Integrated Effect of Parameter Uncertainty in Riverbank Stability Modelling

A. Samadi & E. Amiri-Tokaldany
Department of Irrigation and Reclamation Engineering, University of Tehran, Karaj, Iran

S.E. Darby
School of Geography, University of Southampton, Highfield, Southampton, SO17 1BJ, UK

ABSTRACT: To predict the undesirable effects of bank retreat and to inform effective measures to prevent it, a wide range of bank stability models have been presented in the literature. These models typically express bank stability by defining a factor of safety as the ratio of driving and resisting forces acting upon the incipient failure block. These forces are affected by a range of controlling factors including the bank profile (bank height and angle), the geotechnical properties of the bank materials, as well as the hydrological status of the riverbanks. In this research, to disclose the integrated effect of the various parameters affecting the stability of riverbanks, the impacts of simultaneous changes of parameters affecting bank stability are evaluated in a series of sensitivity analysis test using a simple model of bank stability analysis. In order to investigate integrated effect of parameters uncertainty, all effective parameters have been divided into two groups, i.e., strengthening and weakening parameter groups. The results of similar and converse variations of two groups of parameters on the main factor of safety are shown in the paper.

Keywords: Stability analysis, Riverbank, Planar failure, factor of safety, Uncertainty

1 INTRODUCTION

Due to direct benefits for human civilization (e.g. agricultural and industrial water supply, construction of hydro-electric power stations, navigation improvement) as well as protecting against floods and other river disasters, rivers have long been a subject of interest for scientists and engineers. Among different aspects in river engineering, the stream channel width is one of the most reliable and indicative variables for describing stream characteristics and morphology (Andrews, 1982). Bank retreat is also a key process in fluvial dynamics, affecting a wide range of physical, ecological, and socioeconomic issues in the fluvial environment (Rinaldi and Darby, 2008). Bank erosion processes may be responsible for the delivery of large volumes of sediment, with associated sedimentation hazards in the downstream reaches of a fluvial system, which in turn, may represent a significant problem in river management (Rinaldi & Casagli, 1999). Odgaard (1987) stated that the weight of silt and clay entrained into the water from cut banks is estimated to be 30-40 percent of the suspended load of the East Nishnabotna and Des Moines Rivers in Iowa,

USA. Odgaard (1987) also reported that the U.S. Army Corps of Engineer's (1983) study of the sediment budget of the Sacramento River, California, showed that, of the 11.5 million tons of total sediment inflow to the system, 6.8 million tons (59 percent) is derived from bank erosion.

Although a wide range of individual processes can contribute to riverbank retreat (Thorne, 1982; ASCE Task Committee, 1998), the erosion of bank material through mass wasting is probably the most serious from the perspective of water resources management (e.g., Dapporto et al., 2003). This is because mass wasting involves rapid channel widening and the near-instantaneous delivery of large volumes of sediment to the channel. Several factors can lead to the onset of mass failure. These include reductions in the strength of the bank materials as a result of weathering (e.g., Lawler, 1993; Couper and Maddock, 2001), spatially controlled changes in the geometry of the failure block from fluvial erosion (e.g., Hooke, 1979; Simon et al., 1999; Amiri-Tokaldany et al., 2003; Rinaldi et al., 2004; Darby et al., 2007; Rinaldi and Darby, 2008)**,** the development of tension cracks (Darby and Thorne, 1994), hydrographic characteristics (e.g., Rinaldi et al., 2004),

the presence of certain types of vegetation (e.g., Smith, 1976; Pizzuto and Meckelnberg, 1989; Thorne, 1990; Millar and Quick, 1998; Simon and Collison, 2002; Pollen and Simon, 2005; Pollen, 2007; Van De Wiel and Darby, 2004, 2007), soil moisture conditions and seepage forces (e.g., Thorne, 1990; Rinaldi and Casagli, 1999; Simon et al., 1999) and the presence of negative pore water pressure in the unsaturated portion of banks (e.g., Rinaldi and Casagli, 1999; Simon et al., 2000; Simon and Collison, 2002). These factors have been taken into account in a wide range of models that have been introduced to analyze the stability of riverbanks with respect to a range of specific types of bank failures (e.g., Thorne and Tovey, 1981; Osman and Thorne, 1988; Simon et al., 1991, 2000; Darby and Thorne, 1996; Rinaldi and Casagli, 1999; Simon and Collison, 2002; Amiri-Tokaldany et al., 2003).

