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Assessment of flood damage risk for abutments in river floodplains

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ABSTRACT: The scour development in time during multiple floods and equilibrium stage and the assessment of flood damage risk for bridge abutments have been investigated. The differential equation of equilibrium of the bed sediment movement for clear water conditions was used, and a method for computing scour development in time at bridge abutments during multiple floods was elaborated. The test confirmation of the methods allows us to perform computer modeling of scour processes and estimate the influence of floods with different probability, duration, frequency, and sequence on the depth of scour. The results obtained are presented. A method for computing the equilibrium stage of scour has been elaborated too. The methods can be employed for estimating possible damages of the abutments because of scour after successive floods by using the risk factor, predicting possible failure during the maintenance period, and proving at the stage of design that the structure is able to withstand expected or unexpected extreme flood events.

Keywords: Multiple floods, Scour, Sediment transport, Abutment, Risk factor

1 INTRODUCTION

During the past decade, several high floods that occurred in Europe were the reason for failures of bridges, environmental damages, and economic losses.

The equilibrium, or temporal, stage of scour near hydraulic structures was studied by many authors, and new approaches have been elaborated by Cardoso & Bettess (1999), Kothyari & Ranga Raju (2001), Balio & Orsi (2001), Radice et al. (2002), Hager et al. (2002), Armitage & McGahey (2004), Yanmaz & Celebi (2004), Grimaldi et al. (2006), Gjunsburgs et al. (2001, 2004, 2007, 2008), Tregnagh et al. (2008), and Yanmaz & Kose (2009).

For computing the equilibrium or temporal depth of scour, the discharge on the peak of the flood was used; it is not restricted in time for the whole maintenance period of bridge crossings, but is time-restricted for temporal scour estimation. In field conditions, the scour is formed by multiple floods of different probability, duration, frequency, and sequence.

A method for calculating the general scour development in time during the floods at bridge crossings under conditions of clear water and bed sediment movement was first presented by Rotenburg (1969).

However, no formulae are available for computing the depth of scour formed in time near bridge abutments during multiple floods, predicting the development of scour depth before, during, or after the floods, proving the safety of the abutments, and taking necessary protection measures.

The scour hole parameters (depth, width, and volume) during floods under clear-water conditions in the floodplain are summed up and increase from flood to flood. Hence, it is impossible to predict how multiple floods will affect the scour depth at the abutment and to know whether it will or will not be destroyed after a current or forthcoming event, whether the scour depth will exceed or not the designed equilibrium depth if the floods are higher than the calculated ones, and how long the structure will stay undamaged and safe enough after unexpected multiple flash floods. There are no answers to these and other questions.

Using the differential equation of equilibrium of the bed sediment movement in clear water, a method for calculating the scour development in time at bridge abutments during floods has been elaborated. The agreement between the experimental and calculated results (Gjunsburgs & Neilands, 2004) allows us to use this method for computer modeling of the scour process in nature during floods with different probability, duration, frequency, and sequence. This method enables us to compute the scour depth at any stage of the flood during the maintenance period or at the stage of designing the bridge crossings.

It was found that the scour parameters increase with decreasing probability and with increasing duration and frequency of the floods. The sequence of floods can increase or reduce the scour development in time, depending on their probability. The successive floods of the same probability considerably increase the value of scour depth.

A method for computing the equilibrium stage of scour has also been elaborated. It was accepted that the equilibrium scour depth at the abutments is reached when the local velocity becomes equal to the critical one.

The proposed methods allow us to estimate possible damages of the foundations of bridge abutments, caused by scouring in multiple floods, or prove their stability, by using the risk factor R.

2 METHODS

2.1 *Equilibrium depth of scour*

The discharge across the width of a scour hole before and after the scour is determined as follows:

$$Q_f = Q_{sc} \cdot k \tag{1}$$

where Q_f = discharge across the width of the scour hole with a plain bed, Q_{sc} = discharge across the scour hole with a scour depth h_s, and k = coefficient of changes in discharge because of scour, which depends on the flow contraction (Gjunsburgs & Neilands, 2004).

