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Towards management and regulation of gravel mining in urban areas: application of an optimization/morphological model to Maipo river

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ABSTRACT: Gravel extraction from a natural streambed combined with changes in flow discharge in response to water resource usage may induce modification of instream sediment transport characteristics. Reduced transport capacity and riverbed overexploitation can bring a system off its natural equilibrium, resulting in decreased sediment replacement downstream from the extraction area but also affecting upstream morphological conditions. This, in turn, may affect civil works founded in the riverbed, as well as water catchment infrastructure designed upon minimum river stages. In this paper, a methodology is presented to couple a sediment transport and morphological simulation model with a management model based on genetic algorithm optimization, to search for good gravel mining practices, determining adequate gravel extraction rates and water use in the system. The model is applied to a hypothetic system inspired on the Maipo River, located near Santiago, Chile, using data representative of current conditions for sediment extraction rates and water use in a reach that includes an urban area. A first step toward integrated water and sediment management is proposed, evaluating the economic impact of combined sediment and water use policy in the hypothetic river. The results of the application provide the identification of sensitive areas together with points where gravel exploitation is possible or even beneficial. The model also yields limiting extraction rates for sustainable operations.

Keywords: Management, Gravel mining, Optimization models, Sediment transport, Morphology

1 INTRODUCTION

Gravel mining is a common practice in Chile, based on the demand of this type of material for construction. Indeed, throughout the world, one of the main sources of sedimentary material corresponds to rivers situated near big cities, which are the main demander for aggregates for construction. The main factors that explain the extraction of aggregates from rivers, are: 1) easy access to the source, 2) high quality material and 3) a wide range of material sizes, reducing costs in material processing (Kondolf, 2002a). In central Chile, sources of fluvial sediments are concentrated mainly in the Andean mountain region, from where the sediments are transported downstream into the river basins. Because of this, most rivers in the region, having steep slopes and coarse and poorly graded beds, are considered as rich potential sources of aggregates (Figures 1 and 2).

On the other hand, many rivers are subject to the extraction of large amounts of water for different uses, consumptive and non-consumptive, such as irrigation, drinking water supply, hydropower generation, etc. Hence, since a close relationship exists between the sediment transport capacity and the flow discharge prevailing in rivers, it is not uncommon for the hydrodynamic behavior and morphologic development of many rivers result to be controlled by external activities, which are planned without taking into account the natural dynamics of these systems, in terms of sediment transport processes and the consequent degradation/aggradation of the streambed.

Several authors have described problems associated to aggregates overexploitation in rivers (Abarca, 2008; López, 2004; Marston, 2003), which are related mainly to severe streambed morphology modifications and to channel width variation. These modifications and their effects are far from being a local phenomenon, existing in the literature reports of actions with impacts over several kilometers in both directions (upstream and downstream) from the exploitation zone (Kondolf, 2002b; López, 2004).

Figure 1. Aggregates process industry, located in the southern limit of Santiago (San Bernando).

Figure 2. Pit for gravel extraction in the Maipo river. The photograph shows a section of the Maipo river located in the municipality of San Bernardo.

When considering the existence of both water and gravel extraction at different locations along the river, as well as the presence of infrastructure founded on the riverbed, it is necessary to evaluate the relative and integrated effects of each extraction process upon the system. Apart from that, it is necessary to assess the incorporation of new activities into the system, in order to anticipate their effect on existing operations as well as the short and long-term impacts on the structures founded on the riverbed. Due to this, the study reported in this paper is focused on the creation of a computational tool, able to anticipate future interferences and problems related to the existing exploitation setup or to the entry of new activities into the system. This tool is thought of as an aid for the management and regulation of gravel mining activities in rivers. For the computational tool, the study couples a numerical model for the hydrodynamics and morphodynamics (MOSSEM) with a management model based on optimization. This allows for the assessment of several operation scenarios in rivers with high demands of aggregates.

2 MANAGEMENT OBJECTIVES

The goals for the management tool under development include a) assessment of the existing system operation, in order to forecast its future possible condition, using as state variables the riverbed longitudinal elevation profile, the effects on structure foundations and the interferences between the existing users; b) assessment of effects due the integration of new users into the system; c) river sectorization in order to define the most sensible zones to gravel mining activities. Therefore, the maximization of a river general productive value is proposed, considering all the activities developed in it. Also, the adverse effects associated to all these activities are to be taken into account as costs or constrains of the problem.