In order to apply bank stability models, estimating the value of the controlling parameters that represent the various factors that affect the stability of riverbanks is necessary. These factors include (i) the bank profile (typically represented using the bank height and bank angle); (ii) the geotechnical characteristics of the bank materials (cohesion, friction angle, and density of the soil material); (iii) stream flow characteristics (e.g., water surface elevation and groundwater table elevation) that control the hydrological status (in particular the pore water pressure distribution) of the riverbanks. The degree to which the values of these parameters can be determined accurately varies according to whether they can be estimated either via direct field or laboratory measurements (though even in this case the magnitudes of the associated measurement errors will still vary) or by some other indirect means (e.g., through the use of models to estimate bank pore water pressures, tension crack depths, and the failure plane angle). The varying extents to which these controlling factors can be parameterised accurately suggests that each parameter may exert a varying influence in terms of generating uncertainty in the analysis of bank stability. Samadi et al. (2009) have shown that care should be taken when estimating the values of river bank height, river bank angle, flow depth, bank material cohesion, and the bank material unit weight as these are the most sensitive parameters. Conversely, a cruder estimation of tension crack depth, the soil internal friction angle, ground water level, and the matric suction angle, may still provide a reasonable degree of accuracy in the bank stability analysis, due to the relatively low model sensitivity to this group of parameters. These results are only in partial agreement with the results of a similar sensitivity analysis undertaken by Langendoen and Simon

(2008). Based on their results, they suggested that to evaluate bank stability accurately special attention must be paid to determining the values of the bank material cohesion, in agreement with Samadi et al. (2009) results and groundwater table, in contrast to the Samadi et al. (2009) results. Differences between results of these two studies may partly reflect the differences in the bank stability models employed in the two studies, but they may also be related to the specific baseline parameter values employed in the two sensitivity analyses. This is the problem of contingency. Although separate uncertainties associated with estimating the values of several parameters affecting the stability of riverbanks with respect to planar failure have been investigated recently, we need to know about the integrated effects of parameters uncertainty in the reliability of riverbank stability modelling.

In order to investigate the integrated effect of parameter uncertainty, all effective parameters have been divided into two main groups, including strengthening and weakening parameter groups (see Table 2 for more details). The results of similar and converse variations of two parameter groups on the main factor of safety are shown in the paper. The results together with previous research are used to provide guidelines for parameterizing bank stability models that are based on the planar type failure mechanism, which is the riverbank stability analysis that has most commonly been employed in the geomorphic literature.

2 RIVERBANK STABILITY ANALYSIS THEORY

Among various types of river bank failure, planar failure is the most common type, being associated with steep, relatively low, banks with thin cohesive layers (Thorne, 1999). The stability analysis of these banks can be carried out by computing the ratio of resisting and driving forces applied to the most critical failure surface. This surface can be determined by performing the stability analysis iteratively for different failure surfaces, considering the variation of forces affecting the stability in each case. In Figure 1, the framework for analysing the stability of a natural river bank, together with the forces acting on the incipient failure block, are illustrated. For each iteration, the factor of safety is estimated using:

$$FS_p = \frac{FR_p}{FD_p} \tag{1}$$

where FS_p = the factor of safety against block sliding ($FS_p < 1$ indicating the onset of failure), and