Now, we have

$$mh_s h_f V_l = k\left(mh_s h_f + \frac{mh_s}{2} h_s \right) \cdot V_{lt} \tag{2}$$

where m = the steepness of scour hole, mh_s = width of the scour hole, V_l = local velocity with a plain bed (Gjunsburgs & Neilands 2004), h_f = water depth in the floodplain, h_s = scour depth, and V_{lt} = local velocity at a scour depth h_s.

From Eq. (2), we find the local velocity for any depth of scour

$$V_{lt} = \frac{V_l}{k\left(1 + \dfrac{h_s}{2h_f}\right)} \tag{3}$$

The critical velocity at the plain bed can be determined by the Studenicnikov (1964) formula:

$$V_0 = 3.6 d_i^{\,0.25} h_f^{\,0.25} \tag{4}$$

where d_i = grain size of the bed materials.

The critical velocity for any depth of scour h_s and for the flow bended by the bridge crossing embankment is

$$V_{0t} = \beta \cdot 3.6 \cdot d_i^{0.25} \cdot h_f^{0.25} \left(1 + \frac{h_s}{2h_f}\right)^{0.25} \tag{5}$$

where β = reduction coefficient of the critical velocity at the bended flow determined by using the Rozovskyi (1956) approach; $h_f\left(1 + h_s / 2h_f\right)$ = average depth of the flow at h_s:

$$h_m = \frac{w}{b} = \frac{mh_s \cdot h_f + mh_s \dfrac{h_s}{2}}{mh_s} = h_f\left(1 + \frac{h_s}{2h_f}\right) \tag{6}$$

where w = area of the cross section and b = width of the scour hole.

At the equilibrium stage, the local velocity becomes equal to the critical one, $V_{lt} = \beta V_{0t}$ and $h_s = h_{equil}$:

$$\frac{V_l}{k\left(1 + \dfrac{h_{equil}}{2h_f}\right)} = \beta \cdot 3.6 \cdot d_i^{0.25} \cdot h_f^{0.25} \left(1 + \frac{h_{equil}}{2h_f}\right)^{0.25} \tag{7}$$

where $3.6 d_i^{0.25} h_f^{0.25} = V_0$

or

$$\frac{V_l}{k\left(1 + \dfrac{h_{equil}}{2h_f}\right)} = \beta \cdot V_0 \left(1 + \frac{h_{equil}}{2h_f}\right)^{0.25} \tag{8}$$

or

$$\frac{V_l}{k\beta V_0} = \left(1 + \frac{h_{equil}}{2h_f}\right)^{1.25} \tag{9}$$

From Eq. (9), we determine the value of h_{equil}

$$h_{equil} = 2h_f\left[\left(\frac{V_l}{k\beta V_0}\right)^{0.8} - 1\right] \quad (10)$$

According to Yaroslavcev (1956), the depth of scour depends on the side-wall slope of the construction.

According to Richardson and Davis (1995), the scour depth depends on the angle of flow crossing (k_α). In our study, $\alpha = 90°$ and $k_\alpha = 1$.

The equilibrium depth of scour can be presented by the next equation:

$$h_{equil} = 2h_f\left[\left(\frac{V_l}{k\beta V_0}\right)^{0.8} - 1\right] \cdot k_m \cdot k_\alpha \quad (11)$$

where h_{equil} = the equilibrium depth of scour for the designed probability of floods, for example, for the flood with a return period of 100 years. The equilibrium depth of scour can be exceeded by higher or multiple floods during the period of maintenance of the bridge crossing.

2.2 *Scour development in time during multiple floods*

The differential equation of equilibrium for the bed sediment movement in clear-water conditions has the form:

$$\frac{dw}{dt} = Q_S \quad (12)$$

where w = volume of the scour hole, which, according to the test results, is equal to $1/6\pi m^2 h_s^3$, t = time, and Q_s = sediment discharge out of the scour hole. The volume and shape of the scour hole are independent of the contraction rate of the flow (Gjunsburgs & Neilands 2004).

