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Modeling of near-bank flow velocities during flow events as basis for developing bank erosion equations

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ABSTRACT: Riverbank erosion is known as a complex interaction between several processes, many of them poorly understood and probably some factors not even detected yet. Even though bank stability analyses exist that account for the main factors and couple mass wasting and fluvial erosion, field data at the process-scale is rarely available so that calibration and validation of these models often remain incomplete. Earlier studies showed that bank retreat is mainly fluvially controlled and bank erosion equations were developed by analyzing near-bank flow velocities. Even if calibration of these equations is valid only for a specific field site, also such bank erosion equations may deliver satisfying results without the need of bank parameterization and process analysis. In the Upper Drava Valley several restoration measures were implemented and further measures are planned. Most of the sites exhibit similar bank properties. In Kleblach-Lind, the widening of a side-arm is monitored since its construction in 2002. Bank erosion equation parameters derived there would be of value for planning further measures. Two data sets of bank retreat were available, one of a repeated bank edge survey during the initial widening, and one more detailed bank geometry data set obtained by terrestrial photogrammetry. Near-bank flow velocities at a defined distance from the bank were simulated with a 2D hydrodynamic-numerical model. The flow event hydrographs were divided into discharge classes to allow an approximate simulation of the entire events, and finally provide inclusion of time-variant flow velocities into the formula calibration. Inapplicability of the methodology to the study site highlights the influence sediment transport processes at the river bed during flow events yield on bank erosion.

Keywords: Riverbank erosion, Empirical equations, Numerical modeling, Regression analysis, River restoration

1 INTRODUCTION

Erosion of cohesive banks is widely accepted to be an interaction between fluvial erosion at the bank toe and mass failures in the upper part of the bank, while the resistance to both, fluvial shear and mass failure, is exposed to weakening and weathering processes (e.g., Thorne, 1982; Rinaldi and Darby, 2008).

Fluvial erosion results from detachment of single grains or aggregates of varying size by shear stresses. The erosion rate induced by fluvial erosion depends on the sediment erodibility, which is determined by a variety of sediment and fluid properties (e.g., Grissinger, 1982; Arulanandan et al., 1980). Shear stresses at the bank surface vary strongly due to small scale topographic features. While attempts to model near-boundary flows exist (e.g., Kean and Smith, 2006; Nardi et al., 2008a), monitoring studies of near-boundary flows in the field are lacking, also given the hazardous conditions for monitoring during erosive events.

Mass failures are triggered by gravitation when destabilizing forces (part of the weight of the potential failure block) exceed the resisting forces (shear strengths along the potential slip surface). The weight of the potential failure block is subject to changes by variations of the water content, and may be increased by surcharge of trees (Thorne, 1990). Shear strength is reduced by positive pore water pressures, but may also be significantly increased by negative pore water pressure (e.g., Rinaldi and Casagli, 1999) or by root networks from riparian vegetation (Abernethy and Rutherfurd, 2001). As the bank stability depends on bank hydrology, it is also subject to hydrological influences of vegetation (Simon and Collison, 2002). Additionally, the bank stability varies with the hydrostatic pressure the river stage exerts on the bank surface (Simon et al., 1991).

Weakening and weathering processes such as freeze/thaw cycles may destabilize banks and strongly increase sediment erodibility (Lawler, 2005). Sediment properties may vary strongly in the vertical, and, even more difficult to account for in modeling, in the horizontal direction. Some parameters are difficult to determine and may require detailed investigations, e.g. physical modeling studies from Nardi et al. (2008b) with coarse, partly packed and cemented sediments.

This multitude of processes, factors and parameters involved makes it difficult to investigate single processes in detail. Moreover, it gives to the riverbank erosion a stochastic nature, which is hard to reproduce accordingly in models.

1.1 *Modeling attempts*

Significant progress has been made in the development of general bank erosion models with increasing complexity, which capture the majority of factors and processes identified so far (e.g., Darby et al., 2007; Van de Wiel and Darby, 2007; Langendoen and Simon, 2008). For adequate application to obtain good results, these models require detailed bank parametrization and monitoring, both elaborate and expensive. Model results have to be interpreted with care, as processes integrated in models are subject to simplification and data for verification of single processes are often not available. And, for specific sites, important processes may be missing in a general bank erosion model.