3 MODEL DEVELOPMENT

The model development considers some modification to the Model for Sediment Transport and Morphology, MOSSEM (González, 2006; Abarca, 2008), mainly to include gravel extraction from the streambed in the local sediment mass balance. Thus, the modified model is capable of evaluating the dynamical change in morphology due to gravel extraction. The management model is developed based on optimization, adapting an open source Genetic Algorithm (GA) code. A general scheme of the model is given in Figure 3. Is important to show that the methodology used in this study for coupling the models is applicable to any morphological model that is based on binary files for the inputs/outputs (similar to MOSSEM).

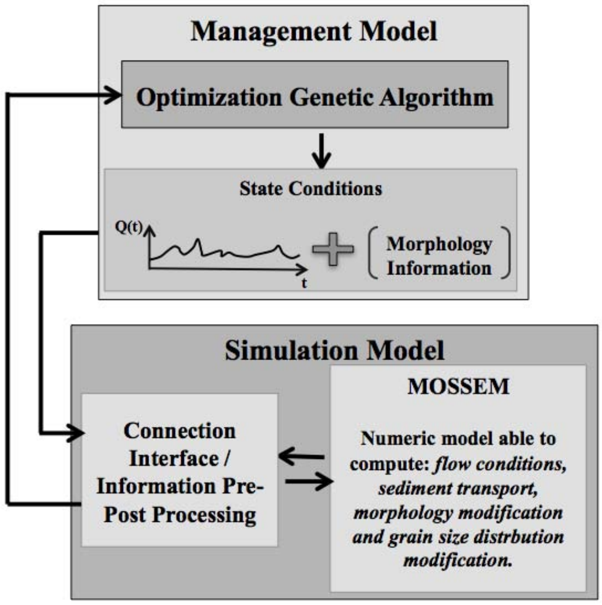

Figure 3. Connectivity implementation between the numerical model (MOSSEM) and the management model.

3.1 *Management Model*

The Management model is based on the optimization of resource usage, by means of the evaluation of an objective function. In this study a genetic algorithm is used to determine the best way to exploit gravel resources in rivers, by analyzing their morphological response; in other words, by tracking changes in bed elevation, due to different gravel extraction rates imposed in each of the reaches in which the river is divided.

The model considers a set of control and optimization nodes, associated to different river reaches. The nodes are divided in 4 categories: Type 1 – Node with presence of large infrastructure, Type 2 – node that can potentially be exploited, Type 3 – node with presence of minor infrastructure, Type 4 – node with existing gravel extraction permits. The main variable to define is the volumetric rate possible to be mined from new extraction zones.

The optimization problem is formulated as:

Objective Function:

$$Max\left[\alpha B - (1-\alpha)C\right] \quad (1)$$

where,

$$\text{Benefits}: B = \sum_{i}\sum_{t}^{N}\delta_i^B X_i\, b_i \quad (2)$$

$$\text{Costs}: C = \sum_{i}\sum_{t}^{N}\delta_i^C C_i(\eta_i) \quad (3)$$

with

$$\delta_i^C \in \{0,1\}\ ;\ \delta_i^B \in \{0,1\} \quad (4)$$

The decision variable $\mathbf{X_i}$ is the gravel volume extracted from node **i**; η_i is the level of the streambed in node **i**; δ_i^C and δ_i^B are step functions to enable the application of costs and benefits for a node; $\mathbf{b_i}$ is the benefit per unit of $\mathbf{X_i}$ in node **i**; $\mathbf{C_i(\eta_i)}$ is the cost function for node **i** and depends on the variability of the streambed level. The parameter α is used in the objective function in order to weight the benefits of the exploitation and the costs derived from it. Other cost functions can be introduced to evaluate different impacts, for example, alteration of natural habitat, affecting a specific biological community, exploitation of zones where other activities take place (e.g., tourism, fishing).

The optimization is performed through a genetic algorithm (GA), which, in general terms, simulates the evolution of possible solutions (cases) to the problem posed from an initial arbitrary case. For this, the GA evaluates and evolves a population (a set of cases previously defined in number) in time, by steps, such that each step is represented by a generation of the population. For the evolution of the population, the GA performs crossing and alteration of the best cases in each generation, using their objective function value, and/or mutation under specific conditions (Marcyk, 2004; Rutkowski, 2008; Wiese, 2009).

