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Mixed alluvial and non-alluvial bed topographies: observations, modeling and implications

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ABSTRACT: The presence of mixed alluvial and non-alluvial beds in rivers (for example exposed bedrock, clay layers, riprap or armored beds) highly influences the river morphological behavior and river functions. In many situations these non-alluvial sections only cover a limited area of the river bed (therefore interact closely with the surrounding alluvial beds), or have a temporal character (e.g., remobilize during high flows). More detailed monitoring techniques, and new modeling approaches that are deployed in mixed areas in the Dutch Rhine branches have revealed some important implications for longterm morphological development of the river bed. It is shown with some examples how techniques to highlight bed forms in multibeam maps, followed by directly oriented sampling, are appropriate to recognize the relevant features. Using special modeling concepts for undersupplied sediment-transport in computational models it is possible to simulate the time-dependent and spatially varying impacts on flow and morphology for navigability. For the break-up of temporary fixed layers, or semi-erodible layers, we have combined the modeling concept with a multi-fraction layer approach in a special way. Specific applications, using 2D (quasi-3D) simulations, reveal the importance of these tools to simulate the impact of these layers on the surrounding alluvial sections and the global river topography.

Keywords: River beds, Erosion, Models, Channel stabilization

1 INTRODUCTION

In many rivers the presence of mixed alluvial and non-alluvial beds has serious impacts on the river behavior and river functions. Non-alluvial sections can for instance be found at exposed rock layers, clay or peat areas, armored beds. They can also be of an anthropogenic origin, for instance introduced for bed stabilization of fairway improvement. In many situations these non-alluvial sections only cover a limited part of the riverbed (short stretches, or parts of a cross section), and therefore interact closely with the surrounding alluvial beds.

The occurrence of these non-alluvial parts is not always recognized, as observations require special interpretation. It is shown on basis of multibeam surveys in the Dutch Rhine branches how visualization techniques have been applied to highlight bed forms and reveal indirectly the nonalluvial sections. The actual (non)-erodibility of the sections can be confirmed by directly oriented sediment sampling.

In many morphological (modeling) studies the presence of material with different erosion characteristics is often ignored and uniform-sediment approaches are applied. However, it is found that the information of under layers is particularly important for the assessment of morphology on a long time scale, for instance requested for climatechange studies and long-term management options. The large changes in a system on such a scale are certainly affected by the presence of less- and more-erodible layers.

The impact of non-alluvial patches on the morphology can be modeled using a special transportreduction approach (Struiksma, 1999) as presented in section 3. Although the classical approach is defined for bed-form average bed topography, an extension towards dune-height prediction in supply-limited conditions is being developed (Tuijnder, 2009). Another extension is related to armoring and the development of temporary (armored) non-alluvial areas. These semierodible layers require a combined modeling concept for non-uniform sediment and non-erodible layers, which is presented in section 3.

With the help of these modeling concepts, implemented in computational models, it has been shown that the presence of a mixed alluvial and non-alluvial riverbed seriously affects the local and global topography of the river. Some results from cases from the Dutch Rhine branches are presented in section 4.

2 OBSERVATIONS OF NON-ERODIBLE SECTIONS

Non-erodible layers in the river bed become apparent as soon as they are exposed to the flow. In that case there are different indicators that give away there location, such as:

- areas with zero bed-level change in a degrading river. Analysis of consecutive soundings of bed levels in an area (preferably using multibeam soundings) reveal stable reaches in certain parts of the Rhine Rivers in Germany and Netherlands.
- areas with unrealistic shapes or elevations within an alluvial bed. For instance rocks that protrude into the channel will show on soundings, and may lead to local scour holes and even attract channels that also reveal there presence.
- in a sand-bed river, non-erodible parts of the bed may show a different more sparse dune pattern (or missing dunes) that indicate undersupplied conditions.