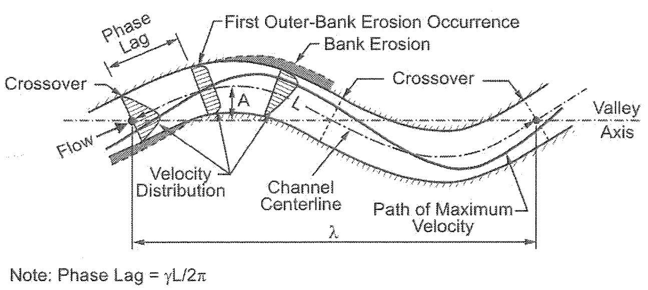

Figure 1. Proportionality of bank retreat to near-bank flow velocity according to Ikeda et al. (1981) (Odgaard and Abad, 2008).

If monitoring data of bank retreat exists for a specific site, the employment of bank erosion equations with empirically derived coefficients from regression analyses is an alternative. Accepting that the derived coefficients are only valid for the originating site and that information about bank retreat is sufficient at a larger scale (for example, without being able to assign eroded sediment volume to fluvial erosion and mass failure), empirical equations have the potential to deliver satisfying results with less effort. Ikeda et al. (1981) related bank retreat to near-bank flow velocities to explain meander migration (Figure 1).Pizzuto and Meckelnburg (1989) used field data for regression and generally confirmed a linear relationship between near-bank flow velocity and bank retreat (Figure 2), originally proposed by Ikeda et al. (1981).

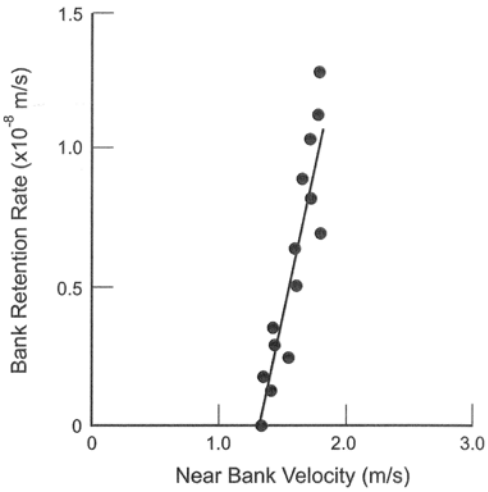

Figure 2. Relationship between rate of bank retreat and near-bank flow velocity for a meander bend of the Brandywine Creek in southeastern Pennsylvania (Pizzuto and ASCE Task Committee on Hydraulics, Bank Mechanics, and Modeling of River Width Adjustment, 2008).

1.2 *Objectives of presented study*

So far, empirical equations were derived by relating mean-flow properties (e.g., near-bank flow velocities during bankfull discharge) to bank retreat (Odgaard and Abad, 2008); in a recent study by Pizzuto (2009), the event scale bank profile evolution was related – amongst other variables – to the maximum near-bank flow velocity per cross section during the event investigated. But near-bank flow velocities and bank erosion vary strongly with discharge during erosive events, and also the duration of erosive flow velocities determines the final bank retreat.

This work seeks to develop and apply a methodology of formula development which includes the time-variant flow velocities in formula calibration. In this study, the applicability at a gravel-bed river will be tested and the limits will be identified. In future studies, an improved methodology would be the basis for the following aims: (a) achieve higher correlation between developed empirical equations and observed bank retreat, (b) find significant differences in correlation also for more complex, non-linear formula types, (c) decouple formula development and coefficient calibration from hydrograph characteristics of the investigated field site, also to (d) make results from different study sites more comparable, (e) identify limits for application, (f) test if a general formula type can be found which fits best when applied to a larger number of study sites, which could (g) satisfyingly predict bank erosion at any study site based on small data sets of bank retreat observations.

The background of the specific investigated case – bank retreat in a restored river section at the Drava River – and the importance of reliable bank erosion models for this site will be explained in the following chapter.

2 STUDY SITE

2.1 *Site characterization*

The study site is located at the Drava River near Kleblach-Lind in Southern Austria in a side-arm within a restored section (Figure 3). There, the mean discharge of the Drava River at a gauging station near the study site is about 74 m^3s^{-1}; the catchment area covers approximately 2561 km². Floods mostly occur during snowmelt in the catchment basin in spring or after thunderstorms in summer (one-year-flood: 320 m^3s^{-1}). The riverbanks are about 3.5 m high and consist of silty and sandy deposits.

