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Scaling properties of laboratory-generated river networks

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ABSTRACT: Selected laboratory experiments were carried out in the Hydraulic Engineering Laboratory at University of Basilicata, to explore scaling properties and self-organization tendencies of river networks. Experiments were made using a 1.5 m by 1.5 m basin-simulator-box with an outlet in the middle of the downslope-end side. The experimental landscape was weakly-cohesive soil mainly made by clay and silt. A system of micro-sprinklers delivered an almost uniform artificial precipitation. Simulations were performed at a constant rainfall intensity of 100 mm/h using different planar slopes. An additional experiment was also made to consider the base-level control. Digital elevation models (DEMs) of the evolving landscape were achieved through detailed soil surveys with a laser distancemeter and/or a laser scanner. The river networks were extracted from the DEMs using the D8 algorithm. Channel networks were ordered using the Horton-Strahler ranking as well as the Shreve topological classification. Hence, scaling laws were analysed to explore whether the generated systems resembled natural river networks. Thus, findings are provided on basin allometry, exceedance probability of basin areas and stream lengths, and slope-area relationship.

Keywords: drainage basins, laboratory tests, river systems.

1 INTRODUCTION

Though simulated drainage networks at laboratory scale may represent highly-simplified models of natural drainages, they can shed light on the complex dynamics governing fluvial systems. Laboratory experiments would also give the advantage to explore transient growth phases drawing on the knowledge of temporal and spatial landform evolution (Oliveto et al. 2009).

Here a short review on laboratory experiments on drainage network evolution is given. More references are provided in previous papers.

Schumm (1977) made experiments using a basin simulator 9 m by 15 m filled with mixtures of clay, sand, and silt. Observations revealed that the interaction of the initial surface slope and the outlet elevation provokes different growing mechanisms. Parker (1977) carried out two experiments testing different relief. For the low-relief experiment the drainage network evolved by headward growth with simultaneous branching while for the high-relief a master rill rapidly formed, later followed by side branching. Wittmann et al. (1991) made a special experiment by using a circular basin simulator of diameter 1.4 m filled with sand. A rotating arm fitted with sprinklers supplied the artificial rainfall. Drainage networks were induced by a sinkhole at the sandbox center. Gomez & Mullen (1992) investigated the evolution of drainage networks by subsurface flow. As for overland flow main channel and tributaries evolved by headward growth, but networks expanded both by lateral retreat of valley walls and divide decay. Hancock & Willgoose (2001) designed a landscape simulator consisting of a rainfall simulator suspended above a box 1.5 m by 1.5 m filled with erodible material. Two rainfall intensities were tested of 120 and 48 mm/h. Pelletier (2003) carried out four experiments in 15 m by 9 m enclosed flume: (i) 3% plane, (ii) 10% plane with accommodation space for an alluvial fan, (iii) plateau with 0.3 m of relief at plateau edge, and (iv) 10% plane at downslope end intersecting 3% plane at upslope end. Generated drainage networks were significantly depending on the initial topography. Finally, Hancock et al. (2006) found that sediment output is strongly related to the initial hillslope profile (linear or concave). They also found that short-term fluctuations would occur in both the erosion and sediment transport processes. Bigi et al. (2006) analyzed potential interactions between migration of knickpoints and hillslope failures, by using an elliptical basin simulator 99 cm in length and 87 cm in width with an outlet 1 cm wide. Finally, Niemann & Hasbargen (2006) discussed scale effects of drainage networks at laboratory scale.

This paper is part of a wider project on the dynamics leading to the typically observed patterns of natural river systems. Here, an analysis of physical experiments simulating the evolution and the development of drainage networks induced by an initial knickpoint is presented. Scaling properties are discussed and compared with those of natural basins.

2 EXPERIMENTS

In this section a synthetic description of the experimental set-up and procedure is given. More details and photographs can be found in Oliveto et al. (2008, 2009).

