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Fluvial sediment transport and morphology: views from upstream and midstream

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ABSTRACT: In the 1970s, the challenge of analyzing fluvial sediment transport and morphology was likened to attempting to treat a chronic skin disease in the absence of effective medications. Guided by the insight of preceding researchers, this paper highlights contemporary advances in measurement and analysis frameworks and methodologies that contribute to the search for effective remedies. The principal aspects of the fluvial engineer's itch are discussed along the way, namely: channel roughness and resistance, initiation of motion, sediment continuity and transport, and bed stability and bedforms. The recent advances in methodologies and understanding reveal great potential for progress to be made in the coming years, especially where research efforts are guided by the vision of those upstream.

Keywords: Fluvial hydraulics, Sediment transport, Threshold, Bedforms, Dunes, Ripples

1 INTRODUCTION

With water being the key to our existence, rivers have always attracted settlement and interest. The dynamic nature and power of the river reminds us of humankind's limitations, however, and provide us with the sorts of challenges and questions that have inspired our development through time.

Lane (1955) identifies the particular importance of river morphodynamics and sediment transport to the engineer, with many of their greatest problems lying in this arena. Half a century later, we are reminded by Gyr and Hoyer (2006) of the gulf between the hopes for progress in understanding of this field and the state of the science. They refer to the following insight of Kennedy (1971), also quoted by Müller (1996), that paints an entertaining picture of difficulties still faced today.

> *"Engineering problems associated with sediment transport by alluvial streams can be likened in many respects to a chronic skin disease. Many treatments (theories) have been developed and tried, but few have produced immunity or lasting cures (confirmed theories or realizable solutions). Meanwhile the itch (the river engineer's problems) goes on and on, while the pharmacist's shelves (the literature on sediment transport) become ever more cluttered up with salves and ointments (papers and reports) that are of no particular value but still are not thrown away (rejected), perhaps because of the pharmacist's (engineer's) languor, perhaps because no better medications (formulations) are available, or perhaps because each doctor or pharmacist (engineer or researcher) has a vested interest in promoting his own compound (theory). Each periodic inventory (survey paper) must take account of the ineffective unguents (theories), with the accompanying danger that if a dermatological wonder drug (valid generalized sediment transport theory) were to be forthcoming, it might well be overlooked on the overfull shelves (journals) of ineffectual remedies (theories).*
>
> *There are five principal aspects to the alluvial river sediment transport "itch": initiation of motion, bed and channel stability (formation of ripples, dunes, meanders, etc.), channel roughness, bed-load transport, and sediment suspension. ... Only in the case of sediment suspension are the understanding of the mechanics and its formulation on relatively secure ground."*

As recognized by Gyr and Hoyer (2006), Müller (1996), and Kennedy (1971), a critic is obliged to suggest corrective measures to remedy the situation judged deficient. Particular observations posed by the last two of these authors include the following:

- Researchers need to be more discerning in choosing avenues to explore;
- Radical new types of experiments are needed to clarify the mechanics of entrainment and suspension, especially utilising revolutions in electronic instrumentation;
- Theories need to relate directly the relevant quantities in the physical processes, e.g. considering instantaneous forces from particle contacts and fluid accelerations and pressures when analysing particle movements, rather than necessarily simply linking the same process to a temporally- and spatially-averaged bed-parallel shear stress; and
- Understanding of processes at small scales needs to be improved, and also correctly upscaled to give the predictive tools the river engineer is looking for.

Our view today is from on top of the shoulders of others such as those quoted above. At the risk of highlighting further salves to clutter our pharmacy shelves, this paper presents and discusses recent research on the hydraulician's itch, particularly regarding some of the above comments concerning possible avenues to explore and advances to utilise. The material is presented in terms of successive aspects of the 'itch': from measurements of the intricacies of turbulent and grain motions, and associated assessments across varying scales of hydraulic roughness and sediment transport; to entrainment at particle to reach scales; to sediment continuity over a range of scales; to bedform dynamics at sub-element to reach scales.