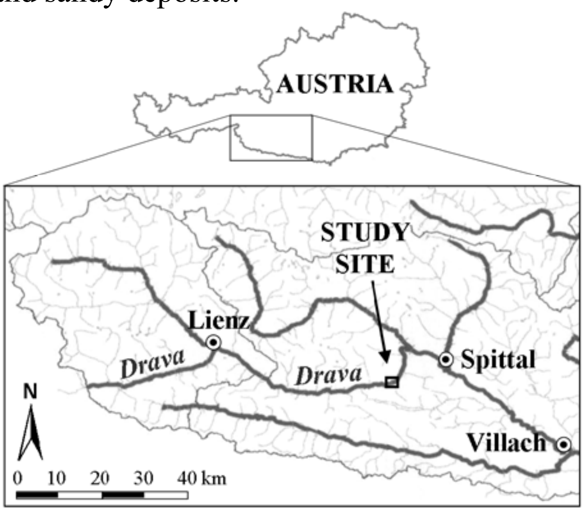

Figure 3. Location of the study site.

2.2 *Historical development*

Historically, the Drava River was a partially braided channel system. After heavy floods at the end of the 19th century and in the 1960s the Drava River was systematically regulated for flood control and for minimizing bed degradation. In combination with catchment-wide changes like torrent control in tributaries, land use changes and intensive gravel dredging, these measures caused a degradation of the riverbed, which led to economical and ecological problems (Habersack and Nachtnebel, 1998).

2.3 *Restoration measures*

In order to improve flood protection, ecological integrity and to stop channel incision, since the 1990s several restoration measures were implemented at the Drava River. Bank protection structures were removed, the riverbed was widened and side-arms were built or reconnected. These measures also initiated self-dynamic bank erosion, for example in the new side-arm at Kleblach-Lind, where the monitoring of bank retreat has been conducted (Figure 4).

So far, the widenings of the riverbed showed stabilizing effects on the riverbed (Habersack et al., 2010). After the side arm had been completed in the year 2002, a small flood with a peak discharge of 286 m^3s^{-1} initiated a widening of the side arm almost doubling its width (from an initial mean width of 29 m to a mean width of 55 m). Since then the bank retreated moderately and allowed more detailed observation of the processes involved in bank retreat.

2.4 *Importance of bank erosion model*

After removal of bank protection structures bank erosion causes a widening of the riverbed, which strongly affects bed morphology. Too strong aggradation in side arms leads to continuous disconnection from the main channel and to reduced morphodynamics, which may partially contradict the original intention to improve ecological integrity. In the side arm investigated repeated dredging is conducted since 2009 to satisfy fishery interests by reconnection at low flow conditions.

Seeking for long-term functioning of side arms, sediment transport models with integrated bank erosion modules would be helpful in testing of different types of side arm construction (for example regarding inlet or thalweg of the initial side arm) and of different types of lateral boundaries limiting bank retreat (for example related to construction type or distance to initial channel) before measure implementation.

In the near future further restoration measures will be implemented in the Upper Drava valley. Bank erosion equations calibrated at the study site Kleblach-Lind would be of great value, as the riverbanks at the other sites exhibit similar bank properties.

Figure 4. Bank retreat after initiation of a new side arm within a restored section at Kleblach-Lind at the Drava River (source: Carinthian government).

3 METHODS

3.1 *Survey methods*

Field work included repeated surveys of channel geometry. In the time immediately following side-arm opening large bank retreat occurred and tachymetric survey was applied for documentation. Later the bank retreated more moderately, which offered the opportunity to observe bank erosion at the single process scale and delivered data of bank retreat close to the onset threshold of bank erosion. In that case, bank retreat would not have been represented sufficiently by tachymetric survey, so that additionally terrestrial photogrammetry was applied at a 30 m long bank section to observe event-scale bank profile evolution.

3.2 *Determination of bank retreat*

In the data set obtained from the first monitoring series when large retreat occurred and bank geometry was measured tachymetrically, bank retreat was defined as the distance between the bank edges before and after flow events.