Figure 1. Multibeam sounding of river bed of the Merwede River (lower Rhine River branch)

A typical example is shown in figure 1 for the Nieuwe-Merwede, which is one of the lower Rhine-delta branches. This river channel is actually a man-made canal, which has been excavated in

1870 through an area with fresh-water marshes with peat and clay, fed by many tidal creeks (Kleinhans et al. 2010). After a period of deposition, sand mining operation in the past decades have resulted in degradation of the bed, such that presently the original excavated bed is now exposed at several locations. This former river bed is composed of compacted (lagoonal) peat and clay (as determined by taking sediment drilling samples from the bed). In figure 1 is shown how the sandy bed upstream of km 964 shows an abrupt transition to the fixed bed with transverse ridges on the left half of the channel. These ridges are presumably traces of the original steam bucketdredgers that were used to dig the channel. On this fixed bed also traces of the course of old creeks (former abandoned and infilled channels) can be found

The bed in figure 1 has been recorded using a multi-beam sounding in 2005. By using a hillshade with fictive light source in ArcGIS software it is possible to make these features on the bed recognizable. The use of artificial lighting is an ideal way to identify the presence of specific dune shapes and other elements. The visual effect of floodlight is for instance illustrated for barchanshaped dunes observed on the planet Mars with sunlight brushing over the dunes. This type of dunes is typical for undersupplied conditions.



Figure 2. Sand Dunes moving over erosion-resistant substrate in Proctor Crater on Mars, from Mars Global Surveyor's Mars Orbiter Camera, Courtesy NASA/JPL-Caltech

Another relevant example of non-erodible layers in the Rhine branches is the bifurcation of the IJssel branch and the Nederrijn branch. The river bed in this reach is composed of a mixture of sand and gravel. In figure 3 a map of the bifurcation is presented, showing the median grain size of the bed as determined from drilling cores of the bed (Gruijters et al, 2003). Due to bend sorting the bed shows a much coarser top-layer on the right half of the channel just upstream of the bifurcation. This leads to supply limitation, particularly in the downstream branch that originates in the outer bend of the main channel of the river bed. The phenomenon was analyzed by Frings and Kleinhans (2008). Undersupplied conditions and winnowing of fines in the upper reach of the IJssel have led to the development of an erosion resistant top layer of gravel between 0.5 to 1.5 m thickness on average. Below this layer sand is found (Gruijters et al., 2003). The thick layer extends over a distance of about 2 km. Sand dunes typically propagate towards the Nederrijn (westward), whereas most sand is transported towards this reach.



Figure 3. Median grain size of the river bed of IJssel (flowing towards the North) and Nederrijn (flowing towards the West) (flow if from bottom to top)

The river channels in this area show a gradual degrading trend. The presence of this non-erodible layer prevents a further degradation during normal yearly hydrographs. The stability of this IJssel branch may affect the discharge distribution of the branches as erosion in the Nederrijn is not hampered. Nevertheless, it is expected that during high floods the armored top-layer of the IJssel can break up, followed by rapid erosion of the under laying sand.

In the Netherlands much knowledge on the behavior of non-erodible layers has been gained from research to the design and construction of fixed layers in the outer bends of the Waal River at Nijmegen (completed in 1988) and Sint Andries (completed in 1999). The aim of these layers is to shallow the outer bend pool with a sustainable riprap layer, in order to create erosion and increased navigation width on the pool bar. The details of these layers are for instance presented in Sloff et al. (2006). The layers were found to be quite effective in reducing the height of the point bar, but because a rather deep scour hole develops just downstream of the layers, in response a rather shallow bar develops in the inner bend next to these scour pools, see figure 4.



Figure 4. Multibeam sounding of bed level in Waal River at Nijmegen, showing non-erodible layer, and downstream pool (scour) and bar development (flow is from right to left)

The bar developing at this location in the inner bend is a bottle neck for navigation, and has to be dredged periodically. Presently plans are drawn to extend the fixed layer by filling the scour hole with coarse material, such that the transition becomes more gradual, and sand bar development occurs in the deeper area further downstream.

The scour holes downstream of these layers are caused by clear-water scour in the alluvial reach just downstream of the fixed layer. Due to helical flow the fine bed-load sediment that runs over this layer is swiped to the inner bend rapidly, such that further down no more sediment is transported over the layer at all (see Sloff et al., 2006). It is found that the scour hole at Sint Andries is still growing (even after more than 10 years), and may eventually destabilize the lower end of the construction.