The left-hand part of Eq. (12) can be written as

$$\frac{dw}{dt} = \frac{1}{2}\pi m^2 h_s^2 \frac{dh_s}{dt} = a h_s^2 \frac{dh_s}{dt} \quad (13)$$

where h_s = scour depth, m = steepness of the scour hole, and $a=1/2\pi m^2$.

The sediment discharge was determined by the Levi (1969) formula:

$$Q_S = AB \cdot V_l^4 \quad (14)$$

where $B = mh_s$ = width of the scour hole, V_l = local velocity at the abutments with a plain bed, and A = parameter in the Levi (1969) formula. At a plain river bed, the formula for A = A_1 reads

$$A = \frac{5.62}{\gamma}\left(1 - \frac{\beta V_0}{V_l}\right)\frac{1}{d_i^{0.25} \cdot h_f^{0.25}} \quad (15)$$

where γ = specific weight of sediments.

The parameter A depends on the scour, local velocity V_l, critical velocity V_0, and grain size of the bed material during the floods:

$$A_i = \frac{5.62}{\gamma}\left[1 - \frac{k\beta V_0}{V_l}\left(1 + \frac{h_s}{2h_f}\right)^{1.25}\right] \cdot \frac{1}{d_i^{0.25} \cdot h_f^{0.25}\left(1 + \frac{h_s}{2h_f}\right)^{0.25}} \quad (16)$$

The sediment discharge also changes with the local velocity V_{lt}. Then, we replace V_l in Eq. (14) with the local velocity at any depth of scour V_{lt} from Eq. (3). The parameter A in Eq. (14) is replaced with the parameter A_i from Eq. (16). The sediment discharge upon development of the scour is

$$Q_s = A_i \cdot mh_s \cdot V_{lt}^4 = b\frac{h_s}{k^4\left(1 + \frac{h_s}{2h_f}\right)^4} \quad (17)$$

where $b = A_i m V_l^4$.

The hydraulic characteristics, such as contraction rate of the flow, the velocities V_0 and V_l, the grain size in different bed layers, the sediment discharge, and the depth, width, and volume of the scour hole, varied during the floods.

Taking into account formulas (13) and (17), the differential equation (12) can be written in the form

$$a h_s^2 \frac{dh_s}{dt} = b\frac{h_s}{k^4\left(1 + \frac{h_s}{2h_f}\right)^4} \quad (18)$$

After separating the variables,

$$\frac{k^4 \cdot a}{b} h_s \left(1 + \frac{h_s}{2h_f}\right)^4 dh_s = dt$$

or

$$D_i h_s \left(1 + \frac{h_s}{2h_f}\right)^4 dh_s = dt \quad (19)$$

where

$$D_i = \frac{k^4 a}{b} = \frac{\pi \cdot m \cdot k^4}{2A_i \cdot V_l^4}$$

After integration of Eq. (19), we have:

$$t = D_i \int_{x_1}^{x_2} h_s \left(1 + \frac{h_s}{2h_f}\right)^4 dh_s \tag{20}$$

where $x_1=1+h_{s1}/2h_f$ and $x_2=1+h_{s2}/2h_f$ = relative depths of scour.

According to the method, the hydrograph was divided into time steps, and each step in turn was divided into time intervals (Fig. 1). It was assumed that D_i was constant inside the time interval.

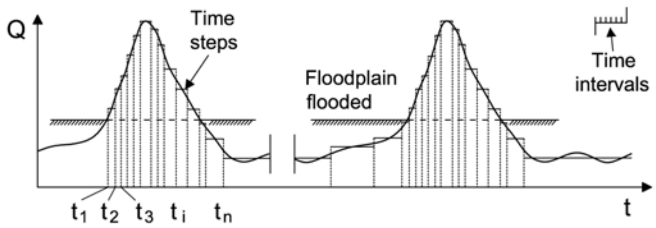

Figure 1. Hydrographs divided into time steps and time intervals.