2 CONTEMPORARY DATA AND SPATIAL AVERAGING

As foreseen by Kennedy (1971), and others, advances in computing and equipment technologies have lead to measurements and simulations of flow, sediment flux and river morphology being made at increasingly finer temporal and spatial resolutions.

In the laboratory, measurements of bed morphology are now obtained at sub-millimetre resolutions using lasers (e.g. Aberle and Nikora 2006; Tuijnder et al. 2009; Haynes and Ockleford 2010) and photogrammetry (e.g. Henning et al. 2009), with particle-image velocimetry (PIV) now commonly providing detailed measurements of instantaneous three-dimensional (3D) velocity fields and boundary dynamics in selected planes of interest (e.g. Adrian 1991; Schlicke et al. 2007; Coleman and Nikora 2009a). High-resolution imaging of particle motions is also providing valuable new insight into sediment-particle dynamics and links with turbulent flow (e.g. Ancey et al. 2006; Radice et al. 2009, 2010). In order to obtain detailed laboratory measurements of bed and flow dynamics associated with developing bedforms, "flying-probe" methodologies (e.g. Bruun 1995) have also been developed and utilized (e.g. Clunie et al. 2007; Coleman et al. 2008).

In the field, GPS-linked sonar systems now provide high-resolution measurements of bed bathymetry, with acoustic Doppler current profilers (ADCPs) commonly used to measure instantaneous 3D velocities along a vertical. ADCPs are also used to provide detailed measurements of fluvial bedload dynamics (e.g. Rennie and Millar 2004), and 3D PIV systems are being developed for field application. In a novel development, multibeam echo-sounding has been used to visualize sediment motions and dynamic turbulence structures (e.g. Best et al. 2008). At larger measurement scales, airborne LiDAR (Light Detection and Ranging) and integrated technologies show great potential for providing high density topographic and bathymetric data (e.g. Kinzel et al. 2007).

From such measurements, we are today gaining detailed pictures of turbulence and associated sediment-transport and bed dynamics (e.g. Best 2005). The challenge arises, however, as to how to interpret the collected fine-scale data in terms of bulk morphological, flow and transport characteristics, e.g. hydraulic resistance, bed shear stress, and sediment transport rate, that can be used for design and management purposes. In this regard, if morphology is described at the bed-roughness (e.g. dune) scale, associated descriptions of flow and sediment-transport properties need to be representative of this spatial domain, i.e. scale-consistent with the roughness description. Appropriate spatial averaging of finer-scale descriptions and measurements provides the requisite tool for this upscaling.

Recent application of spatial averaging of rough-bed flows (e.g. Nikora et al. 2001, 2004, 2007a,b) has led to valuable outcomes that include: strengthened definitions of hydraulic terms such as flow uniformity, two-dimensionality, and bed shear stress; identification of specific flow layers and flow types; knowledge and understanding of the vertical distribution of double-averaged velocity in the roughness layer between the roughness tops and troughs; and explicit accounting for form and surface drag and form-induced stresses and fluxes in flow conservation equations. In particular, Nikora and Nikora (2007) and Nikora (2008, 2009) highlight valuable insight into rough-bed hydraulic resistance that can be gained from the spatial averaging technique, including determination of the relative contributions of, and interplay between, different processes at biota to catchment scales. The value of upscaling through spatial averaging for interpreting flow, sediment and morphological behaviour is further highlighted in the following discussions.

3 SEDIMENT ENTRAINMENT

With improvements in computing capacity and measuring instrumentation, research efforts have intensified in the past few years regarding the process of sediment entrainment (e.g. Niño and García 1996; McEwan and Heald 2001; Nelson et al. 2001; Papanicolaou et al. 2002; Hofland et al. 2005; Hofland and Battjes 2006; Cameron et al. 2006; Schmeeckle et al. 2007; Vollmer and Kleinhans 2007; Detert et al. 2008, 2010; Diplas et al. 2008; Dwivedi 2009). A vast amount of data can now be collected, and the presence and passage of coherent turbulent structures at entrainment can be inferred, but the question remains as to what actually acts to entrain sediments?