3.2 *Sediment Transport and Morphology Model (MOSSEM)*

MOSSEM is a simulation model which integrates: a) solution of 1D Saint-Venant equations, b) computation of bedload sediment transport capacities, c) computation of bed elevation modification through the solution of Exner equation, and d) computation of turbidity currents and fine sediment dynamics (deposition/resuspension) if they are relevant for the problem (e.g., reservoir sedimentation analysis).

The MOSSEM model has been used in several Civil Engineering projects in Chile, giving good results in comparison to commercial models, as HEC-RAS and MIKE11. The advantage of MOSSEM lies in the numerical scheme used (Ying et al, 2004), which allows to solve impermanent fluxes that are close to crisis, with the reduction of the numerical instabilities. Also, the model allows the use of several relations for estimating potential sediment transport rates, including: Meyer-Peter and Müller (1948), Acker & White (1973), Parker (1990), and Wilcock and Crowe (2003); The above allows the simulation of fluvial systems that presents extensive grain-size curves, including coarse and fine sediment material.

4 APPLICATION CASE

The model and proposed methodology is applied to a hypothetic system inspired in the Upper Maipo River basin (Figure 4), which is the main source for water supply of the city of Santiago, with extractions equivalent to about 21 m^3/s. Also, the Maipo River represents one of the main sources of aggregates for construction in the area. Presently, there is no certain information about the total amount of aggregates extracted from the river, due to weak control and the existence of irregular exploitations. Even so, it is known that some authorized operations have exploitation rates close to or even higher than 150 m^3/hr of extracted material. These high exploitation rates, based on equally high demands for aggregates, are endorsed by the fact that, given the proximity of the Maipo River to the city, there is an easy access to the source and the transportation costs are considerably low.

The system considered is hypothetic, because although the morphology and hydrology of Maipo river are included in the present application, several simplifications are introduced to this complex system, particularly with regards to the interactions between gravel mining and water extraction operations. The system flow configuration accounts for the major water extractions, reducing the flow capability for sediment transport. Most importantly, in order to accelerate the natural morphological dynamics of this system, a reduction in the bed grain size distribution was introduced with the aim of enhancing bed deformation. This allowed us to reduce simulation and computational times and hence, helped to develop and test the hypotheses of the current management model. For application to natural fluvial systems, an unmodified grain-size curve should be used. More details on the computational time savings, due the modification of the grain size distribution, is given in Chapter 6.

For the construction of the morphological model, detailed GIS information available from the Regional Government (GORE) was used, together with satellite images. The morphologic model considers a 42 km long reach of the upper part of Maipo river, between the flow gauging station "El Manzano" and the "Ruta 5" bridge, and is divided in 408 cross sections, 168 of which correspond to evaluation nodes for the management model.

Seven control zones were defined as possible new extraction sites. They are distributed along the 42 km of the model, and are located between the "La Obra" town (downstream from the "El Manzano" station) and the "Ruta 5" bridge, which is closer to the downstream end of the model (Figure 4). Therefore, the problem to be solved consists of 7 unknown variables, namely, the extraction rates in each one of these zones. A one-year term exploitation permit is evaluated. The problem constraint corresponds to setting a maximum degradation or descent of the streambed along the river during the period of concession/permission. This constraint aims at including a sustainability criterion on the exploitation and thus at protecting the infrastructure founded in the river.