The non-erodible layers in this section, although there origin and nature may differ, all have an analogous impact on the morphology of surrounding alluvial sections. The response comes from a redistribution of flow and sediment transport. These have been translated to modeling concepts for transport and flow over non-erodible layers. Recently also concepts have been designed to deal with the break-up and formation of temporary non-erodible layers (e.g. gravel layers). In the past several modeling approaches for nonerodible layers have been applied, ranging from laboratory scale models to physically based mathematical models. These studies have led to the development of a computational modeling approach, which has been implemented in onedimensional (1D) and two-dimensional (2D) computations. The basis of the modeling concepts in this paper is the modeling approach that was presented by Struiksma (1999). His model concept for non-erodible layers has been defined in the early 1980s, and is based on a correction of bedload transport capacity.

In general we assume that bed load is modeled with a simple predictor in which sediment transport is a function of the local transport capacity of the flow. However, when transported over a nonerodible layer, no sediment can be supplied from the bed when the capacity increases. The sediment transport thus becomes supply-limited rather than capacity-limited. Instead of defining an approach that tracks the volumes of sediment passing the layer, the model concept of Struiksma (1999) retains the relation with local transport capacity. It accounts for the transport limitations on nonerodible layers through a reduction of the transport layer thickness on the bed.



Figure 5. Definition sketch (Struiksma, 1999)

In this concept the volumetric sediment transport per unit width (according to a customary transport predictor), s_f , is modified with a correction factor, ψ , to obtain the actual (undersupplied) sediment transport per unit width, s, over the non-erodible layer (including pores):

$$s = \psi \left(\frac{\delta}{\delta_a(h)}\right) s_f(u) \tag{1}$$

in which δ is the thickness of alluvium on the nonerodible bed, δ_a is the maximum thickness of alluvium at which the non-erodible layer affects the sediment transport, *h* is the local water depth and *u* is the local depth-averaged flow velocity. The bed is non-alluvial ($\psi < 1$) if $\delta/\delta_a < 1$ and alluvial ($\psi = 1$) if $\delta/\delta_a \ge 1$ (Fig. 4). The correction factor,

 ψ , accounts for the reduction of the transport capacity due to a thinner transport layer and hindered bed-form development. Basically, the information on the reduction is carried by the sediment layer thickness, δ . Local increases in s_f are counteracted by decreases in δ and ψ in such a way that the resulting erosion cannot reach below the surface of the non-erodible layer. This limits the supply of sediment from the bed. The supply of sediment from upstream is limited by the occurrence of similar reductions upstream. The maximum alluvium thickness for influence of the nonerodible layer, δ_a , corresponds to the thickness of the transport layer on an alluvial bed. It can be taken equal to half the bed-form height and is therefore mostly a function of the water depth.

Indicating the level of the top of the nonerodible layer with z_* , the alluvium thickness satisfies $\delta = z_b - z_*$, where z_b represents the bed level which is equal to the level of the top of the layer of alluvium. Using this relation, equation (1) is substituted into the one-dimensional sediment balance

$$\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = 0 \tag{2}$$

where *t* denotes time and *x* denotes the streamwise co-ordinate. This yields

$$\frac{\partial z_b}{\partial t} + \psi \frac{\mathrm{d}s_f}{\mathrm{d}u} \frac{\partial u}{\partial x} + \frac{s_f}{\delta_a} \frac{\mathrm{d}\psi}{\mathrm{d}(\delta/\delta_a)} \left(\frac{\partial z_b}{\partial x} - \frac{\delta}{\delta_a} \frac{\mathrm{d}\delta_a}{\mathrm{d}h} \frac{\partial h}{\partial x} \right) \\ = \frac{s_f}{\partial_a} \frac{\mathrm{d}\psi}{\mathrm{d}(\delta/\delta_a)} \frac{\partial z_*}{\partial x}$$
(3)

which shows that the correction factor, ψ , modifies the free morphodynamic behavior and that the level of the non-erodible layer, z_* , acts as an external forcing of the system.

