]	915	1930	5060
Qb_{min} [kgs^{-1}]	2.4	2.4	28.8
Qb_{max} [kgs^{-1}]	6.3	9.5	95.4
Qb_{avg} [kgs^{-1}]	4.4	6.0	65.8
$Qb_{nonuniform}$ [kgs^{-1}]	4.7	6.0	22.5
$Qb_{uniform}$ [kgs^{-1}]	0.0	0.0	258.2

Abbreviations: Q = flow discharge; Qb_{min} = minimum cross-sectional bed load transport measured; Qb_{max} = maximum cross-sectional bed load transport measured; Qb_{avg} = average cross-sectional bed load transport of all measurements in the range of ±10% of the given characteristic flow discharge; all measurements presented were taken at the bridge cross-section; $Qb_{nonuniform}$ = modeled cross-sectional sediment transport using the non-uniform formula; $Qb_{uniform}$ = modeled cross-sectional sediment transport using the uniform formula.

Bed load transport at the highest discharge analyzed (HSQ) shows a high variability in nature as the measured values vary between 28.8 and 95.4 kgs^{-1}, with an average value of 65.8 kgs^{-1}. While the non-uniform calculations result in transport values slightly lower than the range of measurements (22.5 kgs^{-1}), the application of the uniform formula yields a bed load transport of 258.2 kgs^{-1}. This value is significantly higher than the maximum bed load transport rate measured at any discharge; hence, the uniform formula clearly overpredicts the actual bed load transport. However, when a separate calibration of the uniform sediment transport formula was performed (by adaptation of the critical mobility parameter θ_c), the calculated transport rates for higher discharges could be reduced to fit into the range given by the measurements. Similar findings were also documented in Habersack & Laronne (2002), based on investigations at the Drava River.

3.4 *Implications for morphodynamic and annual load calculations*

Since morphodynamic processes take place mostly during phases of high river discharge, the usual application of sediment transport formulae (uniform and non-uniform), calibrated indirectly on the morphologic changes between several geodetic surveys is not expected to introduce significant errors into the morphological predictions. However, all long-term morphological developments taking place during phases of low and mean flow discharges are consequently disregarded.

When the annual sediment load is calculated, it is important to note that the application of a uniform, single-grain sediment-transport formula inevitably leads to a gross underprediction of the result. Even if bed load transport at high discharges is correctly modeled, discharges of 5,060 m^3s^{-1} (HSQ) and higher occur only during 1% of the year (3.6 days) on the Danube River East of Vienna. During the rest of the year, the calculated load is significantly too low, since the uniform formula predicts incipient motion at approximately 3,500 m^3s^{-1} whereas this value is actually less than 900 m^3s^{-1} according to non-uniform calculations, which are in entire consistence with measurements. Therefore the usage of non-uniform transport formulae is highly important for a correct calculation of annual bed load transport.

4 CONCLUSIONS

An integrated sediment transport and morphology model that uses the results of external 2-D or 3-D hydrodynamic codes has been developed. The model is capable of computing suspended and bed load transport, as well as bed evolution and grain sorting processes for an arbitrary number of grain size classes, employing hiding-exposure corrections in the empirical transport formulae.

The numerical sediment transport model was applied to a stretch of the Danube River East of Vienna, calibrated directly on transport measurements using a basket sampler. Numerical modeling was performed employing both a uniform and a non-uniform formulation of the modified Meyer-Peter/Müller equation including hiding-exposure corrections. It was found that the uniform equation largely underestimates transport rates for discharges lower and equal to mean flow, as it predicts a later onset of bed load transport, while the non-uniform equation yields results in consistence with the measurements. For higher discharges, the non-uniform equation slightly underestimated transport rates, while the uniform equation was found to highly overpredict the actual bed load transport. This aspect could, however, be mitigated if the transport formula was specifically calibrated for high discharges and the corresponding morphological changes on the river bed. In general, the study shows that the conduction of bed load transport measurements using a basket sampler at various discharges is an essential prerequisite for calibration purposes of the sediment transport formula applied.

Even though the underestimation of sediment transport rates for lower runoffs is not so problematic as these discharges are not usually related to large morphological changes, the annual sediment load is consequently underpredicted by the uniform sediment transport formula. Furthermore, longer phases of lower discharges may occur which cause erosion and sedimentation due to the specific conditions encountered; i.e., erosion due to a reduction of the cross-section size and resulting higher flow velocities and bed shear stresses. These processes, as well as the calculation of the annual sediment load, can only be predicted correctly when applying non-uniform sediment transport formulae.

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