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# Comparison of different reliability analysis methods for a 2D morphodynamic numerical model of River Danube

R. Kopmann & A. Schmidt
*Bundesanstalt für Wasserbau, Karlsruhe, Germany*

ABSTRACT: The propagation of uncertainties in morphodynamic numerical models can have serious implications in the reliability of the simulation results. Therefore, it is necessary to identify the various sources of uncertainty and to quantify their contributions to the variance of the model result. Reliability analysis can help to calculate the uncertainties in a very effective way. For a two-dimensional morphodynamic numerical model of River Danube two different methods of reliability analysis were applied: a Monte Carlo method specialised to confidence limits (MC-CL) and the Scatter Analysis (SA). In the presented examples the development of the river bottom is predicted in respect to 13 model parameters. For each model parameter a probability distribution is assumed. The comparison of the methods is focussing on the results and the calculation times. Additionally, the user-friendliness, the error-proneness and the costs for interpretation are discussed.

*Keywords: Reliability, numerical models, movable bed models, Monte Carlo method*

## 1 INTRODUCTION

In morphodynamic numerical models model parameters, initial conditions and input data can be uncertain due to the natural variability, the deficient description of the physical processes in the model and the imprecision of the model parameters. The propagation of these uncertainties can have serious implications in the reliability of the simulation results. Therefore, it is necessary to identify the various sources of uncertainty and to quantify their contributions to the variance of the model result. Reliability analysis can help calculating the uncertainties in a very effective way.

## 2 METHODS OF RELIABILITY ANALYSIS

The influence of uncertain input parameters to the state variables were calculated with two different methods: a Monte Carlo method specialized to confidence limits (MC-CL) and the Scatter Analysis (SA). Compared to the MC-CL the number of required simulation runs is less in the SA but also the ability to handle non-linearities.

The first step for each method is assuming the statistical distribution of the uncertain parameters. In this paper only Gaussian or double Gaussian distributions are used. For a fully description the mean value and the standard deviation or – in case of double Gaussian distribution – two standard deviations are needed.

The result of the reliability analysis is a confidence interval of the state variable, e.g. the evolution of the river bed topography, connected to a given probability. The confidence interval incorporates only the uncertainties of the considered input parameters and takes not into account the various other sources of uncertainties like the imperfection of the numerical model.

### 2.1 *Scatter Analysis*

For the Scatter Analysis the results of different simulation runs are used to determine the uncertainties. Thus, modifications of the program code are not needed. This method is adequate for linear or slightly non-linear problems. From the root mean square (rms) the deviations are assumed. This is done in first order methods like the First-Order Reliability Method (FORM) as well.

Demonstrating the method for the influence of the friction coefficient ks to the state variable evolution, which describes the vertical changes in the bed elevations, a Taylor expansion for the Evolution E can be written as

$$E(ks) = E'(ks_0)dks + \frac{1}{2}E''(ks_0)dks^2 + Odks^3 , \quad (1)$$

where $dks = (ks - ks_0)$, $ks$: gaussian distributed. The root mean square rms(E) it the product of the first derivation of the evolution and the standard deviation plus a higher order term:

$$rms(E) = (E'(ks_0)\sigma + O(\sigma^3)) \quad (2)$$

For the confidence limits only the first order is taken into account

$$E(ks \pm \sigma) = (Eks_0) \pm E'(ks_0)\sigma \pm \frac{1}{2}E''(ks_0)\sigma^2 + O(\sigma^3) \quad (3)$$

so that the confidence interval of the evolution for a 68 % probability is two times the rms and for a 95 % probability 4 times the rms. The distortion

$$\delta E = \frac{1}{2}E''(ks_0)\sigma^2 << rms \quad (4)$$

should be much smaller than the rms, otherwise the function is not slightly non-linear and the method is not adequate for this special problem. However, the distortion can only be used as an indicator for slightly non-linearity in case of symmetric distributions.