The dependence of the correction factor, ψ , on the relative alluvium thickness, $\delta' \delta_a$, is represented by an assumed function which increases monotonously from zero to unity for $0 \le \delta' \delta_a < 1$ with a smooth transition to the constant value of $\psi = 1$ for $\delta' \delta_a \ge 1$. Two different functions for this parameter have been proposed by Struiksma (on basis of laboratory data):

$$\psi = \sin\left(\frac{\pi}{2}\frac{\delta}{\delta_a}\right) \text{ or } \psi = \frac{\delta}{\delta_a}\left(2 - \frac{\delta}{\delta_a}\right) \text{ for } \frac{\delta}{\delta_a} < 1$$
 (4)

Struiksma's approach has been applied and validated for many experimental and actual field cases (e.g. Sloff et al., 2006), but yet appeared to be oversimplifying some of the complex physical phenomena that have been observed in undersupplied conditions. For instance a proper generic sub-model for predicting the alluvial thickness δ_a is missing. Tuijnder (2009a,b) studied in more detail the development of bed forms in undersupplied conditions in laboratory-flume experiments, and extended Struiksma's model to account for the effect on bed friction depending on coverage of non-erodible (gravel) bed and height and shape of the bed forms. The new concepts of Tuijnder have been found to provide a more physically based and hence more generic approach for fixed layers. For more details on the concept we refer to Tuijnder (2009b) and Tuijnder's contribution in this conference.

Another development step was made for simulating semi-fixed layers. These are bed-layers which are supposed to be fixed during low-flow conditions, but can break up and erode during high flows. The approach is applicable for the armored layers in the gravel-sand bed of the upper Rhine reaches in the Netherlands. Also the intention of Rijkswaterstaat to use nourishment of coarse gravel in outer bends to create artificial temporary fixed layers in outer bends and to deal with bed degradation simultaneously, requires such an approach.

The modeling concept for semi-fixed layers is a combination of Struiksma's fixed-layer concept with an extension of the multi-fraction approach that is already available in the Delft3D software system. It includes a multi-fraction (bed-load) sediment transport computation, a multi-layer bed system with an active layer (i.e., transport layer or mixing layer with thickness δ_t) on top, an intermediate exchange layer (defined by Ribberink, 1987, with thickness δ_{ex}) and multiple under layers. In an alluvial bed the active layer and exchange layer are associated to migrating bed forms that produce vertical sorting processes in the bed, and the mixing of size fractions, see figure 6. The sediment-transport rates are computed using the average composition of the active layer. The active layer and exchange layer usually get a thickness based on dune properties (figure 6) or other physical features that determine the mixing processes in the bed surface (see also Sloff and Ottevanger, 2008). The under layers get a thickness that remains constant in time in a fixed reference frame. The active (and exchange) layers remain attached to the changing bed level, whereas they cut into the under layers, or new under layers are added to accommodate bed level rise.



Figure 6. The bed-layer schematization for graded-sediment modeling following Ribberink (1987), where sub-script i is associated to sediment size fraction i

Consider the definition sketch in Figure 7 for an arbitrary semi-fixed layer. Layer δ_1 (i.e. the active layer) contains only fine material, layer δ_2 (exchange layer or first under layer) contains fine material at locations A and B, and coarse material at location C. Layer δ_3 consists of the coarse sediment fraction and underneath it in layer δ_4 there is only fine material. Layers δ_2 to δ_4 are referred to as under layers (there may be an arbitrary number of under layers). For each of these layers, and at each location, the 'mobility' of sediment can be determined by considering the flow shear-stress at the river bed. For instance sediment is considered mobile if Shields values exceed the critical Shields value (including hiding and exposure effects). Otherwise, if sediment is found to be immobile, it is assumed that the respective layer is non-erodible, and Struiksma's approach as defined in figure 5 has to be applied.



Figure 7. Bed-layer schematization for semi-fixed layers

This implies that, if the coarse sediment in figure 7 is mobile, no reduction of transport takes place because the mobile sediment thickness δ_{mob} is greater than δ_a . However, when the coarse sediment is immobile, e.g. at location C, a reduction of the transport rate according to Struiksma (1999) will be applied, because δ_{mob} is less than δ_a ($\delta_1 \leq \delta_a$). At locations A and B, δ_{mob} is still greater than δ_a and therefore no transport reduction will occur.