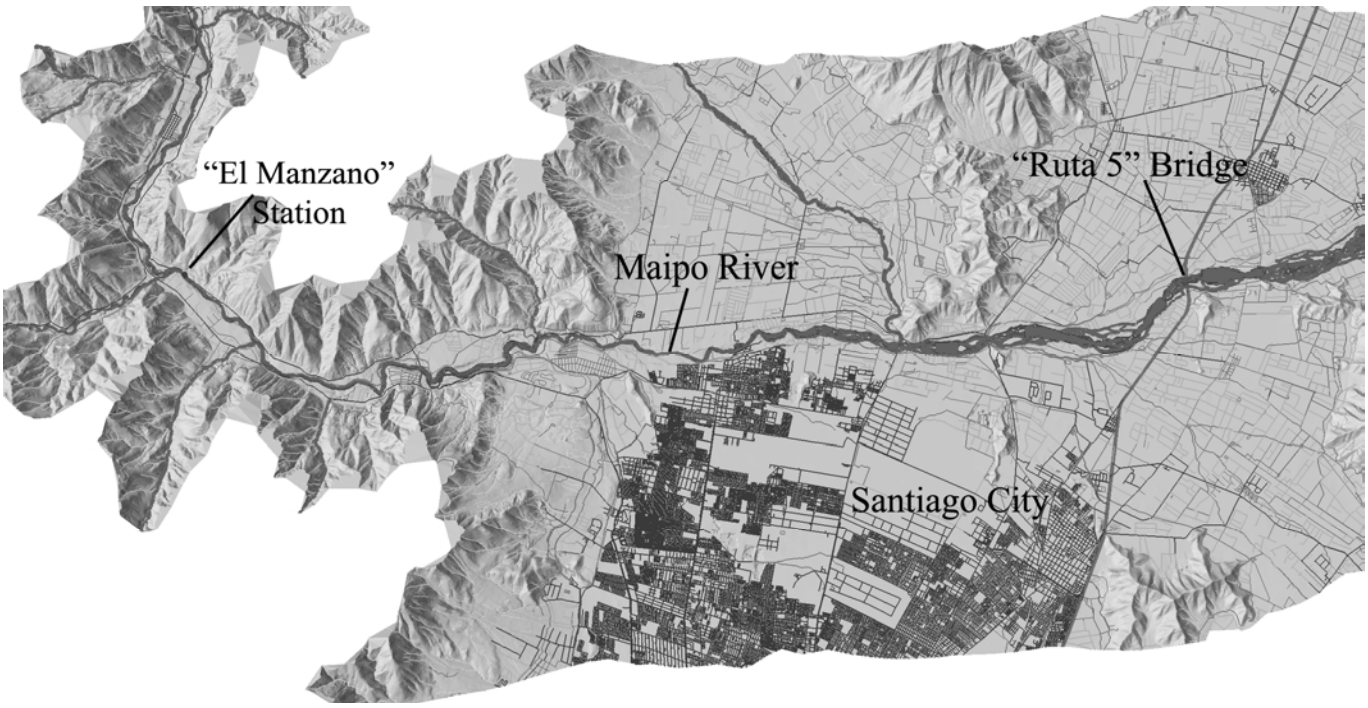

Figure 4. GIS information used for the construction of the Maipo river morphologic mode

4.1 *Modeling Conditions*

A one-year instantaneous discharge time series was used for the simulation, which includes mean conditions of the system and a high flow event close to a return period of 5 years (Figure 5). As already explained, a modified grain size distribution curve of the system was used, in order to reduce the computational burden in the model-testing phase. Both the actual and modified curves are presented in Figure 6. Characteristic diameters for the original curve are D_{50}=2.5 cm and D_{90}=5.67 cm, while for the modified curve these values are D_{50}=0.95 cm and D_{90}=2.85 cm. The bed degradation constraint was fixed at 5 meters of maximum descent anywhere along the river. The optimization is focused in benefit maximization (α=1), eliminating the costs terms in the optimization process. This focus is a consequence of the lack of information for estimating the costs factors that link the degradation of the streambed with the cost associated to repair different types of infrastructure founded in the riverbed or in the riverbanks. The maximization is done over the total amount of gravel extracted, without introducing an economical value (b_i) for the benefits associated with each unit of gravel extracted (X_i).

Thus, with the modification of the grain size distribution curve and the rather lax bed degradation constraint, high rates of extraction are expected to be allowed in the hypothetic system. These results may be far from realistic (excessive amount of sediment is supplied to the system), in terms of the total amounts of transported sediment that is available to be exploited, but at this point of the study, the aim is mainly to validate the concepts and the methodology created to solve the management problem.

Regarding the boundary conditions used in the MOSSEM model simulations, at the upstream boundary, the known hydrograph was imposed together with an estimation of the sediment supply that is expected due to the local conditions of morphology (slope, cross section), grain size characteristics and instantaneous flow discharge. This estimation can be made using different bedload relationships; in this case the Wilcock & Crowe's (2003) equation was used to include the sand fraction of the grain-size curve. At the downstream boundary the corresponding water surface level is imposed, assuming local uniform flow conditions.