FR_p and FD_p = the resultant resisting and driving forces acting on a unit width of the failure block, respectively. Hence, bank failure is predicted to occur once the ratio of resisting and driving forces falls below unity. A large number of riverbank stability analyses exist for planar failures (e.g., Osman and Thorne, 1988; Darby and Thorne, 1996; Rinaldi and Casagli, 1999; Simon et al., 1999; Amiri-Tokaldany et al., 2003; among others), with each model varying in the ways they simulate the resisting and driving force terms in Eq. (1). For the purposes of this research, we elected to employ a bank stability analysis (Amiri-Tokaldany, 2002; Amiri-Tokaldany et al., 2003) that is relatively simple but which is able to account for most of the factors known to influence the stability of banks subject to planar type failures: layered riverbanks, the presence of tension cracks, as well as the effects of pore water and hydrostatic confining pressures.
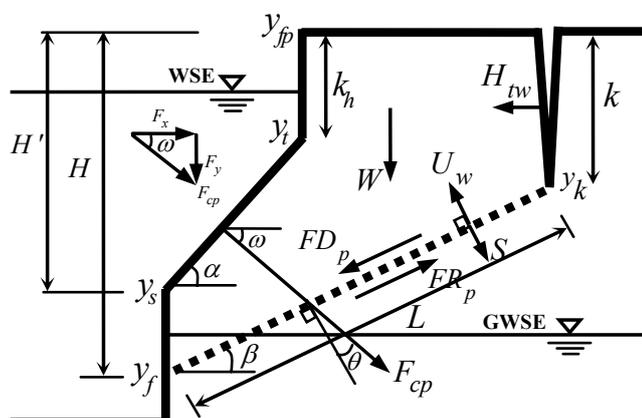

Figure 1. The bank geometry and forces exerted on the incipient failure block.

In the model, the resultant driving force acting on a unit width of the failure block is given by (Amiri-Tokaldany, 2002; Amiri-Tokaldany et al., 2003):

$$FD_p = W \sin \beta - F_{cp} \sin \theta + H_{tw} \cos \beta \qquad (2)$$

where β = the failure plane angle, θ = the angle between the direction of the resultant of the hydrostatic confining pressure and a normal to the failure plane, W = the weight of a unit width of the failure block, F_{cp} = the hydrostatic confining pressure acting on a unit width of the failure block, F_x and F_y = horizontal and vertical component of confing pressure, ω = the angle at which the resultant of the confining pressure acts on the bank surface, and H_{tw} = the hydrostatic force exerted by the water present in the tension crack on a unit width of the failure block. Also, the resultant resisting force acting on the failure block is given by (Amiri-Tokaldany, 2002; Amiri-Tokaldany et al., 2003):

$$FR_p = C'L + S \tan \phi^b + (W \cos \beta + F_{cp} \cos \theta - U_w - H_{tw} \sin \beta) \times \tan \phi' \qquad (3)$$

where C' = the effective cohesion of the bank material acting along the surface of failure plane, L = the length of the failure plane, S = the resultant negative pore water pressure, ϕ^b = the angle expressing the rate of strength increase relating to the negative pore water pressure, U_w = the resultant uplift force or positive pore water pressure acting on a unit width of the failure block, and ϕ' = the effective internal friction angle of bank material. By geometry:

$$W = \gamma / 2 \left(\frac{H^2 - k^2}{\tan \beta} - \frac{H'^2 - k_h^2}{\tan \alpha} \right) \qquad (4)$$

where γ = soil unit weight, H = overall bank height, H' = uneroded bank height, k = tension crack depth, k_h = relic tension crack depth, α = bank angle. Finally, WSE and GWSE = the level of water surface in the river and in the ground, respectively, and y_{fp} = elevation of floodplain, y_s = elevation of base of uneroded bank slope, y_k = elevation of base of tension crack, y_t = elevation of base of relic tension crack, y_f = elevation of most critical failure plane. The model does not directly take into account the effect of shear stress exerted by water flowing in the channel upon the river bank materials. This is not a limitation because in most channels, the magnitude of shear stress exerted upon the bank materials is, compared to the hydrostatic force, negligible. The long-term effects of shear stress on the stability of the slope are, however, indirectly accounted for by changing the characteristics of the bank profile, such as the bank height and bank angle, in response to fluvial erosion.