In the new approach we have introduced a parameter that evaluates the mobility of the upper layers of the bed to detect whether Struiksma's transport reduction has to be switched on. The parameter, so called mobility ratio, is defined as the ratio between the computed sediment transport for size fraction i in layer k and a preset value $s_{b,thresh}$.

$$r_{i,k} = \min\left(\left|s_{b,i,k}\right| / s_{b,thresh}, 1\right)$$
(5)

The mobility ratio of the layer k is then given by the mobility ratios per sediment fraction weighted by the percentage content of sediment fraction i in layer k (i.e. $p_{i,k}$):

$$r_{tot,k} = \sum_{i} r_{i,k} p_{i,k} \tag{6}$$

To determine the total thickness of mobile sediment, the following definition is used:

$$\delta_{mob} = \sum_{k} r_{tot,k}^* \delta_k \tag{7}$$

where $r_{tot,1}^* = r_{tot,1}$ and $r_{tot,k}^* = r_{tot,k} \cdot r_{tot,k-1}^*$ for k>1, to prevent erosion of mobile layers below immobile layers such as layer $\delta 4$ beneath $\delta 3$ shown at location A in Figure 7.

The advantage of the above approach is that the mobility ratio is consistent with the chosen sediment formula. Note that if δ_{mob} becomes less than δ_t , the active layer will be reduced. Furthermore, note that from a physical point of view the active-layer thickness δ_t is best chosen equal to δ_a (related to dune height). It also has to be noted that these equations are only used to detect the presence of non-mobile layers, and are considered as a pragmatic approximation. Alternative parameters for this purpose will be investigated in the future.

In situations with exposed non-erodible layers $(\delta_{mob} < \delta_a)$, the transport rates are reduced using Struiksma's reduction function of equation (4). Note that in this approach the transport rate is still based on the bed composition of the top-layer, and scaled according to the thickness of alluvial sediment on the bed (including indirectly the mobility of sub-layers through parameter ψ). A next step to be considered, could be to include the transport rates of lower layers in the total transport rate as well. Therefore these lower transport rates would have to be scaled with the probability in which they are exposed (the deeper the layer, the lower the frequency), e.g. an approach following Blom (2003). For cases in which δ_{mob} is less than δ_a we can assume that equation (1) based on the toplayer still suffices, but for fully alluvial conditions this is not necessarily true. Including the exposure probability of deeper sediment in the transport rate means that the surface composition is used rather than the top-layer average composition,

which is more natural for dune-covered beds. The associated sorting process is illustrated in figure 8.



Figure 8. Sorting processes in dunes: development of coarse layer in troughs. Courtesy of Astrid Blom (2003)

In dune-covered alluvial beds of the upper Rhine reaches in the Netherlands the sorting process of figure 8 is assumed to cause the formation of non-erodible gravel layers in the dune troughs. The previously presented approach allows for detection and eventual break-up of semi-fixed layers in the bed (depending on flow conditions). However, for the development of semi-fixed layers by these vertical sorting processes, it is necessary to apply an extended approach in which the exchange layer, defined in figure 6, has been applied. This approach has been presented in Sloff and Ottevanger (2008) and is not further elaborated in this paper. Crucial for the development of the coarse layer is the definition of the vertical sorting flux between active layer and exchange layer. These elements are still being investigated.

4 MODEL RESULTS

In Sloff et al. (2006) some of the successful applications of Struiksma's modelling concept for nonerodible layers in flume and in the Waal River are presented. Recently quasi-3D simulations have been carried out for the Merwede branches in the south-west of the Netherlands, see figure 9.



Figure 9. Map of Merwedes in the Netherlands, with the Boven-Merwede bifurcating into the Nieuwe-Merwede and the Beneden-Merwede

For these simulations we have applied a curvilinear grid covering the full width of the river (main channel and flood plains), and considered the different branches as separate domains (using the Delft3D domain-decomposition approach). The initial bed of 1997 has been introduced, as well as spatially varying roughness, groynes (weirs) and sediment characteristics. Simulations have been carried out using a full yearly hydrograph (i.e. including flood and low-flow periods). Although there is some influence of tides in this region, the effect has been disregarded.