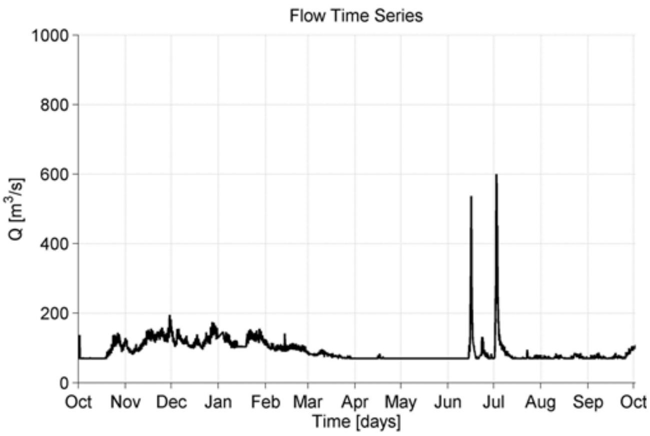

Figure 5. Flow time series used in the simulations.

5 RESULTS

The resolution of the optimization problem takes 137 generations of the Genetic Algorithm, each one containing 10 individuals. This corresponds to 154.2 hours in a 10-node cluster. The results are plotted in Figure 7. The total amount of aggregates extracted, for the best case obtained, is 51.2 million Ton/year. This rather large value results mainly from the lax constraint imposed to the maximum degradation of the system, which permits an over-exploitation of the riverbed and, consequently, the generation of long depressed zones or pits (Figure 8).

On the other hand, the results show that the objective function evolves quickly to the final solution in the first 40 iterations, which indicates a fast learning of the GA. The user has to provide the stopping condition as an initial parameter, and in this application the stopping criterion corresponded to an unchanged objective function for 10 generations.

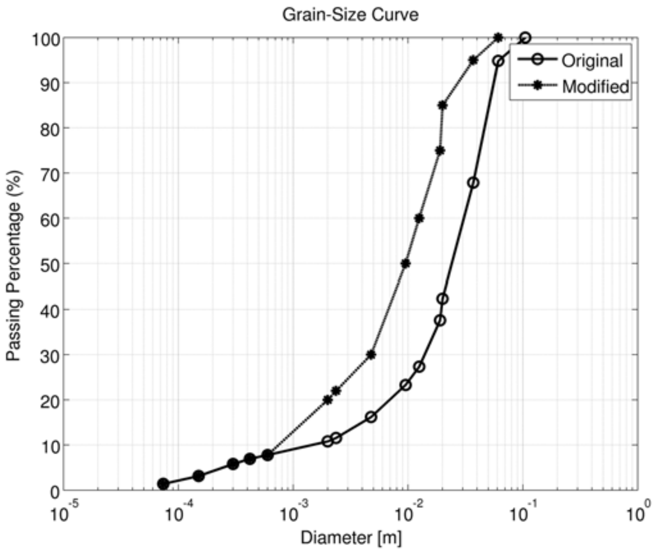

Figure 6. Original and modified grain-size distribution curves for the upper part of Maipo River.

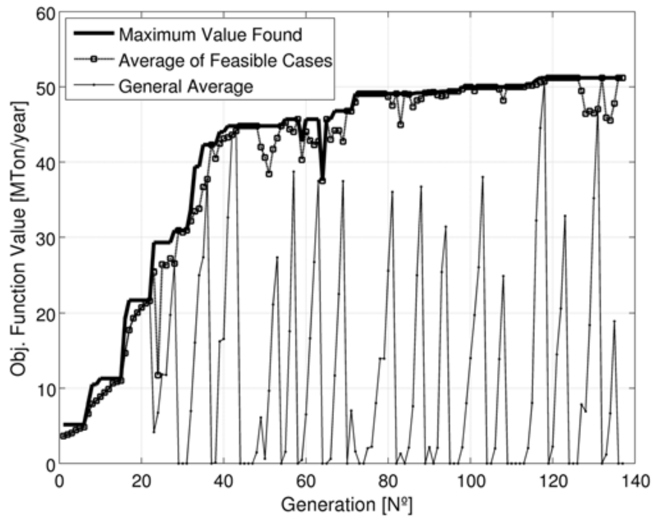

Figure 7. Objective function evolution in terms of the generation number. The feasible average corresponds to the mean of feasible cases that respect the degradation constrains. The general average corresponds to the mean of all the cases. Also the maximum value found in each generation is shown.