After integration with new variables, $x=1+h_s/2h_f$, $h_s=2h_f(x-1)$, and $dh_s=2h_f dx$, we obtain

$$t = 4D_i h_f^2 \left(N_i - N_{i-1}\right) \tag{21}$$

where $N_i=1/6x_i^6-1/5x_i^5$, $N_{i-1}=1/6x_{i-1}^6-1/5x_{i-1}$, and $x=1+h_s/2h_s$ are the relative depths of scour.
From Eq. (21), the value of N_i can be found

$$N_i = \frac{t_i}{4D_i h_f^2} + N_{i-1} \tag{22}$$

where t_i = time interval.

Using the graph $N = f(x)$ for the calculated value of N_i, we find x_i and the depth of scour at the end of time interval:

$$h_s = 2h_f\left(x - 1\right) \tag{23}$$

We assume that the scour depth depends on the slope of the side wall (Yaroslavcev, 1956) described by the coefficient k_m and on the angle of flow crossing (Richardson & Davis, 1995). In our study, the angle of flow crossing was 90° and k_α = 1.
Then, Eq. (23) can be given in the form

$$h_s = 2h_f\left(x - 1\right) \cdot k_m \cdot k_\alpha \tag{24}$$

To determine the scour depth development during the flood or multiple floods, the hydrograph was divided into time steps with duration of 1 or 2 days, and each time step was divided into time intervals up to several hours. In laboratory tests, the time steps were divided into 20 time intervals. For each time step, the following parameters must be determined: the water depth in the floodplain h_f; contraction flow rate Q/Q_b, where Q = discharge of flow and Q_b = discharge in the bridge opening under open-flow conditions; the maximum backwater Δh determined by the Rotenburg (1965) method [a comparison of the values of Δh obtained in the tests with those calculated by Rotenburgh (1965) was illustrated earlier (Gjunsburgs & Neilands, 2004) and gave good results]; grain size d_i; thickness H of the bed layer with d_i; the specific weight γ of the bed material. As a result, we have V_l, V_{lt}, V_0, V_{0t}, A, A_i, D_i, N_i, N_{i-1}, x, and h_s at the end of time intervals and finally at the end of the time step. For the next time step, the flow parameters were changed because of the flood and because of the scour developed during the previous time step.

During the last decade, a lot of bridges were destroyed or damaged due to scour, which means that the designed equilibrium depth of scour was exceeded by unexpected floods.

Using the two above-mentioned methods, the flood-damage risk factor for abutments can be estimated. The time-dependent depth of scour computed for floods with different probability, duration, frequency, and sequence [Eqs. (22) and (24)] is compared with the designed equilibrium depth of scour [Eq. (11)], and the flood-damage risk factor R for the abutments is determined as the ratio of $\Sigma h_s/h_{equil\ des.}$:

$$R = \frac{\Sigma h_s}{h_{equil\ des.}} \cdot 100 \tag{25}$$

where Σh_s = depth of scour development after several floods and $h_{equil\ des.}$ = designed equilibrium depth of scour. The designed equilibrium depth of scour can be exceeded by high flash floods or unexpected multiple floods with a lower probability than the designed flood. Then the value of Σh_s can be greater than $h_{equil\ des.}$, the scour depth can exceed the designed foundation level, and the structure can be damaged.

3 RESULTS

The contraction of the river by abutments leads to streamline concentration, local increase in velocities, vortex structures, increased turbulence, and local scour at the abutments. To study the scour process at abutments, tests in the conditions of different contraction rates of the flow, grain sizes of bed material, Froude numbers of the open flow, different depths of the floodplain, and steady and unsteady flow conditions were carried out.

The test results obtained in the conditions of open flow, rigid and sand bed, and steady or unsteady flow have been published previously (Gjunsburgs & Neilands, 2004). The duration of the tests was 7 hours for a steady and 14 hours for an unsteady flow. The condition that $Fr_R = Fr_f$ was fulfilled, where Fr_R and Fr_f were the Froude numbers for the plain river and for the flume, respectively. The length scale was 50 and the time scale was 7. Relative to the real conditions, the test time was equal to 2 days. This was the mean duration of time steps, into which the flood hydrograph was divided (Fig. 1).