The bed soundings presented before in figure 1 show that in the Nieuwe Merwede a non-erodible layer is exposed from km 964 on the left half of the river. We introduced this layer into the model, and compared the results the measurements and to a simulation without this layer. The results in figure 10 show that without fixed layers rather deep pools can be found, but by using the fixed-layer approach the results fit the data very well.

Note that in figure 10 the reach upstream of km 964 shows a rather strong sedimentation in the simulations. This can be explained from dredging (for sand mining) in this reach, and can be corrected easily in the simulations (not shown in figure 10). The simulations above do not consider break-up of the non-erodible layer, and therefore mainly represent a validation for Struiksma's concept.



Figure 10. Bed level computed and measured for a longitudinal section left of the river axis in the upper reach of the Nieuwe-Merwede (flow is from right to left)

Recently plans have been drawn for sediment management strategies in the Boven-Rijn to stop autonomic bed-level degradation and to improve the navigability by (amongst others) nourishment of coarse sediment in outer bends. By using coarse sediment in the pools it is expected that a semi-fixed layer can be created that temporarily leads to lowering of the point bar in the inner bend (as was successfully achieved in Nijmegen and Sint-Andries in the Waal River).



Figure 11. Map of Boven-Rijn (Rhine) at border of Germany and Netherlands. Legend refers to bed-level in m.

Simulations have been carried out for testing the effectiveness of these plans and evaluation of the Delft3D tool for semi-fixed layers presented in this paper. Here we present the result of a simulation with a one-time nourishment of coarse gravel with a mean grain size of 11 mm in the outer bend indicated in figure 11. A flow hydrograph is imposed with a constant low-flow discharge (1500 m³/s), followed by a flood period starting in august, a peak-flow of 6000 m³/s in the second half of September, and a low flow period again starting from the next January until august (end of the simulation).



Figure 12. Longitudinal profile of river bed of the Boven-Rijn, showing the amount of nourished sediment at time = 0



Figure 13. Longitudinal profile of river bed of the Boven-Rijn, showing the amount of nourished sediment at time = just before break up (rising limb of flood)



Figure 14. Longitudinal profile of Boven-Rijn, showing the available fraction of finest sediment fraction (D<0.5 mm), time = just after break up (flood conditions)



Figure 15. Longitudinal profile of Boven-Rijn, showing the available fraction of finest sediment fraction (D<0.5 mm), time = end of simulation (after 1.5 year)

In figures 12 to 15 a time-sequence of a longitudinal profile (right of the axis) is given that present the fraction of nourished material (tracer 7) in the top-layers and under layers of the bed. In these figures the flow is from left to right. In the simulation the semi-erodible layer remains stable (and is covered with material from upstream) during the low-flow period, but breaks up during the rising limb of the flood. Thereafter the nourished gravel is mixed with the other sediment in deeper layers and downstream of the nourishment location.

In 2010 a first pilot-nourishment will be carried out in the Boven-Rijn. Findings of these simulations are used to design the nourishment strategy. A measurement campaign will provide the necessary data for further improvement of the approaches and for design of efficient nourishments.

5 CONCLUSION

Non-alluvial or non-erodible sections affect the morphology on a local and global scale. The examples in this paper illustrate that a careful interpretation small-scale features of high-resolution bed-level recordings is useful to recognize the non-erodible layers.

For practical purposes stable fixed layers can be modeled in a proper way using a 2D/3D computational model with Struiksma's sedimenttransport reduction approach. In cases with semierodible layers (layers that are only stable during low flows) an extended approach has been introduced based on a combination of Struiksma's concept with a multi-fraction approach with under-layers in the bed. This approach is used to design temporary stable nourishments which are planned in the Dutch-German border region of the Rhine.

It has been observed from modeling application in the Rhine branches that for a long-term assessment of impacts of measures and autonomic processes the non-erodible layers are very important. For instance amplification of degradation of the Rhine branches and surprising stabilization of river bifurcations are some of the implications that demand for careful assessment of the presence of mixed alluvial and non-alluvial river beds.

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