For the second data set obtained by terrestrial photogrammetry a methodology had to be developed to represent the observed state of bank profile evolution by an equivalent value of bank retreat. This modification was required because the actual state of bank profile evolution could not be accounted for as only one value for bank retreat per cross section could enter the further analysis. Using all digital bank elevation models, the mean width of failure blocks at the top of the bank and the smallest and the largest (steepest) bank angle occurring were determined. The actual state of bank profile evolution in a cross section was then characterized by the actual bank angle. Bank profile evolution induced by fluvial erosion from the smallest to the largest bank angle was defined as 100% of a bank retreat with a value of the mean failure block width, while from the largest to the smallest angle no retreat was assigned. This methodology was applied because a simple calculation of eroded volume per bank length and bank height would overestimate mass failures in cross sections used in further analysis. Mass failures remove large amounts of sediment, but only little or no erosion is necessary to make an already relatively unstable bank collapse. The erosive "work" is done by continuous fluvial erosion, which would not be detected when bank retreat was only measured at the bank edge. Figure 5 shows the bank profile evolution in a cross section.

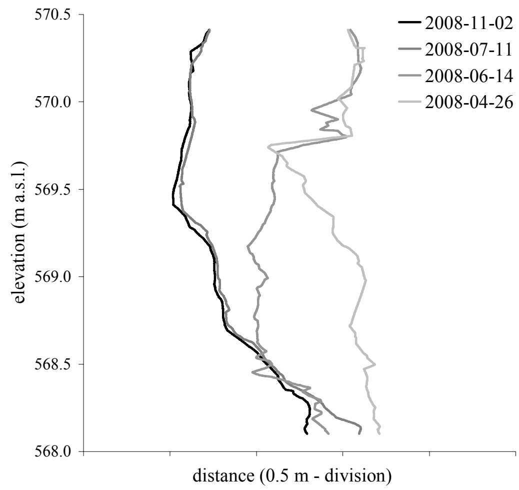

Figure 5. Bank profile evolution in a cross section.

3.3 *Elevation model generation*

The flow events of the time period investigated induced morphological changes due to bank erosion and sediment transport processes. Nevertheless the flow velocity distribution, which caused the investigated bank to retreat between two surveys, had to be modeled with constant channel geometry. In order to represent satisfyingly the hydrodynamic conditions in the channel during the events, intermediate geometries were generated on the basis of the surveys before and after the events investigated. Intermediate banks were generated at half distance between the banks surveyed. The intermediate riverbed was created using a GIS software. Figure 6 demonstrates the generation of the elevation model in a cross section. As an example, elevation models from two channel surveys and the generated intermediate elevation model are displayed in Figure 8.

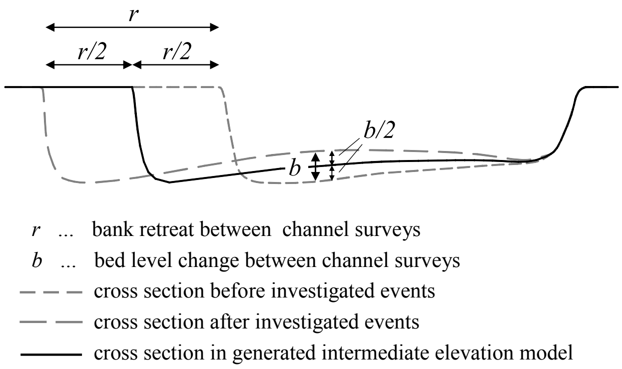

Figure 6. Generation of 'intermediate' elevation models.

3.4 *Modeling of flow velocities*

Hydrodynamics were simulated using the two-dimensional numerical flow model RSim-2D, a part of the RSim river modeling framework (Tritthart, 2005). The applied integrated hydrodynamic-numerical model is based on the Finite Element method, a triangular mesh and the Smagorinsky turbulence closure and delivers depth-averaged flow velocities.

In some cases the 2-D approach may not adequately represent the flow field, in contrast to 3-D approaches. Given the relatively large radii of curvature of the investigated river sections, the application of a 2-D model was considered suitable at the study site.

As flow velocities vary strongly close to the bank, a standardized distance had to be defined where the modeled flow velocities were taken from for further analysis. The depth-averaged flow velocities in a distance of half the mean bank height from the bank toe seemed appropriate to represent the near-bank flow field (Figure 7).