Consistent with the recommendation of Müller (1996) to consider relevant forces when analyzing particle motions, Coleman and Nikora (2008) express Newton's second law of motion separately for sediment particles and fluid flow in their derivation of a rigorous framework for describing particle threshold. Using spatial averaging to provide a scale-consistent coupling of fluid and particles, they then combine the respective expressions of motion to explicitly show that for an individual particle at threshold, particle weight and buoyancy and inter-particle contact forces are balanced by forces arising from instantaneous fluid accelerations, pressure gradients and stress gradients. The derived framework of Eq. (1) given in the Appendix to this paper appropriately reveals bed shear stresses, across-particle differences in pressures and fluxes of momentum, and sediment-bed characteristics to be typical key factors in particle entrainment (Coleman and Nikora 2008). This framework can potentially be used to aid understanding of entrainment mechanics, the design and analysis of further studies of entrainment, the design of parameterizations that lead to the solution of entrainment problems, and numerical modelling of combined fluid and sediment dynamics. As envisaged by Müller (1996), for example, the framework can be used to associate detailed instantaneous three-dimensional flow structures with the concomitant effects on sediment particles that lead to entrainment. The framework is furthermore of more direct use to the practicing engineer who is interested in larger-scale descriptions of erosion. As promoted by Müller (1996), the upscaled expression of Eq. (2) is utilized by Coleman and Nikora (2008) to interpret the form, variability and applicability of relations originating from the work of Shields (1936) that are widely used by river engineers to define threshold conditions in an averaged (at the reach-scale) sense.

4 SEDIMENT CONTINUITY

The Exner (1925) equation of sediment-mass conservation is the foundation of morphodynamic analyses (e.g. Paola and Voller 2005, Parker 2008). In contrast to conventional 'mixture-scale' control-volume approaches to deriving this equation, spatial averaging of the subparticle-scale differential equation of mass conservation gives a general statement of sediment-mass balance that provides insight into considerations of sediment continuity at patch, bedform and larger scales (Coleman and Nikora 2009b). The spatially-averaged form of the Exner equation addresses the need, e.g. identified in Paola and Voller (2005) and Parker (2008), for a general expression that both provides a universal description of the sediment-mass balance and also enables interpretation of the assumptions and limitations implicit in various ad hoc formulations of reduced, combined or improvised terms.

Importantly, the spatial-averaging approach highlights the effects of the scale of consideration on defining and interpreting macroscopic (mixture-scale) sediment and layer properties such as averaged densities, volume concentrations or fractions, velocities, transport modes and rates, interfaces and bed layers (e.g. Figure 1). Double-averaged sediment-mass transport rate (per unit area), for example, is explicitly shown to be given by the product of volume concentration, solid density, and sediment velocity.

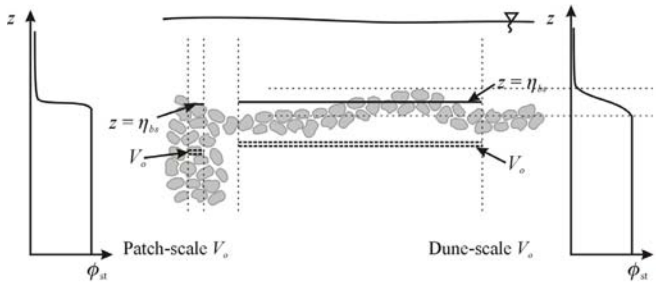

Figure 1. Schematic variations of sediment concentration ϕ_{st} for patch- and dune-scale averaging volumes V_o applied to the same riverbed. Also shown are potential definitions of the bed surface $z = \eta_{bs}$ based on these distributions.

The spatially-averaged form of the Exner equation enables analyses in terms of individual or successive layers, including bed and suspended loads, where layer interfaces (e.g. the bed surface) are clearly shown to be defined based on isosurfaces of sediment concentration, or other sediment properties (e.g. densities or transport rates) within regions of constant concentration. This general equation also novelly includes the effects of fluctuations in sediment properties (e.g. density, velocity, and concentration or volume fraction) within analysis volumes, and readily enables calculations in terms of size fractions.