5.1 *Analysis of the improved system – Maipo River*

The behavior of the undisturbed system is estimated through the simulation of a base case without sediment extraction. In this situation, the system tends to generate aggradation on the upper and middle sections of the river (Figure 9). This behavior occurs due to the slope change in the system, and the high flow event included in the flow discharge time series. Also, given that the sediment supplied to the system is estimated through the empirical bedload relationship of Wilcock & Crowe (2003), the estimated transport rates may differ substantially from the actual transport rates produced in the system. It should be noted that Figure 9 shows the final state for the system without extractions, but this streambed profile is consequence of a time-varying aggradation, from the initial point of slope change towards the upstream part of the river.

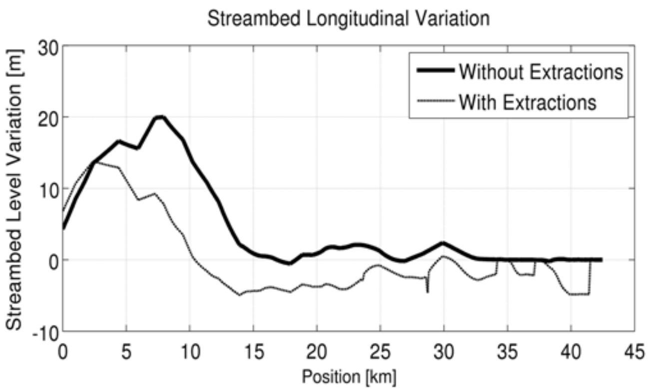

Figure 8. Longitudinal variation of the streambed for cases with and without sediment extraction.

The final solution shows that sediment extractions from zones in the upper part of the system prevent a natural excessive aggradation of this zone. The optimal extraction volumes are higher in the upper zones, which is expected due the larger supply of sediment coming from the riverhead.

An analysis of the system capacity for recovery was conducted by means of a 5-year simulation without extractions, starting from the final condition of the optimal solution found by the management model. The recovery of the system is shown in Figure 10, where it is observed that the most affected areas (upper and middle part of the river) are those that have a faster recovery. Also, the severely degraded zone at the lower reach of the river has the slower recovery, as is expected, due the high retention of supplied material in the upstream affected zones. Thus, mean recovery time scale is much longer than the period of intensive exploitation (one year in this application), especially for the extraction zones located in the lower part of the study reach. Therefore, constraints imposed to the extraction zones located on the lower part of the river have to be stricter.

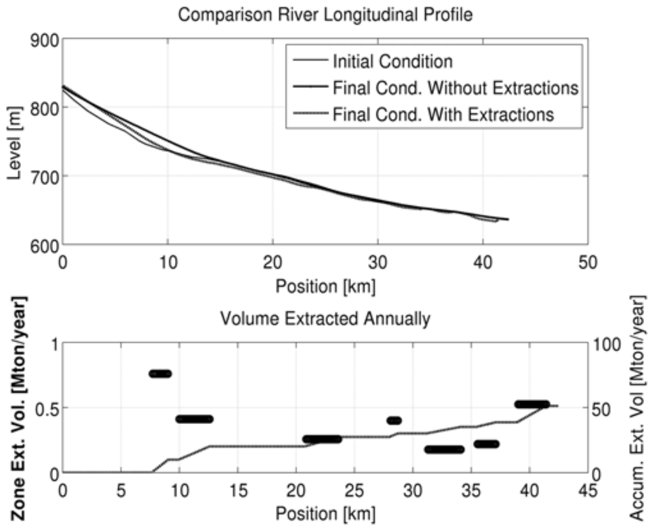

Figure 9. (Upper panel) Comparison between the base case~~s~~ without extraction and the best extraction scenario obtained. (Lower panel) Recommended aggregates extraction rates: DOT – Annual volume of aggregate extracted in zone. LINE – Accumulated volume of aggregate extracted along the river.