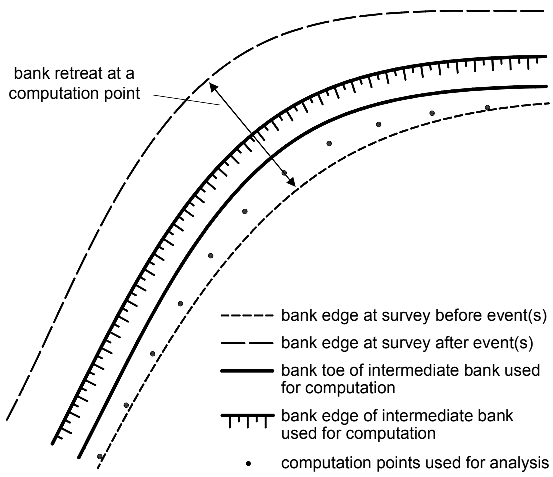

Figure 7. Placing of near-bank computation points at a standardized distance to the intermediate riverbank.

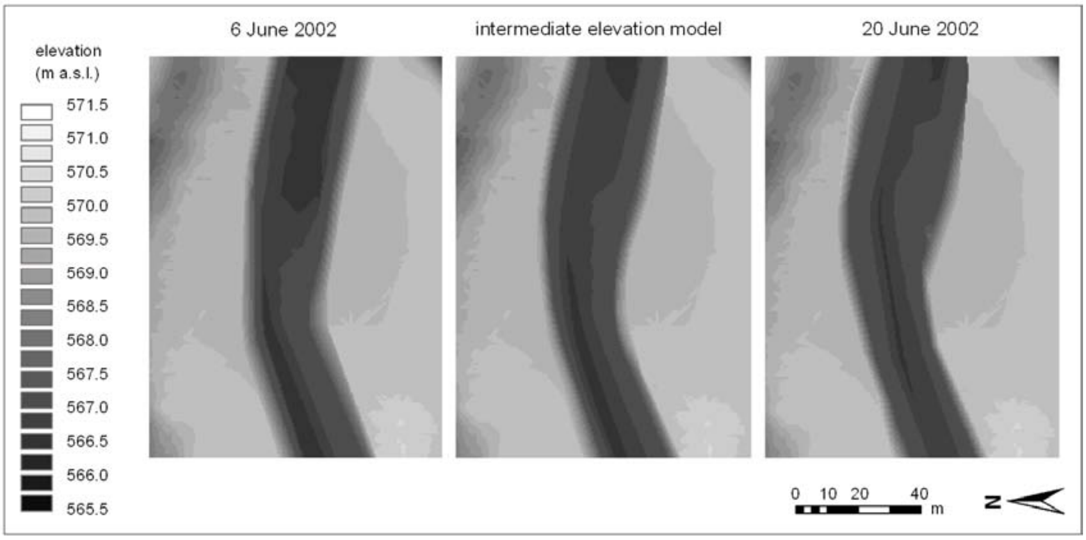

Figure 8. Detail from elevation models from 6 June 2002 and 20 June 2002 and generated intermediate elevation model.

3.5 *Hydrograph discretization*

The hydrograph between two surveys was discretized using uniform discharge classes. For every discharge class its duration in the investigated events could be determined (Figure 9).

All discharge classes were modeled and resulted in different flow velocities in the computation points observed. This way the time-variant flow velocities and the duration of every flow velocity could be included in the regression analysis. Ten discharge classes were considered sufficient for representation of hydrograph and flow velocity.

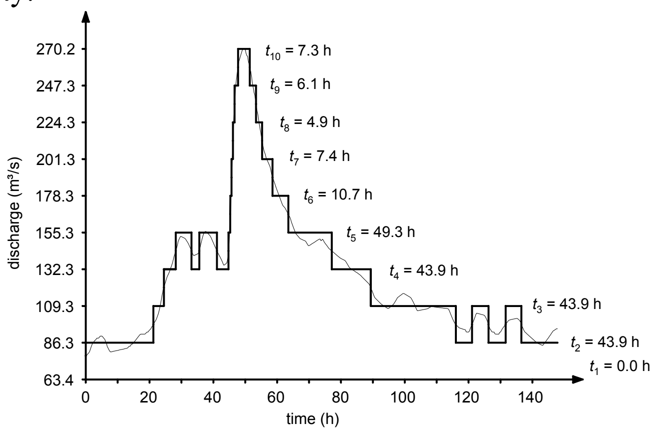

Figure 9. Hydrograph discretized into discharge classes with assigned durations (section for illustration).