The experimental data for open flow conditions, as well as comparisons between the values of local velocities and scour depth at the abutment obtained in tests and calculations have also been presented previously (Gjunsburgs & Neilands, 2004). Some comparison results between the experimental and calculated scour depth at the abutments are shown in Table 1, where Q/Q_b = flow contraction rate, Q = flow discharge in tests, Q_b = discharge in bridge opening under open flow conditions, h_f = water depth in the floodplain, L_b = length of the bridge model opening, Δh = maximum backwater value, V_l = local velocity at a plain bed, and t = duration of tests.

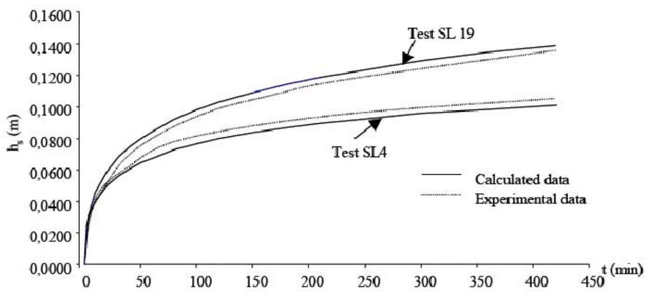

Figure 2. Scour development in time under steady flow conditions.

The patterns of the scour development in time, both determined in tests and computed by the suggested method, are similar, namely the rapid development at the start of the scour process and gradual reduction with time (Figs. 2 and 3) was observed.

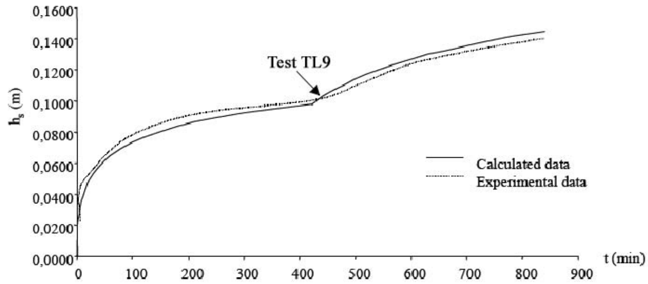

Figure 3. Scour development in time under unsteady flow conditions.

Based on the results obtained in laboratory tests, a computer modeling of the time-dependent scour during multiple floods with different probability, duration, frequency, and sequence was performed, the equilibrium stage of scour was examined, and the risk factor for estimating the safety and stability of the abutments was determined.

Figure 4 shows the scour development in time for the discharge with a return period of 1 and 4 times over 100 years. The scour hole at the abutments is deeper for the flood of a lower probability.

Table 1. Comparison between the experimental and calculated values of scour depth at the abutments

Tests	Q (l/s)	$\frac{Q}{Q_b}$	h_f (cm)	L_b (cm)	Δh (cm)	V_l (cm/s)	t (hours)	$h_{s\,exp.}$ (cm)	$h_{s\,calc.}$ (cm)	$\frac{h_{s\,exp.}}{h_{s\,calc.}}$
SL1	16.60	5.27	7	50	2.20	36.81	7	13.30	12.91	1.03
SL2	22.70	5.69	7	50	3.60	43.20	7	16.70	16.40	1.02
SL3	23.60	5.55	7	50	3.95	46.61	7	17.30	17.61	0.98
SL4	16.60	3.66	7	80	1.19	33.04	7	10.10	9.79	1.03
SL5	22.70	3.87	7	80	1.79	39.76	7	12.80	13.03	0.98
SL6	23.60	3.78	7	80	2.35	45.85	7	16.22	15.45	1.05
SL7	16.60	2.60	7	120	0.60	26.76	7	6.40	5.94	1.077
SL8	27.70	2.69	7	120	0.99	33.87	7	9.81	9.22	1.06
SL9	23.60	2.65	7	120	1.28	38.77	7	11.62	11.28	1.03
SL13	35.48	4.05	13	80	1.42	34.65	7	9.44	9.68	0.97
SL14	41.38	3.99	13	80	1.80	39.26	7	12.30	12.62	0.97
SL15	47.10	4.05	13	80	2.70	49.15	7	18.30	17.82	1.03
SL17	22.70	3.87	7	80	1.79	39.76	7	9.60	9.68	0.97
TL3	23.60	3.78	7		2.35	45.85		12.53		
	47.10	4.05	13	80	2.70	49.15	14	24.20	23.14	1.05