2.1 Limitation of the stability model

Although the model selected for use in this research is able to account for all the key factors influencing the stability of banks subject to planar failures, all models are idealisations and as such recognising the inherent limitations of this one is helpful. Similar to many existing models, vegetation is not considered. Consequently, the effects of vegetation can only be accounted for by adjusting the parameter values (e.g., soil cohesion and unit weight) appropriately. In calculating the pore water pressures, we assume that the phreatic surface is parallel to the floodplain surface and its level changes with respect to the variation of the water table. Moreover, the distribution of water pressure in the channel adjacent to the bank is assumed to be hydrostatic. Because of the lack of

data regarding the relationship between soil moisture and the matric suction for most soil types, the possibility also exists of inaccurately estimating the effects of negative pore water pressure in the current model.

3 SENSITIVITY TESTS

Young (1999) highlights that since the inherent uncertainty associated with modelling most environmental systems is often acknowledged, it is surprising that many models are completely deterministic in nature. Research surrounding slope stability has revealed that the heterogeneity of soils provides a major source of uncertainty in estimations of operational shear strengths within all slope design applications (El-Ramly et al., 2005), and therefore is a well recognised issue within geotechnical research (El- Ramly et al., 2002).

Samadi et al. (2009) to determine the impacts of the independent parameters on bank stability, evaluated the effects of the parameters on the factor of safety (FS_p) in a series of model sensitivity analyses, following the approach adopted by Van de Wiel and Darby (2007). In this approach, a real riverbank located in northern Mississippi (hereafter referred to as the reference bank, Table 1) has been selected to provide a factor of safety representing marginal stability conditions ($FS = 1.091$), so that an opportunity exists for variations in the parameter values to either stabilize or destabilize the bank (Van de Wiel and Darby, 2007). Next, using the same riverbank, the impacts of variations in each individual parameter on the factor of safety are investigated, while keeping all other parameter values constant. In these sensitivity tests, the range over which each parameter was varied (Table 2) was selected as follows:

(i) The reference bank height (3.83 m) was varied, in the range from 1.4 to 7.3 m. These limits were arbitrarily selected, but they are sufficiently wide to encompass a wide range of riverbanks that are likely to experience the planar failures that are the subject of the current analysis. Banks that are lower than 1.4 m were not investigated because these tend to be stable across a wide range of geotechnical characteristics, while very high banks are often subject to rotational failures (Thorne, 1982).

(ii) The range of tension crack depth was selected on the basis that the maximum depth of the tension crack is half the bank height (Taylor, 1948; Thorne and Abt, 1993). Based on the reference bank height of 3.83 m, this gives an overall range of 0.0 to 1.9 m for this parameter.

(iii) The range of bank angle values (36° – 90°) used in the sensitivity tests was again arbitrarily

selected, while noting that the upper limit is the maximum possible and that bank slopes that are shallower than 36° were not investigated because they tend to be stable, even for very weak bank materials.

Table 1. Properties of the reference riverbank used in the simulations* (cited in Samadi et al., 2009).

	Variable	Value
Input variables	Bank height (m)	3.83
	Tension crack (m)	0.74
	Bank angle (°)	86
	Flow depth (m)	1.95
	Groundwater level (m)	1.95
	Bank material cohesion (Pa)	15500
	Soil unit weight (N/m³)	20860
	Friction angle (°)	15.42
	Matric suction angle (°)	18
Failure variables	Factor of safety (-)	**1.091**
	Slope of incipient failure plane (°)	50.3
	Width of failure block (m)	2.3
	Volume of failure block (m³)	5.36

* Note: Values of tension crack depth, river flow depth, groundwater table, and the matric suction angle were not measured, so their values were instead selected to ensure that the reference bank's stability is marginal (i.e., to give $FS_p = 1.091$).

Table 2. Range of parameter values used in the sensitivity analyses.