5 BEDFORM INITIATION AND DEVELOPMENT

It is intriguing to consider how trains of highly-ordered sediment waves can arise from the chaos of the underlying turbulence and grain-motion dynamics, and what mechanisms act to limit the growth of these forms. For the river engineer, understanding of these bedforms is pragmatically required in regard to design of structures in the fluvial environment (e.g. Amsler and García 1997; Coleman and Melville 2001), as well as analysis and management of the transport of sediments and attached micro-organisms and chemicals (e.g. nutrients, contaminants). Nevertheless, in spite of strenuous efforts over the previous half century, the ASCE Task Committee on Flow and Transport over Dunes (2002) laments that *"... the engineering prediction of flow and sediment transport over bedforms, in general, and dunes, in particular, still presents a major obstacle in the solution of sedimentation problems in alluvial channels. ... Even for the simplest case of a well-sorted sediment in a straight prismatic channel with rigid banks, a general answer [to defining the expected flow depth and the amount of sediment being transported] can only be given with an uncomfortably high degree of uncertainty."*

In terms of the initiation of bedforms on a sediment bed, Coleman and Nikora (2009a) conjecture that the nascent seed waves from which fluvial bedforms develop are generated on planar mobile sediment beds in a two-stage process. The first stage comprises the motion of random sediment patches that reflect the passage of attached-eddy-generated sediment-transport events. In the second stage, interactions of the moving patches result in a bed disturbance that exceeds a critical height and interrupts the bed-load layer. Quasi-regular seed waves are then generated successively downstream via a scour-deposition wave that arises from the requirement of sediment mass conservation and the sediment-transport and bed-stress distributions downstream of a bed perturbation (e.g. Raudkivi 1966; Jones 1968; Smith 1970; Bradshaw and Wong 1972; Fredsøe 1986; McLean and Smith 1986). Seed waves are thereby of preferred lengths that scale with the grain size, i.e. length = O(130) grain diameters, agreeing with compiled measurements (Coleman and Melville 1996; Coleman et al. 2003; Coleman and Nikora 2009a).

Once seed waves have been generated, their heights and lengths increase through sediment continuity and the sediment-trapping nature of the bedform lee region. The growing sand waves also coalesce as smaller faster waves approach and merge with larger slower waves (e.g. Exner 1931; Simons and Richardson 1960; Führböter 1967; Jain and Kennedy 1974; McLean 1990; Raudkivi and Witte 1990; Coleman 1991; Ditchfield and Best 1992; Coleman and Melville 1994). Instability of the fluid-sediment flow system (e.g. Coleman and Fenton 2000) furthermore gives periods of accelerated growth for the developing bedforms, where these periods are typically accompanied by multiple successive instances of bed-form coalescence (Coleman and Melville 1994).

The growth of sediment waves from plane-bed conditions can be described by the power law $(P/P_{ss}) = (t/t_{ss})^{\gamma}$, where t_{ss} is the time t to achieve steady-state magnitude P_{ss}, P is the average value of a sediment-wave parameter (length λ or height h), and the growth exponent γ = 0.28-0.37 (e.g. Grinvald and Nikora 1988; Nikora and Hicks 1997; Coleman et al. 2005).

Due to an approximate invariance in bedform steepness during development (e.g. Coleman et al. 2005), the rapidly-adjusting bedform-associated boundary layer is found to essentially pose an equilibrium property for developing bedforms, i.e. to be self-similar in time (Coleman et al. 2006). For this boundary layer, the double-averaged longitudinal-velocity distribution is found to be linear below the crests of developing dunes, which is useful to know for field measurements of discharge over dune beds (e.g. Nikora et al. 2004; Coleman et al. 2006; McLean and Nikora 2006; McLean et al. 2008). In addition, increases in double-averaged Reynolds stresses in the vicinity of the dune crest are found to be balanced by equivalent negative form-induced stresses at these levels.

An interesting variation on the studies that have lead to this understanding of bedform initiation and growth has been consideration of the role of turbulence in morphology generation and control through studies of bed morphology processes in laminar flows (e.g. Coleman and Eling 2000, Cameron et al. 2006, Lajeunesse et al. 2010).