For the reliability analysis shown here the 95 % confidence limits and the related tolerance range were used. These can be calculated from the rms by applying a central difference scheme

$$rms(E) = \frac{1}{2}\left[E(ks_0 + \sigma) - E(ks_0 - \sigma)\right] \quad (5)$$

where $E(ks_0+/-\sigma)$ are results from simulation runs with $ks_0+/-\sigma$. The calculations of the deviations or the tolerance limits for n uncertain parameters need only n*2 + 1 simulation runs. (Nikitina, 2008)

## 2.2 *Monte Carlo-CL method*

In the MC-CL method the confidence limits are determined approximatively. In case of strong non-linearities the confidence limits can not be deduced from the root mean square (rms) any longer. As well, it is not possible to calculate the rms from the deviations. A connection between the confidence limits and the root mean square only exists in case of non-distorted gaussian distribution, as in linear functions. For strong non-linear functions the root mean square and the confidence limits are not equivalent, not proportional and furthermore there is no functional connection between them.

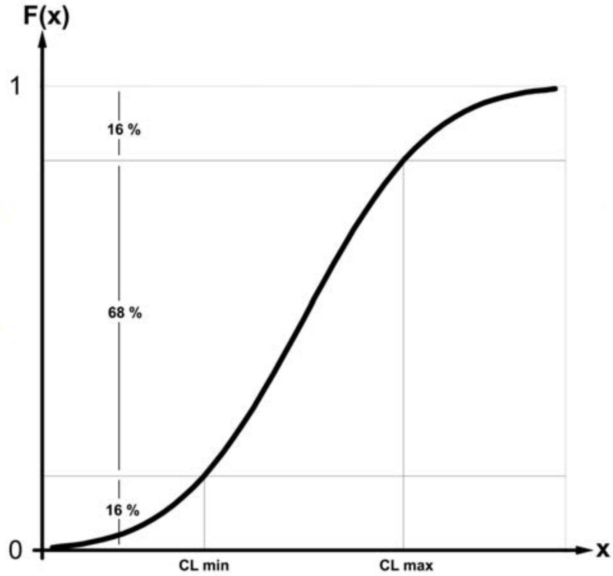

Figure 1. CDF (bold line) and its confidence limits

Because of that the confidence limits (CL) have to be derived from an empirical distribution function (EDF). Figure 1 shows a cumulative distribution function (CDF) and its confidence limits (CL). The CDF has to be inverted in the two points $(1+/-\alpha)/2$ (here $\alpha$=68%) in order to get the values of the confidence limits.

Instead of inverting the unknown CDF F(x) the EDF FN(x) is used (see figure 2), which is a step function with step length of 1/n. If n is big enough the EDF passes over to the CDF (law of large numbers). A design of experiment (DoE) generator chooses from all possible combinations of the input parameters a parameter set for a given number of simulation runs, which has a "reasonable" statistical distribution.

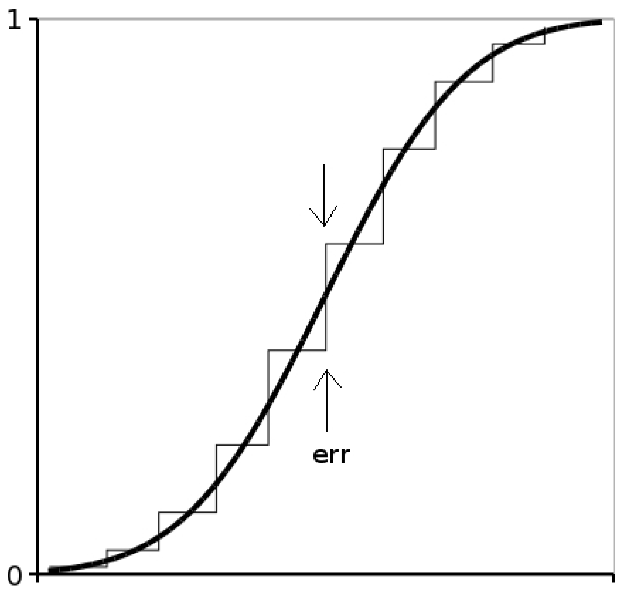

Figure 2. CDF (bold line) and EDF (step function line) of a random variable

In case of infinity number of simulation runs the approximation error of the EDF is normally (gaussian) distributed and can be given as