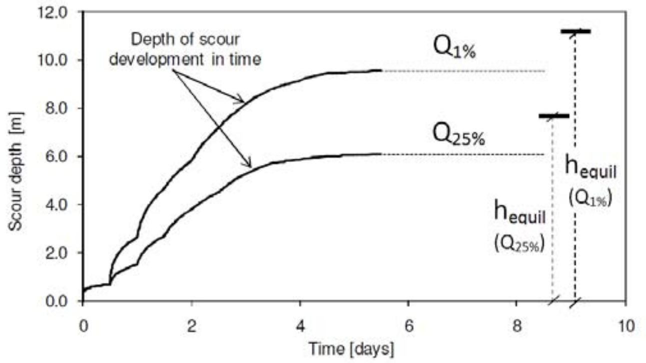

Figure 4. Time development of scour depth with discharges of different probabilities.

In the case of SL14 test (Table 1) with a discharge of 41.38 l/s, after 7 hours, the scour depth was 12.30 cm compared with the calculated value of 12.62 cm; but, in the SL17 test with a discharge of 22.7 l/s, these values were 9.60 cm and 10.0 cm, respectively. The contraction rate of the flow, grain size of the bed material, and thickness of the bed layers were the same. It was found that the higher discharge, the deeper the scour hole.

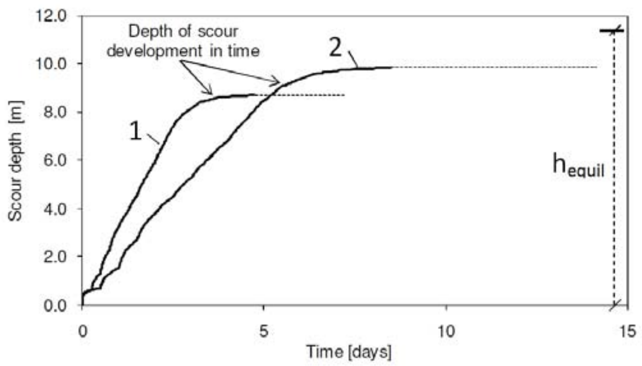

Figure 5. Scour development in time: 7-day duration (1) and 14-day duration (2).

The scour development in time for the floods of different duration is illustrated in Fig. 5. It is seen that the scour depth increases with the flood duration, i.e., the greater duration of the tests, the deeper the scour depth. In the TL3 test (Table 1), the scour depth was 12.53 and 24.2 cm after 7 and 14 hours, respectively.

To investigate the influence of the flood frequency on the scour development in time, we choose a period of, for example, 5 years and suppose that, during this period, we have two or four floods of the same probability.

It is obvious that an increase in the frequency of the floods is accompanied by an increase in the scour depth, and it follows from Fig. 6 that the scour depth after two floods at an accepted period of time h_{s1} is less than that after four floods occurred during the same period h_{s2}. After every flood, the depths of scour are summed up, and finally the equilibrium stage can be reached.