6 BEDFORM CHARACTERISATION

As reflected in recent research (e.g. McElroy et al. 2008; van der Mark et al. 2006, 2008; Tuijnder et al. 2009, Bartholdy et al. 2010), there remains a real need to determine agreed reliable methodologies for estimating bedform characteristics. This present situation is over two decades since a 1987 symposium was held with the purpose of classifying large-scale subaqueous bedforms (Ashley 1990), and furthermore four decades since focused efforts to define bedform characteristics were taking place around the world (e.g. Simons and Richardson 1960; Yalin 1964 1972; Bogardi 1965; ASCE 1966; Nordin and Algert 1966; Vanoni and Hwang 1967; Hino 1968; Crickmore 1970; Nordin 1971; Jain and Kennedy 1971, 1974).

Bedform heights are typically calculated based on differences in bed-elevation extremes, between positions of zero-bed-level crossing (e.g. Crickmore 1970) or detected signatures such as lee slopes (e.g. Coleman 1991; Coleman et al. 2005) for example. Bedform heights have also been related to bed-level variance (e.g. Nikora et al. 1997). Characteristic streamwise bedform lengths have been calculated using a range of approaches, e.g. using the distance between zero crossings (e.g. Crickmore 1970; Nordin 1971), bed-level correlation and structure functions (e.g. Nordin and Algert 1966; Nordin 1971; Nikora 1982; Coleman and Melville 1996; Nikora et al. 1997; Butler et al. 2001; Coleman et al. 2003; James et al. 2007), bed-level spectra (e.g. Nordin and Algert 1966; Hino 1968; Jain and Kennedy 1974; Nakagawa and Tsujimoto 1984; Nikora et al. 1997), and roughness functions (Nikora and Hicks 1997; Jerolmack and Mohrig 2005; McElroy et al. 2008). A number of authors have furthermore considered distributions of bedform lengths in advance of a single characteristic value (e.g. Ashida and Tanaka 1967; Wang and Shen 1980; Raudkivi and Witte 1990; Coleman and Melville 1994; van der Mark et al. 2008). Each of these approaches outputs characteristics of the bed, although importantly, these may not appropriately describe the intended bed aspect. Predictions of the various methodologies can consequently vary widely owing to the different approaches reflecting different physical aspects of the bed.

Owing to the density of data utilised, bed-level variance (or standard deviation σ) provides an advantageous means of estimating bed-form height for the digital elevation models (DEMs) measured today, where height h = 2.83σ for a single-frequency sine wave, h = 3.43σ for a train of identical triangles, and h = 1.7-2σ for natural sand waves (Nikora et al. 1997).

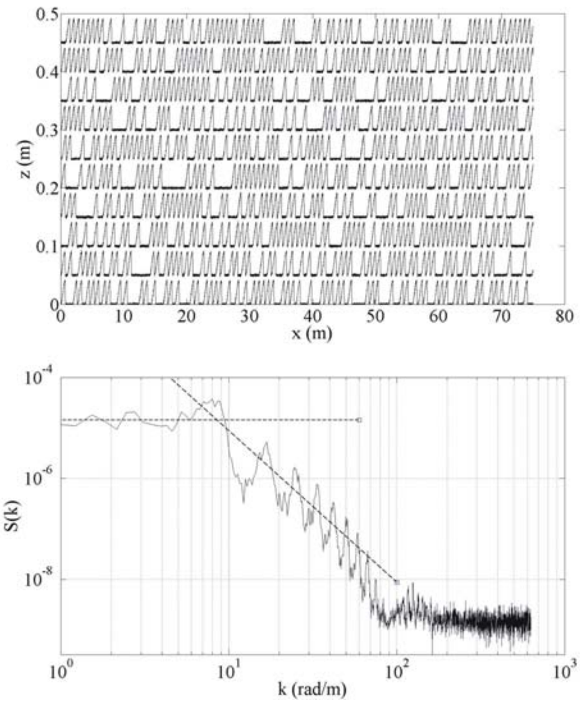

Figure 2. (a) Ten bed profiles (offset vertically by 50mm), and (b) the corresponding autospectrum (with shown fitted lines of $S(k) \propto k^{-3}$ and $S(k) \propto k^{0}$)