$$err_{CDF} = \sqrt{\frac{(1-\alpha^2)}{4N_{\exp}}} \,. \tag{6}$$

This formulation is only valid if n is large enough (n(1-α)/2>>1) and the larger n, the smaller is the approximation error (central limit theorem).

The method is independent of the number of uncertain parameters but only dependent on the chosen confidence level. The reason is that the DoE generator works with a probability distribution in the space of the input parameters. The simulations convert this probability distribution in the space of the output values. Each output value (e.g. the evolution in time and space) can be seen as a single random number while its CDF determines the CL. Such a CDF is independent of the number of parameters but should have enough simulation runs in order to be “filled” sufficiently. (Nikitina, 2009)

## 3 APPLICATION OF RIVER DANUBE MODEL

A 10 km long stretch of the river Danube including a 270° bend (see figure 3) is modelled with the morphodynamic program Sisyphe (Villaret, 2006) coupled with the hydrodynamic program Telemac2D (Hervouet, 2007).


Figure 3. River Danube model area with a 270° bend of Mühlham

Nearly 100 000 grid elements were used for the discretization with mean node distances of about 6 m in the river channel and up to 30 m at the flood planes. A synthetic hydrograph of 9 days was simulated including two high flood events (see figure 4). During the reliability analysis more than 1000 simulations were needed, therefore the simulation time had to be comparably small. Using 32 processors one simulation run needed about 45 min. With a natural 9 days hydrograph the discharge dynamic would be too small for extrapolating the results.

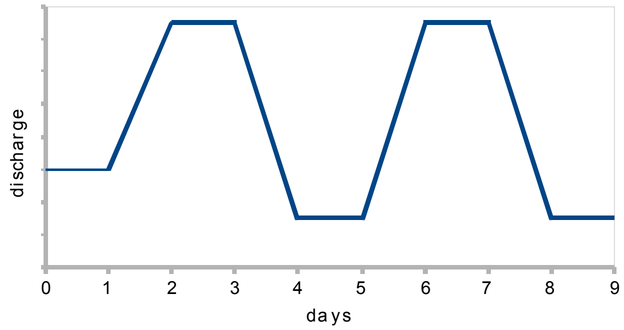


Figure 4. Synthetic hydrograph

For the reliability analysis 13 parameters were assumed to be uncertain: The active layer thickness, the coefficient for the slope effect β described by Koch & Flokstra (Koch & Flokstra, 1981), which changes the value and the direction of the solid transport rate:

$$\left(1-\beta\frac{\partial z}{\partial s}\right) \qquad \tan\alpha = \tan\delta - \beta\frac{\partial z}{\partial n} \tag{7}$$

where s / n are the co-ordinates in the current / orthogonal current direction and α / δ are the angles between flow direction and solid transport / bottom stress,

the coefficient for the bedload formulation of Meyer-Peter & Müller, the coefficient for the secondary current effect α according to Engelund

$$\tau_{secondary\_current} = 7h / R\, \tau_{main\_flow} \tag{8}$$

and using the following formulation of the slope of the free surface zs in bends

$$g\frac{\partial zs}{\partial n} = \alpha\frac{u^2}{R}\,, \tag{9}$$

the friction coefficient of the Nikuradse roughness law for 4 areas and the grain size of 5 classes.

The mean values and the standard deviations, assuming a Gauss or double gaussian distribution are listed in the table 1.