The experimental stand consisted of a rainfall simulator 2.8 m above a two-dimensional experimental basin. The whole equipment was enclosed in an external Plexiglas housing. The basin simulator was a box of projected dimensions of 1.5 m by 1.5 m with an outlet in the middle of the downslope-end side. A smaller box under the outlet collected both sediment and runoff outputs. The base of the basin simulator was a steel mesh normally lined with geotextile fabric to allow free drainage without soil losses. Infiltration water was saved into an underlying box with a piezometric tube. The rainfall simulator used 25 equidistantly spaced sprinklers on an area of 1.7 m by 1.7 m delivering micro-rain (drop size <0.5 mm) with negligible or no rainsplash effects. The rainfall distribution was found suitably uniform (Christiansen's coefficient equal to 86.6%). The experimental landscape was made of a weakly cohesive soil with a particle size distribution where 84% by weight was silt, 9% clay, and 7% fine sand.

Each experiment was carried out by ensuring consistent initial conditions (initial outlet elevation, as well) except for the landscape planar slope. In particular, the initial outlet elevation was always 4 cm above the base level. The soil surface was smoothed by a movable straight-edge leveler realizing surfaces with only small imperfections. Rainfall was then started and allowed to continue until significant drainage patterns formed. The base level and the rainfall intensity were kept constant through time and among experiments. Three experiments were carried out with a landscape (planar) slope of 8.5%, 5%, and 0.6%. The rainfall intensity was always 100 mm/h.

For the 8.5% experiment, three growing steps were surveyed at 41 minutes, 1 hour and 41 minutes, and 3 hours and 50 minutes after the rainfall start. The maximum (downslope) lengths of the well-dissected basins were 63, 95, and 120 cm, respectively. For the 5%, three growing steps were surveyed at 1, 11 and 22 hours. The maximum lengths of the well-dissected basins were 25, 72, and 120 cm, respectively. Finally, for the 0.6%, two growing steps were surveyed at 5 and 12 hours with maximum observed basin lengths of 97 and 110 cm, respectively. After the last survey, each experiment was protracted for additional 24 hours. In the case of the 5% and 0.6% tests, only minor drainage adjustments were observed. Thus, the last survey could be confidently considered as a steady-state condition or the final stage of the drainage pattern evolution. Conversely, for the 8.5% changes in topography were observed with even some influence of basin simulator boundaries on drainage patterns. Thus, in such case the last survey could not be considered a finale stage.

Planimetric and altimetric data were meticulously collected at different stages of growth by a laser pointer, for the 8.5% and 5% experiments, and a laser scanner for 0.6%. Measured points were interpolated using the Kriging method. Then, DEMs with resolution of 2 mm for the 8.5% and 5% experiments and even 1 mm for the 0.6% experiment were generated. Drainage networks from the DEMs were processed through the D8 algorithm using a threshold area of 2000 mm^2.

Figure 1 shows in the top the generated river networks for the 8.5% experiment at last survey. Sub-parallel drainage patterns were formed spanning the whole width of the basin simulator, despite the central outlet constraint. Just for comparison, in the bottom is shown the drainage patterns for the same initial landscape slope, but different initial outlet elevation coinciding, in this case, with the base level. The widespread alluvial fans in the lower-relief basin were induced by water submergence effects. However, this latter experiment and, in general, the effects of the outlet elevation, will not be discussed in this paper.

3 EXPERIMENTAL RESULTS

Several properties of the experimental drainage networks were derived from DEMs (Figure 2). Channel networks were ordered using the Strahler's stream ordering system as well as the Shreve's topological classification. Scaling laws were analysed to explore whether the generated systems resembled natural river networks.