The spectral representation of a bedform-covered riverbed is typically similar to the form of Figure 2b. The characteristic bed-surface scaling at higher wavenumbers of $S(k) \propto k^{-3}$ has been related to various physical aspects of bedforms, including the angle of repose nature of lee slopes (Hino 1968), discontinuities in bed slopes associated with the dune lee-side (e.g. Plate 1967, 1971; Engelund and Fredsøe 1971, 1982; Kennedy 1980), bedform coalescence through bedform speeds decreasing with increasing size (Jain and Kennedy 1974), geometric self-similarity of bedforms (Nikora and Hicks 1997; Nikora et al. 1997), and energy dissipation considerations (Nikora and Goring 2000). Although the physical nature of this scaling can be debated, the scaling remains a characteristic signature of bedforms (e.g. Aberle et al. 2010). As suggested by Tuijnder (2009), the wavenumber defining the lower limit of the bedform-related scaling region can be taken to provide a good estimate of the principal bedform length for a bed. This is because the limiting plateau region at low wavenumbers (Figure 2b) simply arises from a redistribution of spectral energy across adjacent wavelengths due to random spacing of the bedforms (Figure 2a). The indicated bedform length of Figure 2b is then 0.74m (wavenumber $k = 2\pi/\lambda = 8.5$), where the underlying bed configuration of Figure 2a consists of 10 profiles of random-length plane-bed sections combined with triangular dunes of a cosine stoss slope, 40mm height, 0.75m length, and 1mm surface roughness. This spectral approach to determining characteristic bedform length takes advantage of the data density that can be recorded today, with rigorous guidance furthermore available to determine associated statistical uncertainties.

Modern means of recording bed surfaces mean that methodologies for quantifying the three-dimensional nature of bedforms can now be trialled and refined, e.g. for interpreting the hydraulic resistance of bedforms (e.g. Sirovich and Karlsson 1997; Maddux et al. 2003a, b; Venditti 2007), for classification of bedforms (e.g. ASCE 1966; Southard and Boguchwal 1990; Ashley 1990; Venditti et al. 2005), and for understanding and interpreting local flow dynamics (e.g. Best 2005). The need to quantify dune three-dimensionality is clearly apparent for considerations of deformable mobile beds of waves that vary markedly in space and time (e.g. Inglis 1949; ASCE 1966; Allen 1968, 1969). Following on from the earlier work of Nordin (1971), the geometry of the 2D autocorrelation function (or the related second-order structure function) when applied to sand-bed elevation fields is found to provide an effective means of assessing the three-dimensionality of sand waves (e.g. Goring et al. 1999, Butler et al. 2001, Coleman et al. 2008, Aberle et al. 2010). Assessment of the 3D structure of gravel-bed surfaces has similarly recently been advanced by viewing the measured topography as a three-dimensional random field as an alternative to the characteristic-particle-size approach (e.g. Nikora et al. 1998, Goring et al. 1999, Butler et al. 2001, Nikora and Walsh 2004, Aberle and Nikora 2006).

Uncertainty in bedform quantification also impacts effective determination of types of bedforms and their associated governing mechanisms, e.g. ripples, dunes, and larger low-angle dunes (e.g. Holmes 2003, Best 2005). Ripples and dunes are identified as separate bedforms in most classification schemes, where ripples are recognised to be limited to forming in sands of up to about 0.6mm in diameter, and ripple and dune lengths scale principally with grain size and flow depth respectively (e.g. Inglis 1949; Bagnold 1956; Bogardi 1965; ASCE 1966; Engelund and Hansen 1967; Kennedy 1969; Yalin 1972, 1977, 1992; Davies 1982; van Rijn 1984; Ashley 1990; Southard and Boguchwal 1990; Baas 1993; Julien and Klaassen 1995; Raudkivi 1997; Watanabe et al. 1997; Schindler and Robert 2004). A separate school of thought, however, contends that there may be no statistical or fundamental differences between ripples and dunes, especially for natural river flows (e.g. Kennedy 1969, Nordin 1971, Flemming 2000, Jerolmack and Mohrig 2005, and Jerolmack et al. 2006). In order to definitively identify any differences between ripples and dunes, it is necessary to correctly define the characteristics of the bedforms.