Table 1. Mean values and standard deviations of the uncertain parameters

| | Mean value | min σ | max σ | Probab. distrib. |
|---|---|---|---|---|
| Active layer thickness AL [m] | 0.1 | 0.0166667 | 0.3 | Double gaussian |
| Coefficient for slope effect β | 1.3 | 0.43333 | 1.23333 | Double gaussian |
| Coeff. for MPM formulation $f_{MPM}$ | 4 | 1 | 1.3333 | Double gaussian |
| Coeff. of secondary current effect α | 10 | 3 | 3.3333 | Double gaussian |
| friction coeff. river channel $ks_{RC}$ [m] | 0.05 | 0.011 | 0.0116667 | Double gaussian |
| friction coefficient groynes $ks_G$ [m] | 0.15 | 0.033333 | 0.1 | Double gaussian |
| friction coefficient nature experiment $ks_{NE}$ [m] | 0.5 | 0.06667 | 0.06667 | Gaussian |
| friction coefficient sandbank with trees $ks_{ST}$ [m] | 0.4 | 0.1 | 01 | Gaussian |
| Grain size class 1 [m] | 0.0005 | 0.00013333 | 0.00033333 | Double gaussian |
| Grain size class 2 [m] | 0.0025 | 0.0003333 | 0.00126667 | Double gaussian |
| Grain size class 3 [m] | 0.01 | 0.0013333 | 0.0023333 | Double gaussian |
| Grain size class 4 [m] | 0.024 | 0.0063333 | 0.004 | Double gaussian |
| Grain size class 5 [m] | 0.048 | 0.004 | 0.004 | Gaussian |

With the Scatter Analysis a sensitivity analysis is possible. Using only three simulations runs with mean value and mean value +/- sigma the partial effect of separate parameter changes can be estimated for each uncertain parameter. The most sensitive parameters to the evolution with descending weight are the slope effect parameter (beta), the secondary currents coefficient (alpha), the bedload transport coefficient (MPM), the active layer thickness and the grain sizes of the first ($dm_1$) and the forth class ($dm_4$). The first three parameters are strongly influenced by curve effects (see figure 5).

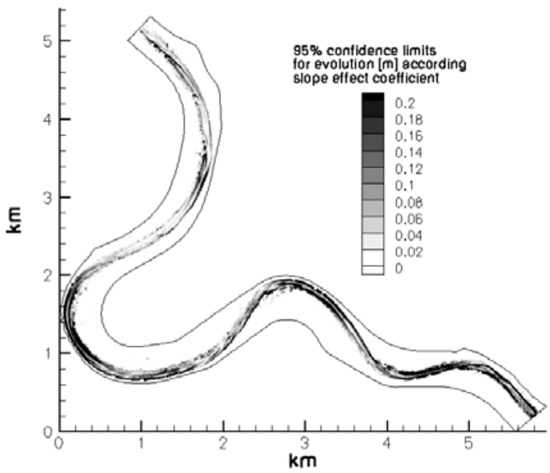


Figure 5. 95 % confidence limits of slope effect coefficient beta influenced by curve effects

The friction coefficients create only local and not very strong effects (see figure 6). The tolerance ranges for 95 % probability exceed 20 cm only for the first three most sensitive parameters.

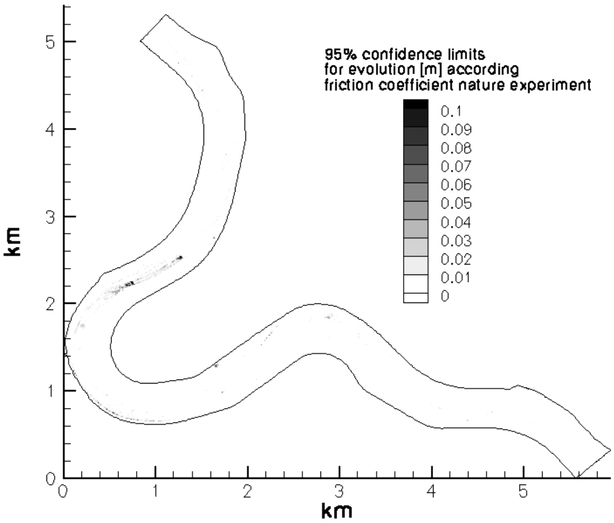


Figure 6. 95 % confidence limits of friction coefficient nature experiment $ks_{NE}$