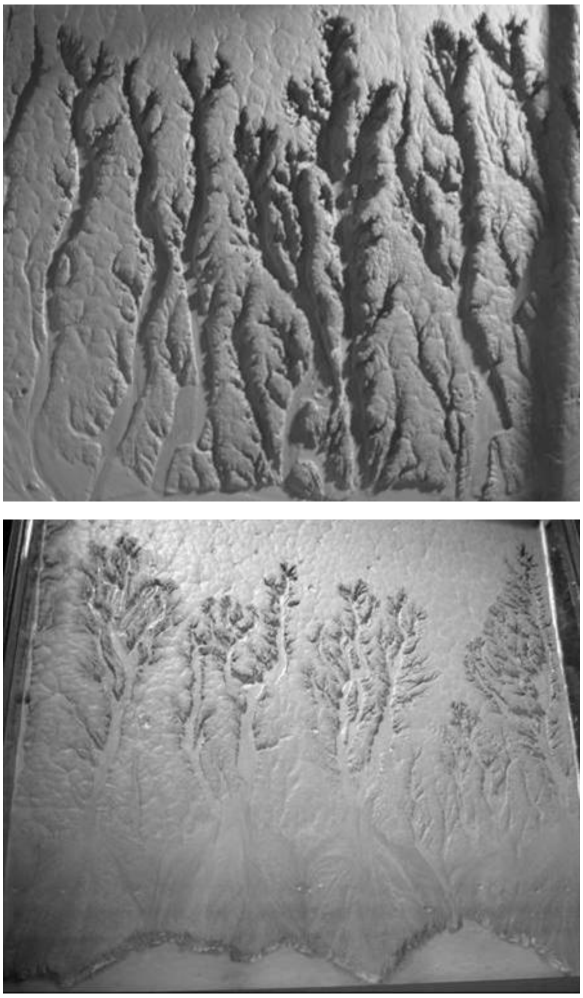

Figure 1. Observed drainage patterns for the 8.5% experiment (top) and for an additional experiment with the same initial planar slope but initial outlet elevation coinciding with the base level (bottom).

In previous papers (Oliveto et al. 2008, 2009) the Horton's laws, Hack's relation and self-similarity of the topological branching structure were considered. Here further investigations are carried out to deeper explore the tendency of the generated networks in simulating the reality. This section would accomplish this by considering both planimetric and altimetric features.

3.1 *Hurst coefficient*

According to Maritan et al. (1996), basins are self-similar when the Hurst coefficient, H, is 1 while are self-affine when $H < 1$.

Following Rigon et al. (1996), H was estimated by through the following relationship

$$L_{\perp} \propto L_{\parallel}^{H} \quad (1)$$

where $L_{\parallel}$ and $L_{\perp}$ are respectively the diameter and the width of a given sub-basin. $L_{\parallel}$ was identified as the longest side of the rectangle enclosing the sub-network (i.e. the straight line from the outlet to the farthest point of the sub-basin) while $L_{\perp}$ as the shortest side.