Series	Bank property	Parameter range	Factor of safety range	Parameter class *
1	Bank height (m)	1.4 $\xrightarrow{Increase}$ 7.3	5.55 $\xrightarrow{Decrease}$ 0.68	⇩
2	Tension crack depth (m)	0.0 $\xrightarrow{Increase}$ 1.9	1.26 $\xrightarrow{Decrease}$ 0.94	⇩
3	Bank angle (°)	36 $\xrightarrow{Increase}$ 90	3.03 $\xrightarrow{Decrease}$ 1.01	⇩
4	Bank material cohesion (Pa)	0 $\xrightarrow{Increase}$ 41000	0.15 $\xrightarrow{Increase}$ 2.43	⇧
5	Soil unit weight (N/m³)	10500 $\xrightarrow{Increase}$ 24000	2.21 $\xrightarrow{Decrease}$ 0.86	⇩
6	Friction angle (°)	9.5 $\xrightarrow{Increase}$ 50	0.83 $\xrightarrow{Increase}$ 1.71	⇧
7	Matric suction angle (°)	10 $\xrightarrow{Increase}$ 26	1.08 $\xrightarrow{Increase}$ 1.11	⇧
8	Groundwater level (m)	0.0 $\xrightarrow{Increase}$ 3.8	1.23 $\xrightarrow{Decrease}$ 0.80	⇩
9	Flow depth (m)	0.0 $\xrightarrow{Increase}$ 3.8	0.91 $\xrightarrow{Increase}$ 2.19	⇧

*Symbols: ⇧ = strengthening parameters,
⇩ = weakening parameters.

(iv) Geotechnical parameter values (bank material cohesion, unit weight, and friction angle) were initially defined according to the range of values reported in Darby's (2005) bank material database, but with the parameter ranges extended by a factor of ± 25% to ensure that the sensitivity tests conservatively encompass a wide range of natural riverbank material types.

(v) According to Rinaldi and Casagli (1999) and Simon et al. (1999), the magnitude of ϕ^b ranges from 10° to 26°, so this was the range used herein.

(vi) Based on the reference bank height, we assumed that both the water level in the river and the groundwater level change from their lowest

level (zero relative to the river bed) to the bankfull discharge level.

In the last research (Samadi et al., 2009), the implications of uncertainty associated with estimating the values of independent parameters affecting the stability of riverbanks with respect to planar failure have been investigated by undertaking a series of model sensitivity analyses. Based on an arbitrarily selected threshold precision of ± 15% as an index of significant impact on simulated factors of safety, results were presented in terms of the degree to which typical parameter uncertainties affect the reliability of simulated factors of safety (Table 3) for the relatively simple case of an arbitrary reference riverbank. Based on this analysis, uncertainties in estimating bank height, bank angle, bank material cohesion, and soil unit weight are large enough to significantly impact the reliability of bank stability modelling, at least at the ± 15% threshold, and for the specific conditions encountered at the reference riverbank employed herein. In contrast, typical uncertainties associated with the other controlling parameters (tension crack depth, friction angle, groundwater level, matric suction angle and flow depth) were not initially found to be large enough to adversely affect the reliability of bank stability modelling.

Table 3. Summary of the sensitivity analysis results indicating the significance of parameter uncertainty in affecting the reliability of simulated factors of safety* (cited in Samadi et al., 2009).

Bank parameter	Typical parameterization uncertainty (%)	Parameter value range needed to induce a 15% change in factor of safety (%)	Effect of parameter uncertainty on reliability of bank stability estimate
Bank height	± 72	-14.4 to +22.7	Highly Significant
Tension crack	± 72	-100.0 to +156.8	Insignificant
Bank angle	± 54	-8.6 to +4.7	Highly Significant
Bank material cohesion	± 220	-19.4 to +20.0	Highly Significant
Soil unit weight	± 26	-14.2 to +20.3	Significant
Friction angle	± 40	-65.6 to +67.3	Insignificant
Matric suction angle	± 48	>> ± 48%	Insignificant
Groundwater level	± 25	-100.0 to +61.5	Insignificant
Flow depth (m)	± 1	-66.7 to +30.8	Insignificant

* Note that the typical parameterization uncertainty is here taken as the largest of the two sources of uncertainty (measurement error and natural variability).