For the evaluation of the model result the influence of all uncertain parameters are important. Figure 7 shows the 95 % probability tolerance range calculated with MC-CL and SA taken into account all 13 as uncertain declared parameters after the simulated 9 days. For the MC-CL method 1000 simulation runs were used, which provided a small estimation error of 0.005. Theoretically, 41 simulation runs would be enough to calculate the 95 % probabilities but the estimation error of 0.024 seemed too high. The SA needs only 27 simulation runs.

As expected the tolerance ranges are larger than for one uncertain parameter (see figure 5). The most uncertain areas are located at the inner and the outer boundaries of the bends. This is due to the dominance of the curve influenced uncertain parameters. For a classical numerical morphodynamic simulation the calculated values for the evolution were believed in a range of +/- 10 cm. With a reliability analysis this very global range can be stated more precisely. At the straight sections the tolerance range is often only 10 cm but in the bends it increases up to 0.5 m. This result shows that the simulation of the bedload transport perpendicular to the direction of the main current has to be enhanced in the models.

In the considered section of the river Danube bed level changes of some decimeters up to 1 m can be observed after high flood events. Thus, confidence intervals of +/- 10 cm in a morphodynamic simulation are significant and are in the same range than the inaccuracy of river bed soundings.

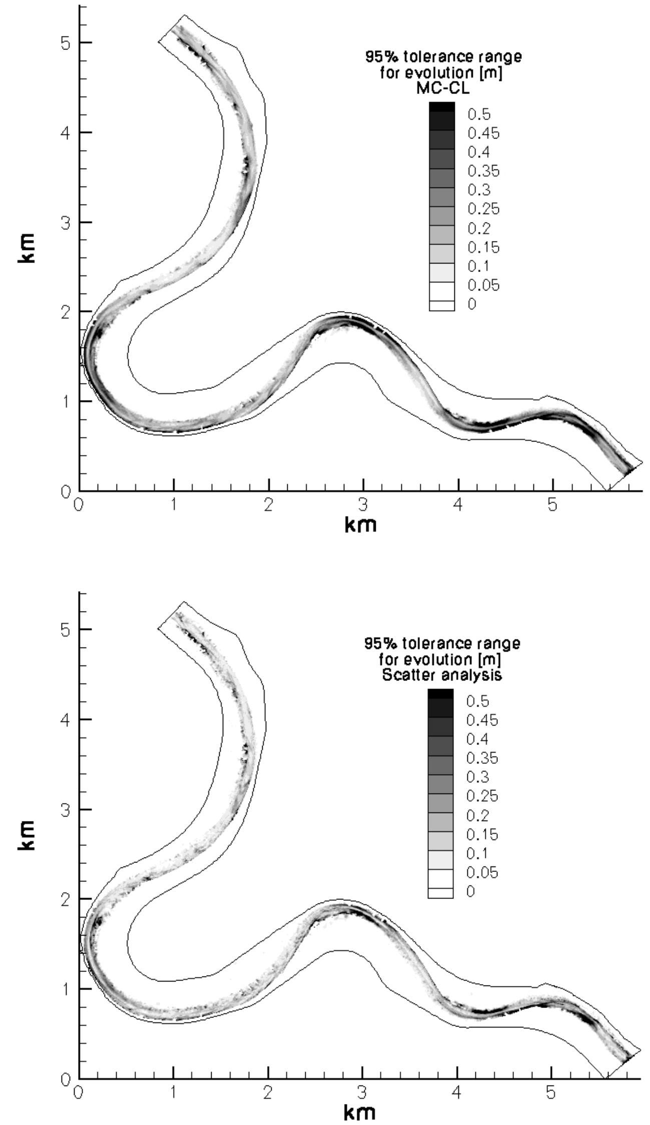


Figure 7. 95 % tolerance range calculated with MC-CL (above) and SA (below) after 9 days simulation.