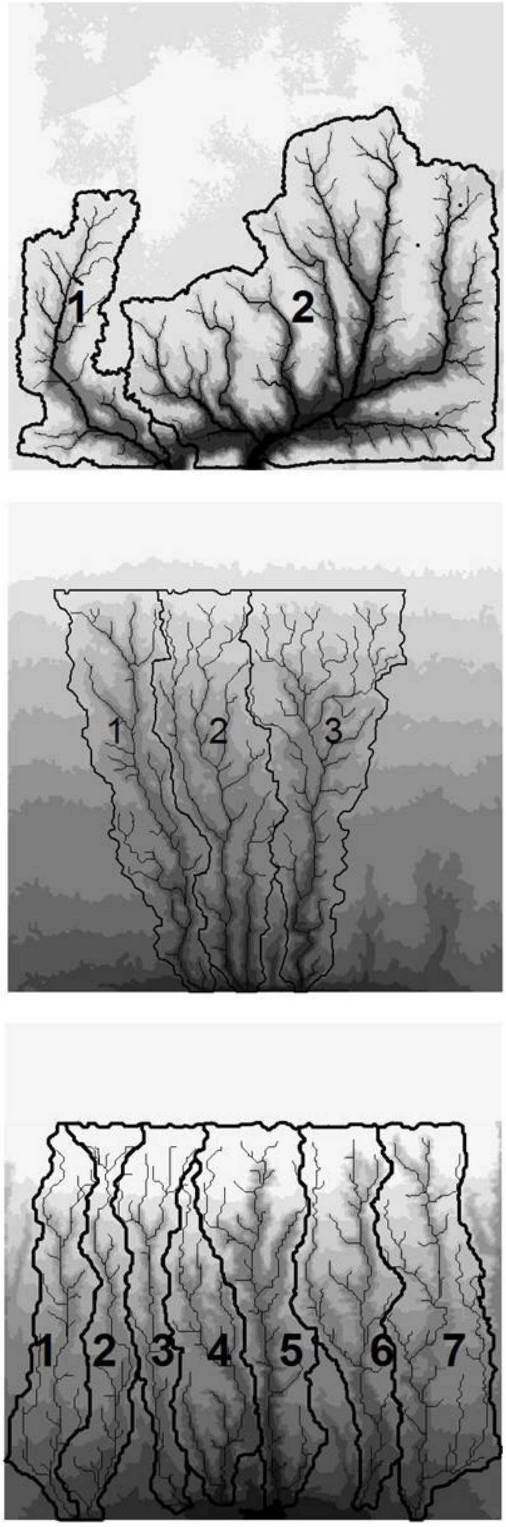

Figure 2. Experimental drainage networks derived from DEMs with delimitation of main sub-networks and related basins. 0.6% (top), 5% (middle), and 8.5% (bottom) experiments.

Figure 3 shows $L_{\perp}$ as a function of $L_{\parallel}$ for a number of well defined sub-basins of the three experiments at last survey. Each point represents ($L_{\parallel}$, $L_{\perp}$) for almost all sub-basins embedded within each main sub-basin shown in Figure 2. The x-axis was truncated at $L_{\parallel} = 10^3$ mm, losing thus the view of a few experimental points, for a better appreciation of data trends and scattering. However, regression lines refer to the complete data-set.

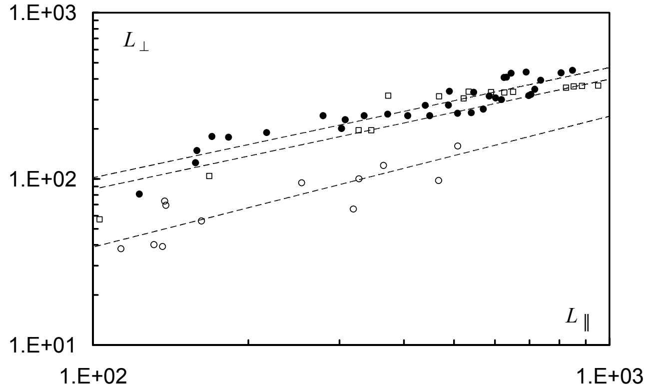

Figure 3. $L_\perp$ versus $L_\parallel$ for (●) 0.6%, (□) 5%, and (○) 8.5% experiments. Dashed lines are the related regression lines. $L_\perp$ and $L_\parallel$ are given in mm.

The regression analysis yielded the following relationships for 0.6%, 5%, and 8.5% experiments, respectively:

$$L_\perp \propto L_\parallel^{0.66} \qquad r^2 = 0.86 \tag{2}$$

$$L_\perp \propto L_\parallel^{0.66} \qquad r^2 = 0.81 \tag{3}$$

$$L_\perp \propto L_\parallel^{0.79} \qquad r^2 = 0.85 \tag{4}$$

Thus, the Hurst coefficient was found close to 0.7 and slightly higher for 8.5% which would match the typical values found in nature (0.7÷1.0), although the lower ones.

It should be noted that in this analysis only sub-basins adequately far from the outlet of the basin simulator were considered. This would allow a more proper comparison among the three experiments. Indeed, for the 5% experiment and even more for 8.5% it would be inappropriate considering sub-basins with the outlet close to the outlet of the basin simulator, owing to the marked sub-parallel morphology of the networks in their wholeness. Whereas, when such sub-basins are also considered for the 0.6% experiment, the relation (1) would exhibit a multi-scaling behavior with H close to 1.