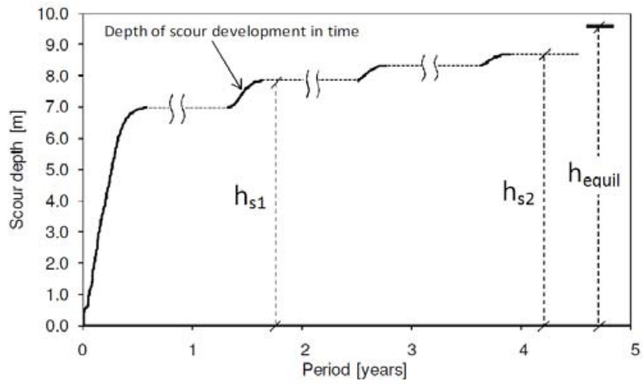

Figure 6. Scour development in time for floods with different frequencies, where h_{s1} and h_{s2} are the scour depths after two and four floods, respectively.

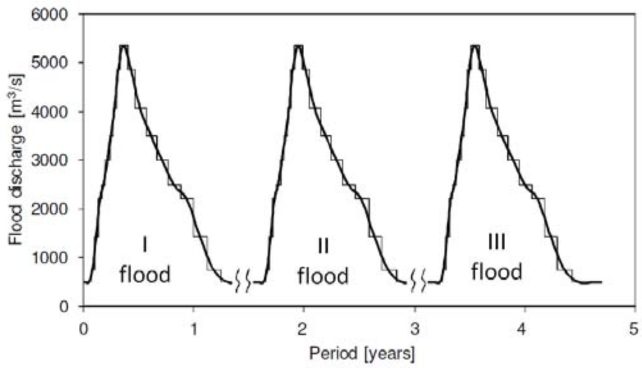

Figure 7. Sequence of three floods with the same probability.

The influence of the sequence of floods with a different probability on the scour development in time was examined according to three scenarios (Figs. 7-9).

Figure 7 shows a scheme of three floods of the same probability. The scour starts when the floodplain is flooded and increases rapidly. Because of the scour hole developed, in the second flood, the scour process starts at the step of hydrograph when $V_{lt\,II} \geq \beta V_{0t\,II}$ and has less duration, while for the third flood the velocities change due to the scour developed after the two previous floods, and it begins at $V_{lt\,III} \geq \beta V_{0t\,III}$ (Fig. 10).

Figures 8 and 9 present different sequences of multiple floods. For a river basin with certain parameters, accepted as an example, the floods with return periods of 1 and 4 for 100 years were calculated.

Figure 8 shows the sequence of floods: in the first example, the high flood was followed by two lower floods. In the TL3 test, the discharge of 23.60 l/s was followed by that of 47.1 l/s, and the depth of scour was 24.20 cm.

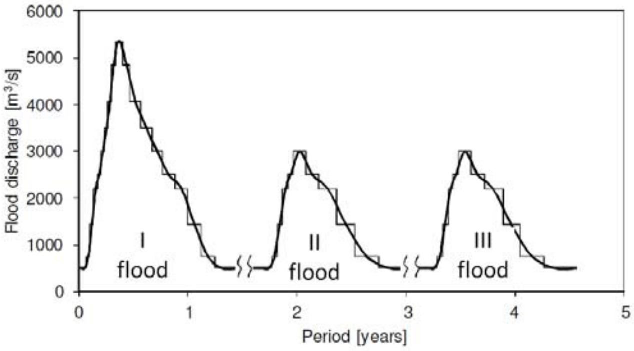

Figure 8. Flood sequence: the first flood with a less probability than the next two.

As seen from Fig. 8, during the first flood, the scour depth develops and remains the same till the next flood. The local velocity V_{lt} reduces [Eq. (3)] but the critical velocity V_{0t} increases [Eq. (5)] because of the scour depth developed during the previous flood. In the next flood, the capacity of the flow is not sufficient to remove sediments out of the scour hole, and V_{lt} is less than βV_{0t}. In the second and third floods, the scour depth remains the same, as after the first flood (Fig. 10).

During the scour process, the parameters V_0, V_l, V_{0t}, V_{lt}, A_i, D_i, N_i, N_{i-1}, and x change.