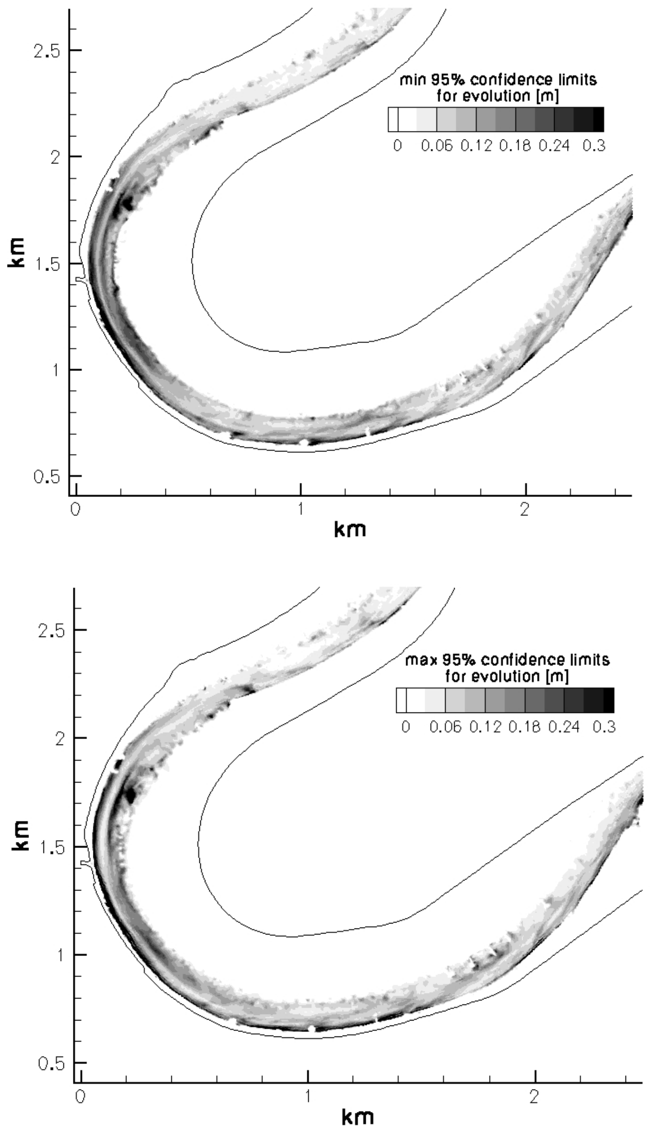


Figure 8. Minimum (above) and maximum (below) 95 % confidence limits at Mühlham bend.

The MC-CL method allows to compare the minimum with the maximum confidence limit. It can be seen in figure 8 that the output distribution is not symmetric. Two effects act in the opposite direction: the central limit theorem combines asymmetric input distributions into symmetric ones whereas non-linear effects deform symmetric input distributions to asymmetric ones. This is a sign for the presence of non-linearities and can only be detected with methods like MC-CL, which consider non-linearities.

The uncertainties increase in time. As expected the deviations grow stronger during high discharges. The peaks of increase occur during the floods after 2.5 d and 6.5 d, while the first peak is bigger than the last one (see figure 9). So probably the uncertainties will not increase to infinity over time if the river system is not changed basically, e.g. as a result of the construction of a new barrage.