In addition to the characterisation of lengths and shapes, relations for bedform speeds (e.g. Simons et al. 1965; Coleman 1996; Nikora et al. 1997; Raudkivi 1997, 1998) are central to linkages between bedform movements, sediment-transport rate and predictions of bed development (e.g. Hubbell 1964; Simons et al. 1965; Crickmore 1967, 1970; Nordin 1971; Führböter 1979; Willis and Kennedy 1980; Engel and Lau 1980, 1981; van den Berg 1987; Gomez et al. 1989; Mohrig and Smith 1996; García 2008b; McElroy and Mohrig 2009). With the improved ability to track individual bedforms that is available today, the effectiveness of determining transport rate from bedform dimensions and propagation speeds can now be reliably investigated.

7 CONCLUSIONS

In reviewing the observations and questions of Leonardo da Vinci regarding water and its motions, Levi (1995) writes *"Let whoever said that hydraulics, as a science, is old and has no longer anything to discover, try to interpret all this and see how great our ignorance still is."* Falvey (1999) echoes this comment, concluding *"... that the field of hydraulics has not stagnated and that we are still in an age where ideas and fundamental concepts are being developed."* In particular, this can be said of fluvial hydraulics today, with a mass of understanding having been accumulated (e.g. García 2008a), and yet so many interesting questions still to answer, and itches requiring adequate treatment. With today's means of generating and collecting data, we are now in the privileged position of being able to test and build on the visions of those who have preceded us. In presenting the research thoughts of previous decades along with recent efforts to test or address these thoughts, this paper has sought to encourage the efforts of researchers that contemporary advances in instrumentation and understanding reveal great potential for progress to be made in the coming years, especially where we learn from the views of those upstream.

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APPENDIX

Coleman and Nikora (2008) show that for an individual particle at threshold (Figure 3)

$$0=\left(m_p-m_{fp}\right)g_i+\sum_k F_{ci}^k$$
$$+\rho V_f\left[g_i-\left(\frac{\partial\langle u_i\rangle}{\partial t}+\frac{\langle u_j\rangle}{\phi_s}\frac{\partial\phi_s\langle u_i\rangle}{\partial x_j}\right)\right] \qquad (1)$$
$$-V_o\frac{\partial\phi_s\langle p\rangle}{\partial x_i}+V_o\frac{\partial}{\partial x_j}\left[\phi_s\left(\mu\left\langle\frac{\partial u_i}{\partial x_j}\right\rangle-\rho\langle\hat{u}_i\hat{u}_j\rangle\right)\right]$$