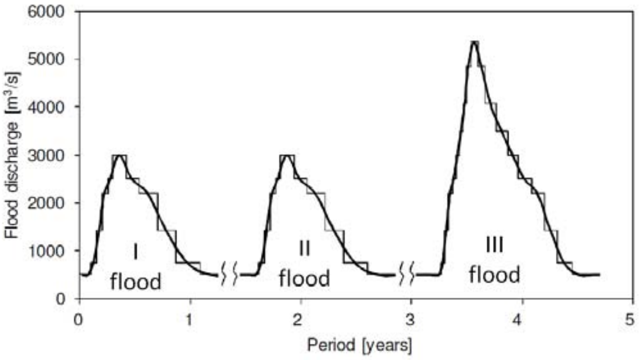

Figure 9. Flood sequence: the first two floods with a higher probability than the third flood.

As seen from Fig. 9, two floods with a return period of 25 years are followed by the flood with a return period of 100 years. The scour depth develops during the first and the second floods; in the third flood, the scour starts at the step of hydrograph when $V_{lt} \geq \beta V_{0t}$ and develops rapidly due to the increased discharge of the flow (Fig. 10).

Thus, the depth of scour during expected or extreme flood events depends on their probability, duration, frequency, and sequence.

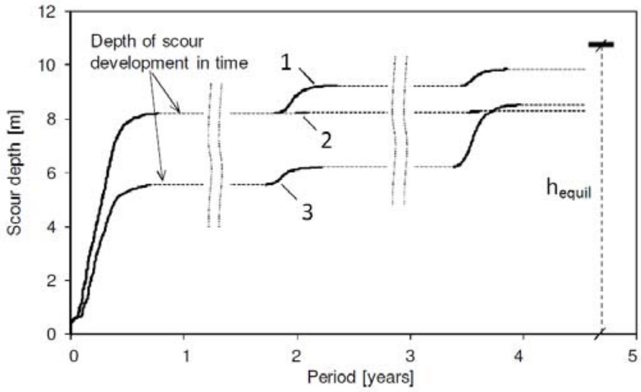

Figure 10. Scour development in time for floods of different sequences: 1- as in Fig. 7, 2- as in Fig. 8, and 3- as in Fig. 9.

4 CONCLUSIONS

During the last decade, a lot of bridges were destroyed or damaged because of scour, and this means that the designed equilibrium depth of scour was exceeded by unexpected floods. The reason was high floods which come more frequently and have different sequence and duration. In some cases, the designed equilibrium depth of scour was exceeded, and bridge crossings were destroyed.

A new approach has been elaborated for estimating the safety and stability of the abutments under the loads of multiple floods with different probability, duration, frequency, and sequence.

The differential equation of equilibrium of the bed sediment movement in clear water conditions was used, and a method for computing scour development in time at the abutments during multiple floods was elaborated. The method was confirmed by experimental results.

A computer modeling of the scour process based on the method suggested [Eqs. (22) and (24)] was performed, and the influence of floods with different probability, duration, frequency, and sequence on the scour depth at the abutments was determined. It was found that, with a less probability, increased duration and frequency of the floods, and certain sequences of different probability, the scour depth at the abutments increases.

A method for computing the designed equilibrium stage of scour has been elaborated, too.

Using the method for computing the designed equilibrium stage of scour and the method for computing scour development in time during multiple floods, the flood-damage risk factor for abutments can be estimated. The time-dependent depth of scour computed for floods with different probability, duration, frequency, and sequence [Eqs. (22) and 24)] is compared with the designed equilibrium depth of scour [Eq. (11)], and the flood-damage risk factor R for the abutments is determined [Eq. (25)]. The designed equilibrium depth of scour can be exceeded by high flash floods or unexpected multiple floods with lower probability than the designed flood. Then the value of Σh_s can be greater than $h_{equil\ des.}$, the scour depth can exceed the designed foundation level, and the structure can be damaged.

The flood-damage risk factor is suggested for assessment of possible damages of the abutments during unexpected multiple or flash floods with different probability, duration, frequency, and sequence. This factor can help to predict possible failures during the maintenance period and, at the stage of design, make sure that the structure is able to withstand expected or unexpected flood events.

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