Although the specific quantitative results clearly depend on the model that Samadi et al. (2009) have selected for use in their study, their general findings are likely to be transferable to a wide range of other stability models. As such these findings present a number of important implications for investigators interested in applying bank stability models to analyse problems of riverbank retreat and width adjustment. First and foremost, greater attention should be paid to estimating the input parameter values accurately if the reliability of model predictions results are not to be con-

founded by those parameter uncertainties. Most previous studies have simply ignored these uncertainties. The key source of input parameter uncertainties is the inherent natural variability of the bank morphology and/or sedimentology, rather than measurement error per se. This implies that investigators should pay greater attention to the careful sampling of bank morphological and sedimentological parameters, such that the statistical variation can be defined more clearly. In particular, sampling strategies should seek to define the variability of these parameters by undertaking multiple replicate measurements. It is important to understand that this natural variability encompasses both spatial and temporal dimensions, presenting a challenging problem of sampling design. We recognize the logistical difficulties involved, but our findings show that comprehensive parameter sampling is necessary, given the magnitudes of the unreliabilities predicted herein. Indeed, these magnitudes are so high that they appear likely to confound the substantial improvements to the process basis of bank stability models that have been made in recent years (e.g., see review by Rinaldi and Darby, 2008). Only by adopting this practice can future bank stability modelling applications be made more rigorous than those undertaken up until now.

In order to determine uncertainties associated with estimating the values of effective parameters in riverbank stability modelling, integrated effect of parameters changing on the reliability of predicted factor of safety have been studied. Based on the previous sensitivity analysis results of independent parameters (e.g., Langendoen and Simon, 2008, and Samadi et al., 2009), the effective parameters have been categorized into two groups of strengthening and weakening parameters. In Table 2, increasing and reduction of strengthening parameters including material cohesion, internal friction angle, matric suction angle and river flow depth, increase and reduce riverbank stability respectively. But weakening parameters including bank height and angle, tension crack depth, soil unit weight and groundwater level, have a reverse effect on the riverbank stability, i.e., increasing and reduction of weakening parameters, reduce and increase riverbank stability, respectively.

Figure 2 shows the reverse changing effect of these two parameter groups on the factor of safety. In this figure, the effect of strengthening parameters increase and weakening parameters reduction along with strengthening parameters reduction and weakening parameters increase on the factor of safety is shown. The same rate of change is assumed for strengthening and weakening parameters in Figure 2, i.e., all strengthening and weakening parameters are changed with the same rate

of 2.5 percent on rising limb (top right) of curve, respectively. However, they are changed with a rate of 0.3 percent on falling limb (down left) of curve, respectively.
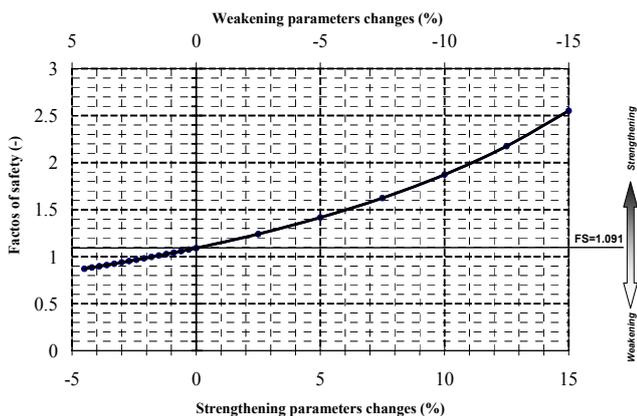

Figure 2. Simulated bank stability (factor of safety) as a function of reverse variations in a range of weakening parameters and strengthening parameters (see table 1 for definition of the reference bank).

In addition, for better understanding of parameter changes, the effects of separating changes of these parameters are shown in Figures 3-4. When we used the same rate for changing all parameters (like Figure 2), one can reach their limit value (min or max), while other parameters have not yet enough value. So to solve this problem in Figures 3 and 4, different rate of change has been given to the parameters. As can be seen, increasing the strengthening parameters accompanying with decreasing the weakening parameters from their main values, caused factor of safety rising with a steep slope, but reduction the strengthening parameters accompanying with increasing the weakening parameters from their main values, caused factor of safety decreasing with a gradual slope.
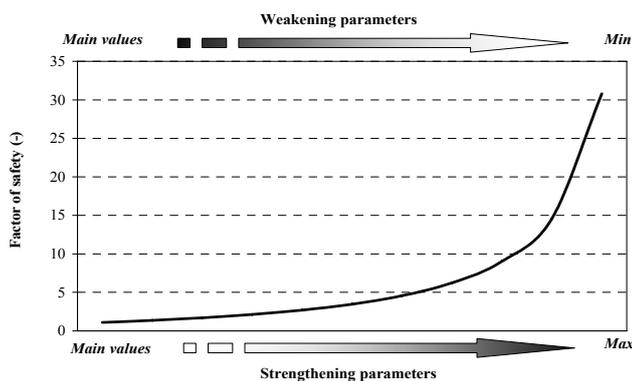