3.2 *Probabilities of exceedance of total cumulative area and stream length*

According to Rodriguez-Iturbe et al. (1992) in real basins the exceedance probability $P[A \geq a]$ that a link has cumulative drainage area A larger than some class area a follows a power law:

$$P[A \geq a] \propto a^{-\beta} \tag{5}$$

with the exponent β close to 0.45, independent of size, vegetation, geology, climate, or basin orientation.

Rodriguez-Iturbe et al. (1992) also showed that real basins obey a scaling form of the type:

$$P[L \geq l] \propto l^{-\gamma} \tag{6}$$

with L stream length and l a given class length. The exponent γ was found close to −1.8.

Figure 4 shows the probabilities $P[A \geq a]$ for 0.6%, 5%, and 8.5% experiments at last survey.

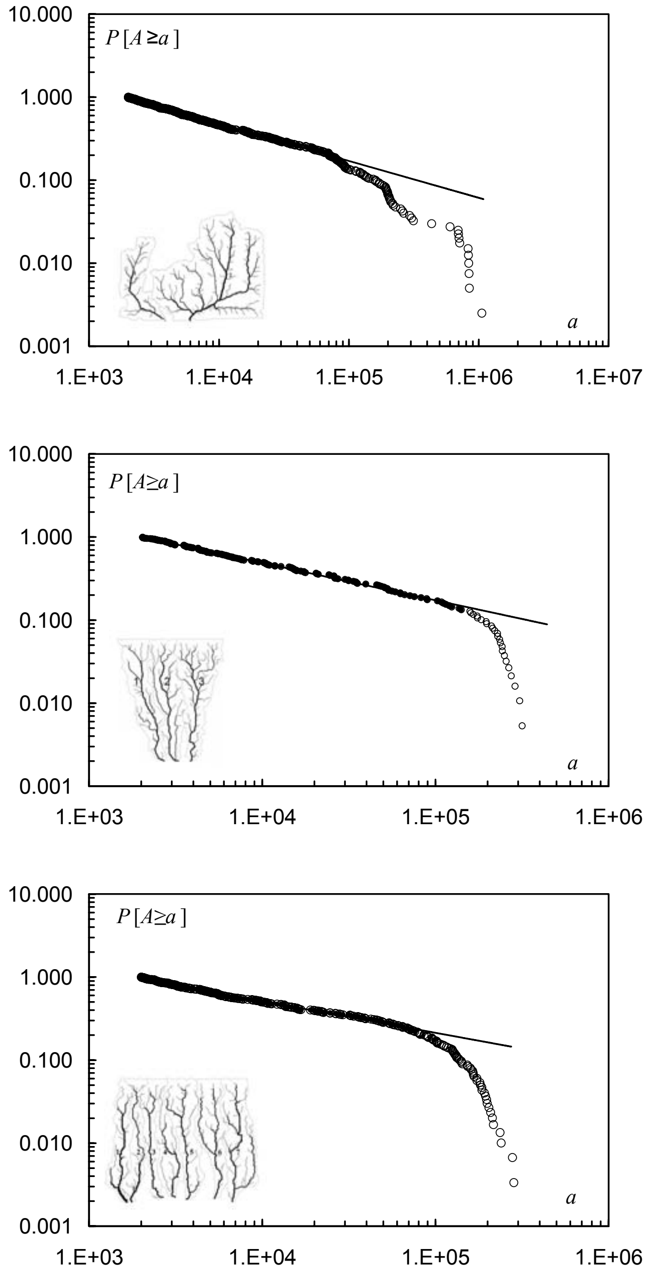

Figure 4. Probability of exceedance $P[A \geq a]$ of total contributing area for 0.6% (top), 5% (middle), and 8.5% (bottom) experiments. Areas are given in mm^2.