3.6 *Regression analysis*

Ikeda et al. (1981) presented the following equation to calculate bank retreat at meandering rivers:

$$v = e(u_\mathrm{b} - U) \tag{1}$$

where v ($\mathrm{ms^{-1}}$) = rate of retreat, u_b ($\mathrm{ms^{-1}}$) = flow velocity near the bank, U ($\mathrm{ms^{-1}}$) = reach-averaged velocity and e = dimensionless erosion coefficient representing all bank properties, mechanisms and processes which determine bank erodibility. With equation (1) Ikeda et al. (1981) suggest that bank migration is proportional to the near-bank velocity (Figure 1) and occurs when u_b is greater than U. Pizzuto and Meckelnburg (1989) decoupled the equation from U and found that erosion already started at values of u_b lower than U.

According to the hypothesis that bank retreat is related to the near-bank flow velocity, the bank retreat observed at every near-bank computation point is the cumulative result of all flow velocities and their durations during the events. Assuming that a critical value for near-bank velocity as an onset threshold exists, the excess shear stress formula for calculation of fluvial bank erosion (equation (2), Partheniades, 1965) may serve as an example for a formula type:

$$\varepsilon = k_\mathrm{d}(\tau - \tau_\mathrm{c})^a \tag{2}$$

where ε ($\mathrm{ms^{-1}}$) = fluvial bank erosion rate, k_d ($\mathrm{m^2skg^{-1}}$) = erodibility parameter, τ_c ($\mathrm{Nm^{-2}}$) = critical boundary shear stress, a (dimensionless) = empirically derived exponent.

Given ten discharge classes and hence ten flow velocities per computation point, the bank retreat r in the cross section of the near-bank computation point i is calculated by:

$$r_i = k\sum_{j=1}^{10}\left[(u_{ij} - u_\mathrm{c})^a t_j\right] \tag{3}$$

where r_i (m) = bank retreat in the cross section of near-bank computation point i, k (dimensionless) = regression parameter no. 1, u_{ij} ($\mathrm{ms^{-1}}$) = flow velocity in near-bank computation point i of discharge class j, u_c ($\mathrm{ms^{-1}}$) = critical flow velocity (regression parameter no. 2), a (dimensionless) = regression parameter no. 3, t_j (s) = duration of discharge class j.

In equation (3) three parameters are unknown. As this equation can be set up for every cross section analyzed, the system of equations is strongly over-determined and the parameters can be calculated using a multivariate regression. For calibration of erosion coefficients the river sections analyzed have to be characterized by equal bank erodibility, or differences in bank properties may at least be assumed to be negligible. For this reason only cross sections where the banks exhibited similar properties can be subject to one regression. At the study site banks with and without woody vegetation had to be separated.

4 PRELIMINARY RESULTS AND DISCUSSION

The modeled flow velocities showed a strong discharge-related variation (Figure 10) and the near-bank flow velocities mostly increased with stage.

However, at banks where highest retreat was measured, the modeled near-bank flow velocities mostly decreased with stage and were generally low (Figure 11).

Data regression would deliver unrealistic parameters and result in inapplicable equations predicting highest retreat rates at lowest velocities. One possible general reason for this fact could be observed at a bank section during monitoring at a bank section of 60 m length, situated between two groins which limit further channel widening. During low flow conditions most of the discharge in the side-arm occurred beside a gravel bar in a run along the bank. At higher discharges, the gravel bar was submerged and the flow velocities along the bank were smaller due to the groins which strongly influenced the flow field. At highest discharges the flow along the bank was even recirculating. Surveys proved that this resulted in sedimentation along the bank during the event. When the discharge went back to mean flow condition, most discharge occurred between bank and gravel bar again, where it was opposed to a reduced cross section because of the bed aggradation. The flow velocities at this time were much higher than during the same discharge on the rising limb of the hydrograph and resulted in bed degradation and large bank retreat until the initial cross section along the bank was re-established.

This change of bed geometry during the event and its effects on the flow velocities could not be accounted for, as all flow velocities have to be modeled with a constant geometry to make the presented data regression analysis possible. At the investigated banks bank retreat appeared to be generally strongly influenced by non-equilibrium sediment transport during the flow events, so that the presented methodology for developing and calibrating bank erosion equations could not be applied.

These findings also have implications on general modeling of bank erosion, for instance when shear stresses modeled using constant geometry are taken as input data for calculation of fluvial erosion and bank stability modeling. Equilibrium sediment transport on the riverbed along the bank is therefore a pre-requisite.