Figure 3. Schematic variations of bank stability (factor of safety) in regard to reduction of weakening parameters and increasing of strengthening parameters.

In addition, the effect of similar changes of all effective parameters on the riverbank stability analysis (i.e., the strengthening and weakening parameters) is shown in Figure 5. In this figure, increased and reduced effect of parameters on the factor of safety is clear. Based on this figure, it is concluded that integrated reduction of all parameters, increases all simulated factor of safety values. So, if in the riverbank stability modelling, all parameters were estimated less than the real value, this would have an undesirable effect on the stability analysis results and increased the simulated factor of safety incorrectly. The integrated increase of all parameters, reduces factor of safety partially to the limit equilibrium amount (i.e., FS = 1.0) and therefore it is concluded that integrated increase of parameters, will not influence the reliability of riverbank stability modeling results.
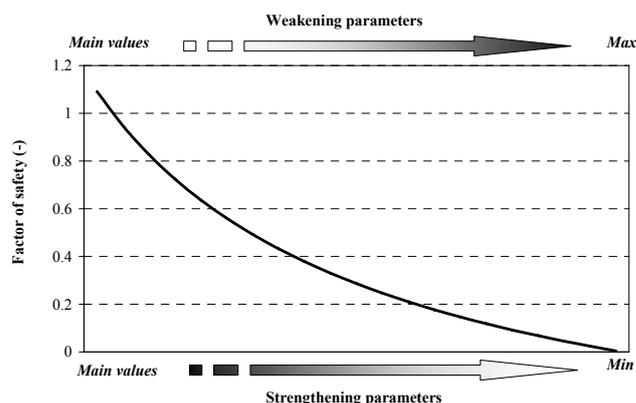

Figure 4. Schematic variations of bank stability (factor of safety) in regard to increase of weakening parameters and reduction of strengthening parameters
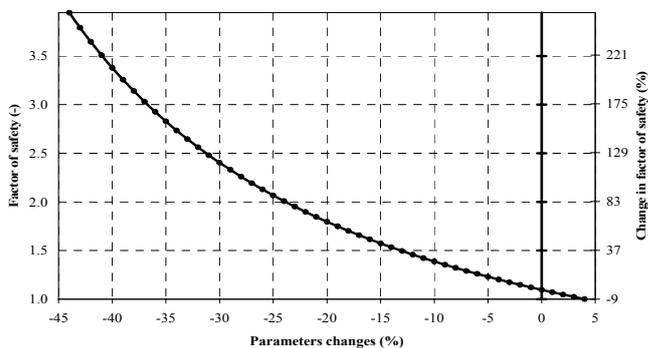

Figure 5. Simulated bank stability (factor of safety) as a function of integrated variations in a range of effective parameters (see table 1 for definition of the reference bank).

4 CONCLUSION

In this research, the integrated effect of uncertainties associated with estimating the values of all effective parameters on the reliability of riverbank stability modelling, has been investigated. For this purpose, all parameters were divided into two different groups; i.e., strengthening and weakening parameter groups. The results of similar and converse variations of two parameter groups on the main factor of safety are as following:
- By increasing and reducing the weakening parameters including bank height and angle, tension crack depth, soil unit weight and groundwater level, a reverse impact was observed on the factor

of safety; i.e., decreases and increases of riverbank stability, respectively.

- Increasing and reduction of strengthening parameters including material cohesion, internal friction angle, matric suction angle and river flow depth, increases and decreases riverbank stability, respectively.

- Reverse changes of strengthening and weakening parameters from their main values, caused factor of safety rising with a steep slope and decreasing with a gradual slope.

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