The y-axis was extended to 10 just for a better view of data points. Obviously, its maximum value should be 1. Interestingly, the data trends well resemble those of real networks and in the power-law region the regression analysis yielded the following relationships for 0.6%, 5%, and 8.5% experiments, respectively:

$$P[A \geq a] \propto a^{-0.44} \qquad r^2 = 0.997 \tag{7}$$

$$P[A \geq a] \propto a^{-0.45} \qquad r^2 = 0.997 \tag{8}$$

$$P[A \geq a] \propto a^{-0.38} \qquad r^2 = 0.995 \tag{9}$$

Overall, the exponent β was found close to 0.45, independently of the initial landscape slope, but the 0.6% and 5% experiments would appear as those properly representative of natural basins.

In the same way, Figure 5 shows the probability of exceedance $P[L \geq l]$ of Strahler's stream length. Also in this case the data trend looks similar to those of real networks with the extreme data regions suggesting hillslope processes (lower values of l) and the typical cut-off features of real basins, as well.

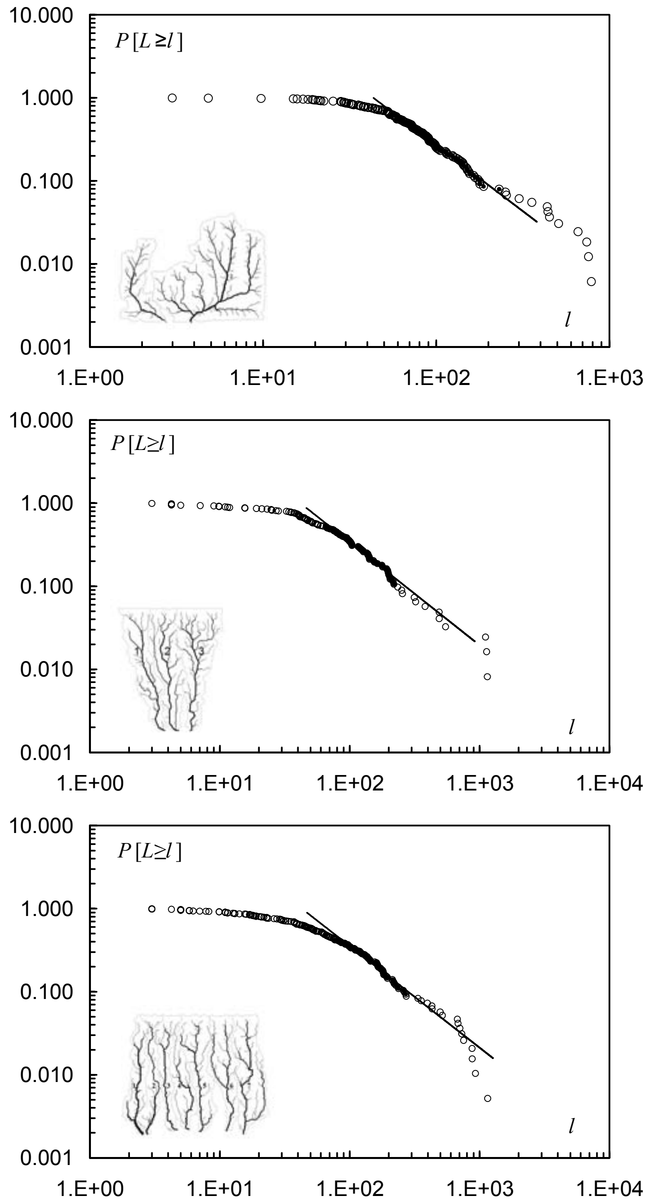

Figure 5. Probability of exceedance $P[L \geq l]$ of stream lengths for 0.6% (top), 5% (middle), and 8.5% (bottom) experiments. Stream lengths are given in mm.