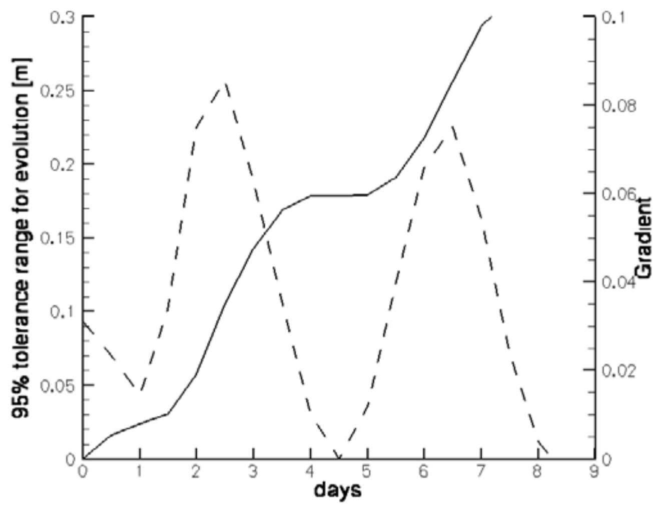


Figure 9. Time series of deviation (solid line) at one point (fairway of Mühlham bend) and the gradient of deviation (dashed line).

Figure 10 shows the comparison for the 95 % probability tolerance range for the evolution calculated with MC-CL and Scatter Analysis. The results pattern are quite similar (see figure 7) but the tolerance ranges calculated with MC-CL are at the most locations bigger. The effects of non-linearities seem to be too important to be neglected. For a significant analysis only the MC-CL method is precise enough to calculate the tolerance ranges.

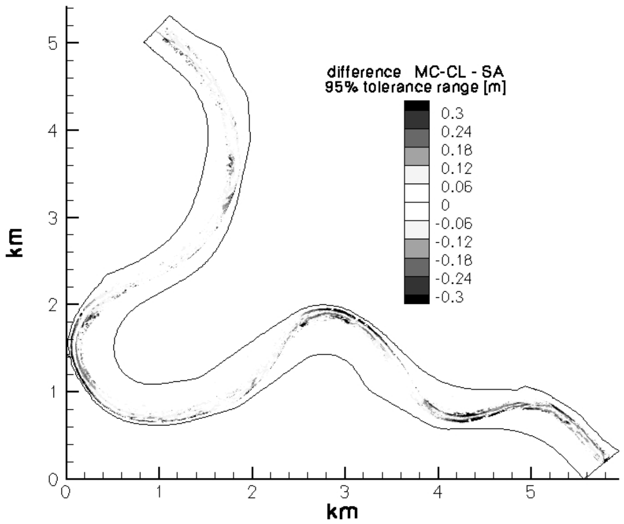


Figure 10. Comparison of MC-CL and SA for the 95 % tolerance range after 9 days simulation.

## 4 CONCLUSIONS

The Scatter Analysis is a very simple and efficient tool for calculating the sensitivities and should always be used before other more sophisticated methods are applied. For the shown example the simulation time is 37 times higher for the MC-CL method than for the SA. For the interpretation of the results a special post-processing program is needed for both methods. The run time for the SA post-processor is also much less time consuming (SA: 4 min, MC-CL: 4 h). With these post-processors the user-friendliness and the error-proneness are the same for both methods. The only but strong argument for the MC-CL method is that it allows for non-linearities, which is important for calculating the tolerance ranges precisely.

Instead of presenting just one value in time and space with a general uncertainty range the reliability analysis allows to specify a tolerance range with a given probability, which varies in time and space. Therefore, there can be areas of high and low result uncertainty, which may give reason to a deeper investigation or to choose another site being appropriate for a monitoring station. The results of the reliability analysis are quite satisfying but the authors see a big need for further development mainly on two topics. Firstly, the reliability analysis comes from the mathematic/statistic field and has to be adapted to an engineering field. This means to choose a suitable reliability method as well as modifying it to the needs of the morphodynamic numerical models. Secondly, presenting the results becomes not easier if they are distributions instead of single values in time and space. Thus, helpful diagrams of the results are needed in order to explain them because for most users these kind of results from numerical models are new and unusual. Still a lot of work has to be done to achieve that reliability analysis is an established part in morphodynamic numerical modelling.

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