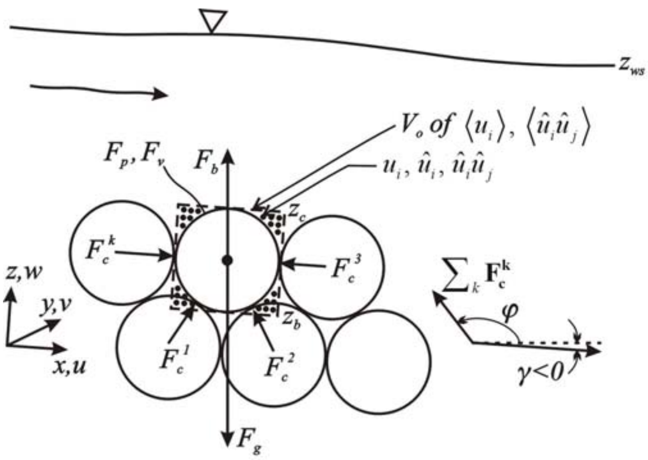

Figure 3. Forces on a sediment particle at threshold

where $m_p = \rho_b V_b$ is the particle mass; $m_{fp} = \rho V_b$ is the mass of fluid displaced by the particle; ρV_f is the mass of fluid within the spatial averaging domain V_o; ρ_b and ρ are particle and fluid densities, respectively; V_b and V_f are the respective particle and fluid volumes within V_o; g_i is the i^{th} component of gravitational acceleration in direction $x_i = (x,y,z)$; k interparticle forces of components F_{ci} act on the particle surface in addition to the fluid forces; u_i is the i^{th} component of the fluid velocity vector (u, v, w); t is time; p is pressure; μ is the dynamic fluid viscosity; $\hat{u}_i = u_i - \langle u_i\rangle$ denotes the deviation of the instantaneous variable u_i at a given spatial point from its instantaneous spatially-averaged value $\langle u_i\rangle$; $\langle u_i\rangle = \left(1/V_f\right)\iiint_{V_f} u_i dV$; $\langle\hat{u}_i\rangle = 0$; $\phi_s = V_f/V_o$ is the roughness geometry (or porosity) function; F_g and F_b are particle weight and buoyancy forces; F_p and F_v are form and skin-friction hydrodynamic forces; and the angles γ (bed slope) and φ (to the line of action of the summed interparticle forces) are positive anti-clockwise from the downstream-pointing axis.

Through enlargement of the averaging volume V_o of Figure 3 to include a broad area of the bed surface, the framework of Eq. (1) reveals that averaged en masse movement of particles by steady uniform 2D flow is described by (Coleman and Nikora 2008)

$$\theta_c^a = f\left(\begin{array}{l}\dfrac{\sum_k \mathbf{F}_\mathbf{c}^\mathbf{k}}{\gamma_* V_o},\ \gamma, \phi_s, (s-1),\\ \dfrac{1}{\gamma_*}\dfrac{\partial}{\partial z}\left[\phi_s\left(\mu\left\langle\dfrac{\partial w}{\partial z}\right\rangle-\rho\langle\hat{w}\hat{w}\rangle\right)-\phi_s\langle p\rangle\right]\end{array}\right) \qquad (2)$$

where $\theta_c^a = \tau_0 A_o/\left[\gamma_* V_o\left(1-\phi_s\right)\right] = \theta_c/\left(1-\phi_s\right)$ is a physically-consistent (threshold boundary force relative to the submerged particle weight) alternative to the traditional Shields parameter θ_c; the second and final terms of (2) represent normalized interparticle forces and across-particle gradients in momentum flux and pressure; $s = \rho_b/\rho$ is sediment specific gravity; $\gamma_* = (s-1)\rho g$ is submerged weight per unit volume; g is the gravitational acceleration constant; $A_o = V_o/d$ is the area in plan of the spatial averaging volume; $d = z_c - z_b$ is the particle height (from the averaging domain base level of z_b to the crest level of z_c, Figure 3); the summed interparticle forces are expressed in vector form as $\sum_k \mathbf{F}_\mathbf{c}^\mathbf{k}$; the bed shear stress is clearly shown to be

$$\tau_0 \equiv -\frac{1}{V_o}\int_{z_b}^{z_c}\iint_{S_{\text{int}}}\left[pn_x-\left(\mu\frac{\partial u}{\partial z}\right)n_z\right]dSdz = \rho g_x\int_{z_b}^{z_{ws}}\phi_s dz\,;$$

$\tau_0 \equiv \rho u_{*0}^2$; u_{*0} is bed shear velocity; n_i is the i^{th} component of the unit vector normal to the surface element dS and directed outward from the bed and into the fluid; S_{int} = extent of water-bed interface within the thin (in the bed-normal direction) averaging volume; z_{ws} defines the water surface; and spatially-averaged quantities are effectively averaged in space and time for averaging domains of lateral extent encompassing all turbulent scales, with $\langle u_i\rangle = \langle\overline{u}_i\rangle$, and steady uniform 2D flow characterised by $\langle v\rangle = \langle\overline{v}\rangle = \langle\overline{w}\rangle = \langle w\rangle = \partial/\partial y = 0$.

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