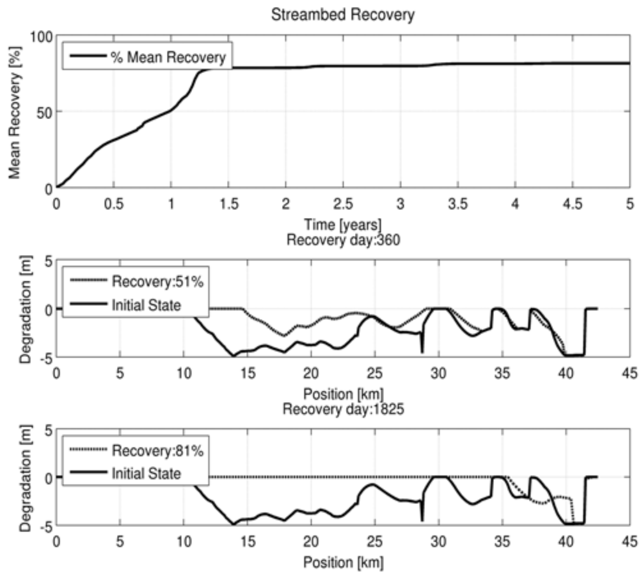

Figure 10. Streambed recovery evolution in time. (upper panel) Mean streambed percentage recovery in time. (middle panel). The recovery is around 51% in day 360. (lower panel) The recovery is around 81% in day 1825 (5 years).

6 ANALYSIS OF MORPHOLOGICAL CHANGE TIME SCALE

A discussion of the time scale associated to the morphological changes in the studied system is presented in order to assess the effect of the sediment size reduction introduced in the analysis reported in previous sections.

Exner's equation, with presence of only bedload sediment transport and streambed modification is given by:

$$\frac{\partial\eta}{\partial t}+\frac{1}{\left(1-\lambda_p\right)}\frac{\partial q_s}{\partial x}=0 \quad (5)$$

where, **η** denotes the riverbed elevation over a datum; $\mathbf{q_s}$ is the bedload sediment flux per unit width; **t** is the time variable; **x** represents the longitudinal position along the channel; and $\mathbf{\lambda_p}$ denotes bed porosity.

Considering the dimensionless variables

$$x^*=\frac{x}{h};\eta^*=\frac{\eta}{h};t^*=t\frac{\sqrt{gRds^3}}{h^2};q_s^*=\frac{q_s}{\sqrt{gRds^3}} \quad (6)$$

where **h** is a length scale, **g** is the gravity acceleration, **R** is the submerged specific gravity of the sediment, $\mathbf{d_s}$ is a characteristic diameter of the sediment. The variables $\mathbf{x}^*$, $\mathbf{h}^*$, $\mathbf{t}^*$ and $\mathbf{q_s}^*$ denote the dimensionless version of the Exner's equation variables. We can rewrite Eq.5, using the corresponding scales, as:

$$\frac{\partial\eta^*}{\partial t^*}+\frac{1}{\left(1-\lambda_p\right)}\frac{\partial q_s^*}{\partial x^*}=0 \quad (7)$$

Considering now a river reach of length L, with a hydrograph of duration T, we can integrate Eq.7 over these temporal and spatial limits as follows,

$$\left\langle\Delta\eta^*\right\rangle_0^L L^*+\frac{1}{\left(1-\lambda_p\right)}\left(\left\langle q_{ss}{}^*\right\rangle_0^T-\left\langle q_{se}{}^*\right\rangle_0^T\right)T^*=0 \quad (8)$$

where the variables $\mathbf{q_{se}}^*$ and $\mathbf{q_{ss}}^*$ represent the dimensionless bed sediment fluxes per unit width that enter and leave the reach, respectively, $\mathbf{L}^*$=L/h represent the river reach dimensionless length, T^* represents the dimensionless hydrograph duration, and $\Delta\eta^*$ represents the dimensionless time-average local bed variation. The triangular brackets represent either time or space averages.