The regression analysis in the power-law region yielded the following relationship for 0.6%, 5%, and 8.5% experiments, respectively:

$$P[L \geq l] \propto l^{-1.58} \qquad r^2 = 0.991 \tag{10}$$

$$P[L \geq l] \propto l^{-1.24} \qquad r^2 = 0.982 \tag{11}$$

$$P[L \geq l] \propto l^{-1.22} \qquad r^2 = 0.975 \tag{12}$$

Overall, the exponent γ was found significantly lower than the typical ones for real networks for 5% and 8.5% experiments and only slightly lower for 0.6%.

3.3 *Slope-area relationship*

The slope-area relationship has been theoretically and empirically studied by many investigators (e.g. Tarboton et al. 1989, Willgoose et al. 1991) in the form of the power law $S \sim A^{\theta}$, where A is the contributing area to the network-node of interest and S is the mean slope of the segment above the node of interest. The analysis of data from real basins has shown that the scaling coefficient θ is between 0.3 and 0.7. For low contributing areas, diffusive hillslope processes are dominant, while for high contributing areas, where the slope-area relationship strictly holds, fluvial incision is the main shaping process (Tarboton et al. 1989, Montgomery & Foufoula Georgiou, 1993).

The above power law would imply that basins are self-similar as the mean (first moment) of slopes scales with contributing area. But self-similarity would also imply that the second moment (i.e. slope variance) should decrease for high contributing areas with an exponent twice the one obtained for the mean slope (Gupta & Waymire 1989). Thus self-similarity implies:

$$E[S] = kA^{-\theta} \tag{13}$$

$$Var[S] = k_1 A^{-2\theta} \tag{14}$$

where $E[\cdot]$ is the mean operator, $Var[\cdot]$ the variance, and k and k_1 proportionality coefficients. Tarboton et al. (1989) and Rigon et al. (1994) tried to verify this second order self-similarity for real basins finding otherwise. The scaling exponent of the variance of the slope was found approximately half of that in Equation (14).

Figure 6 shows the relationships between the link slope and contributing area estimated for the three experiments. The plots show significant scatter indicating that slope is highly variable, as for real basins. However, by grouping 20 links with similar area and averaging (circles in the plots), the mean slope is seen to follow a fairly smooth trend. The power law curves are fitted to the circles using two phase regression. The line with the positive slope refers to hillslope or unchanneled valleys zone, while that with the negative slope to the fluvial zone.

For the fluvial zone, the regression analysis yields the following power laws for 0.6%, 5%, and 8.5% experiments, respectively:

$$E[S] = 3.31 A^{-0.41} \qquad r^2 = 0.96 \qquad (15)$$

$$E[S] = 1.32 A^{-0.28} \qquad r^2 = 0.94 \qquad (16)$$

$$E[S] = 3.67 A^{-0.36} \qquad r^2 = 0.87 \qquad (17)$$

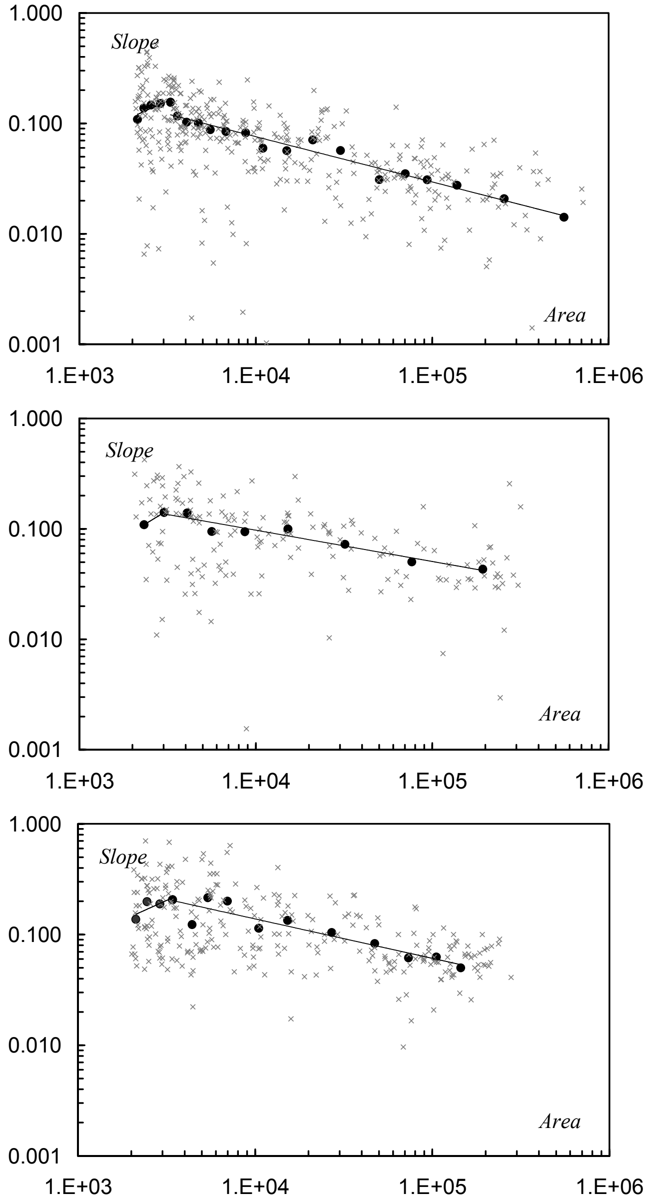

Figure 6. Link slope versus contributing area for 0.6% (top), 5% (middle), and 8.5% (bottom) experiments at last survey. (×) Individual links, (●) mean of 20 links, and (——) two-phase regression fit. Areas are given in mm^2.

Interestingly, the θ exponent belongs to the range of the values observed in nature, although closer to the lower value of 0.3. Moreover, the power laws would appear independent of the initial planar slope with a nearly coincidence between the relationships for 0.6% and 8.5% experiments.

Also, the following power laws were respectively found for the slope variance:

$$Var[S] = 0.49 A^{-0.60} \qquad r^2 = 0.81 \qquad (18)$$

$$Var[S] = 3.40 A^{-0.72} \qquad r^2 = 0.94 \qquad (19)$$

$$Var[S] = 57.5 A^{-0.94} \qquad r^2 = 0.87 \qquad (20)$$

The exponents significantly differ from 2θ, as Rigon et al. (1994) found for real basins. It is worth noting that from these results the 0.6% experiment would better resemble real cases because its scaling exponent is approximately θ, while the other two differ finally with exponents even larger than 2θ.

4 CONCLUSIONS

Three selected laboratory experiments driven to explore scaling properties of growing drainage networks were carried out at University of Basilicata.

For 0.6% experiment the landscape temporally evolved by headward growth with simultaneous intense branching. Whereas, for 5% and 8.5% experiments nearly parallel rills rapidly developed by spanning the whole width of the basin simulator despite the central outlet constraint.

In this paper basin allometry, exceedance probabilities of contributing areas and stream lengths, and the slope-area relationship were analyzed, integrating thus previous findings on Horton’s and Shreve’s descriptors. Results can be summarised as follows with real network values from Rinaldo et al. (1998) provided in parenthesis: (i) The Hurst exponent was found close to 0.7 or slightly higher independently of the initial landscape slope and in satisfactory agreement with the typical values encountered in nature (0.7 ÷ 1.0); (ii) The coefficient of probability of contributing area and stream lengths were respectively found 0.44, 0.45, and 0.38 and 1.58, 1.24, and 1.22, which compared to those of real networks (0.41÷0.46 and 1.8±0.1) would suggest the 0.6% experiment as that closer to the reality; (iii) Also the θ exponent of the slope-area power law was found belonging to the range of the values observed in nature. But from the slope variance analysis it arises that the 0.6% experiment would better resemble real cases because its scaling exponent is approximately θ, while the other two experiments finally differ with exponents even larger than 2θ.

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