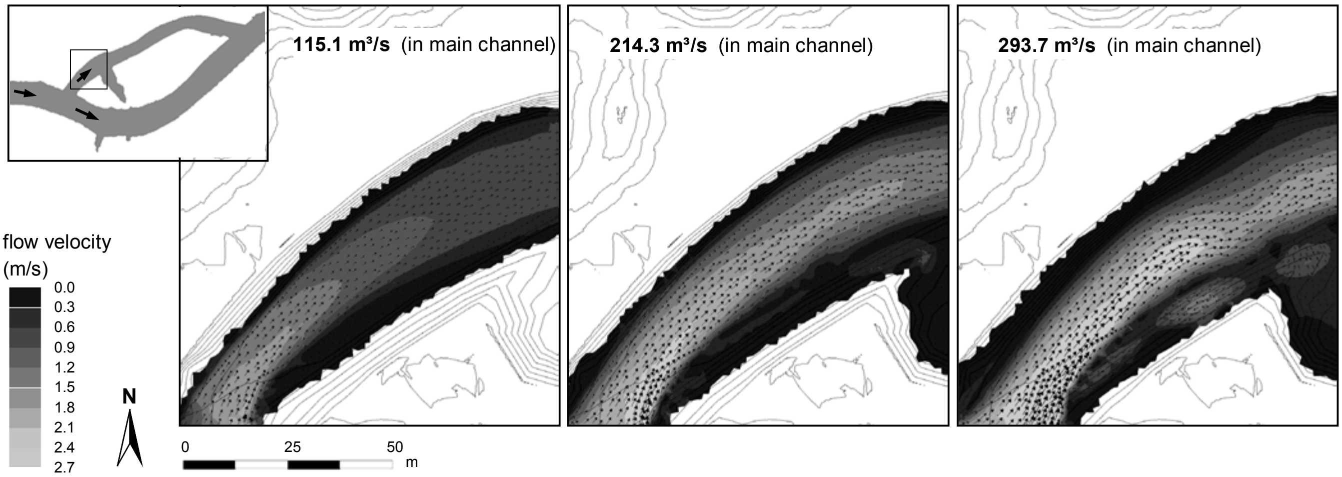

Figure 10. Modeled flow velocity distribution in a river bend at three different discharges.

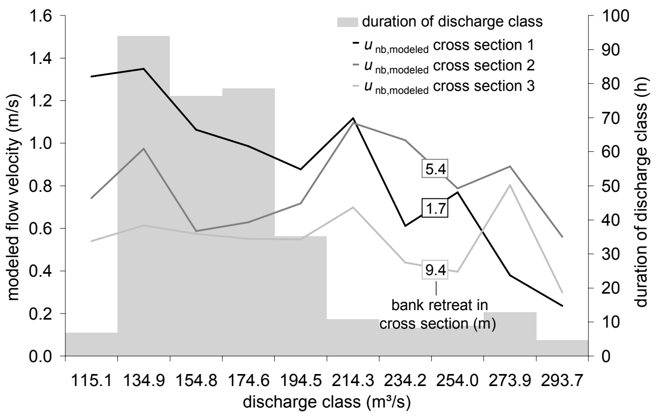

Figure 11. Modeled near-bank flow velocities in computation points along a bank of a river bend with duration of flow velocities and measured bank retreats.

5 CONCLUSIONS

A multitude of processes, mechanisms, parameters and variables is involved in riverbank erosion. Hence, bank erosion modeling is associated with high effort for parametrization of bank properties, monitoring and model calibration. Recognizing this, methods were looked for which deliver reliable results of bank retreat with little effort. In meander migration studies bank retreat was related to near-bank flow velocities. A methodology was developed to include time-variant flow velocities in the development and calibration of empirical bank erosion equations.

The presented methodology was not applicable to the investigated banks of the study site due to non-equilibrium sediment transport processes during flow events. The results highlighted the importance of sediment transport processes during flow events for bank erosion. At the study site, the effects of the groins on the flow have been identified as the main reasons for bed level changes during flow events. There bank erosion and sediment transport would have to be modeled simultaneously to reproduce the observed bank retreats.

If empirical bank erosion equations are developed based on modeled near-bank flow characteristics and retreat data measured before and after events, equilibrium sediment transport on the riverbed along the bank would be a pre-requisite. This would have to be verified by measurements of bed levels during the event or by calibrated sediment transport models.

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