Defining a dimensionless volume variation of the streambed along the river reach, $\mathbf{\Delta V}^*$, given by the morphological modification generated by the variation of the sediment transport capacities during the time interval $\mathbf{T}^*$, as:

$$\Delta V^*=\left\langle\Delta\eta^*\right\rangle_0^L L^* \quad (9)$$

and replacing this result in Eq.8, we can write:

$$\Delta V^*=\frac{1}{\left(1-\lambda_p\right)}\left(\left\langle q_{se}{}^*\right\rangle_0^T-\left\langle q_{ss}{}^*\right\rangle_0^T\right)T^* \quad (10)$$

Now, equating the dimensionless volume variation, $\mathbf{\Delta V}^*$, between a system with a characteristic grain size distribution curve ($\mathbf{\Delta V}^*_\mathbf{o}$) and another which has a modified version of the same grain-size distribution curve ($\mathbf{\Delta V}^*_\mathbf{m}$), all the other parameters remaining equal, yields:

$$\frac{T_o^*}{T_m^*}=\frac{T_o\dfrac{\sqrt{g\,R\,ds_o{}^3}}{h_o{}^2}}{T_m\dfrac{\sqrt{g\,R\,ds_m{}^3}}{h_m{}^2}}=\frac{\left(\left\langle q_{sem}{}^*\right\rangle_0^T-\left\langle q_{ssm}{}^*\right\rangle_0^T\right)}{\left(\left\langle q_{seo}{}^*\right\rangle_0^T-\left\langle q_{sso}{}^*\right\rangle_0^T\right)} \quad (18)$$

In the previous equation, the **o** and **m** subindexes, refer to variables corresponding to the system with the original characteristic grain size distribution and to that with the modified version, respectively.

Now, considering the absence of large perturbations to the system morphology, that is, assuming not so large bed level variations in both cases, we can consider a mean flow height scale, such that $\mathbf{h_m \approx h_o}$, which yields:

$$\frac{T_o}{T_m}=\frac{\left(\left\langle q_{sem}\right\rangle_0^T-\left\langle q_{ssm}\right\rangle_0^T\right)}{\left(\left\langle q_{seo}\right\rangle_0^T-\left\langle q_{sso}\right\rangle_0^T\right)} \quad (19)$$

This results shows that the ratio of time scales associated to a given volume change in bed elevation in two systems with different bed sediment sizes is inversely proportional to the ratio of reach net mean sediment inflow (or outflow) in both systems. This opens the idea of reducing the computational time for a given volume change in bed morphology by creating an equivalent system with a modified grain size distribution.

Two experiments were conducted for the analysis of the reduction of computational time given by the modification of the grain size distribution curve. The original curve corresponds to the mean grain size distribution for Maipo River in its upper part. This distribution was altered to enhance the sediment dynamics, and to reduce the time scale for a given bed deformation, by means of the reduction of bed size material, mainly the coarser fractions. The original and modified grain size distribution curves are shown in Figure 6. The modified curve is that used in the analysis reported in previous sections.

The morphological model was run in the same river reach explained in Section 4, for the same hydrological conditions, considering the situation without gravel mining, for both grain size distribution curves.

To obtain an average value over time for the sediment fluxes per unit width that enter and leave the system, different averaging methods were applied to the model output information. The methodology considers a simple mean, a geometric mean, a harmonic mean and a trim mean. The latter method calculates the mean of the sample excluding the highest and lowest 5% of the observations. Using these results, the estimated values of the time scale ratio are presented in Table 1.

Table 1. Average value for sediment fluxes per unit width.

	Value for T_o / T_m
Simple	1.9060
Geometric	1.9901
Harmonic	2.0453
Trim (5%)	1.9523

The results show that the two experiments have a ratio of about 2 between their times of simulation. This suggests that a reduction close to a half in the computational time can be made by means of the grain size curve modification, under the consideration of having similar results in terms of a dimensionless volumetric morphological variation.

7 DISCUSSION

The previous section shows how the modification of the grain size distribution in the simulation of the Maipo River yields a considerable time reduction over the simulation time, by enhancing the morphological dynamics. Nonetheless, nonlinear effects on sediment transport processes can also create distortions in the morphological process that cannot be easily accounted for by the analysis presented in the previous section. It is clear that the bed response obtained in the base case (i.e., without gravel extraction) is rather exaggerated and this may also be a consequence of uncertainties in the estimated sediment supply rates to the river reach under analysis, as it was already pointed out.

Given that the management tool proposed requires running the morphological evolution model hundreds of times, it is clear that strategies to reduce computational time are needed, which should not, however, introduce important distortions in the predicted morphological response of the system. Further avenues for achieving this goal are being explored and are part of future research.

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