

Solving open channel flow problems with a simple lateral distribution model

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ABSTRACT: A simple lateral distribution model is shown to be a useful tool for analyzing a range of practical problems in river engineering. The model is based on the Shiono & Knight method (SKM) of analysis that takes into account certain 3-D flow features that are often present in many types of water-course during either inbank or overbank flow conditions. The paper demonstrates the use of this model to predict lateral distributions of depth-averaged velocity and boundary shear stress, stage-discharge relationships, as well as indicating how to deal with some vegetation, sediment and ecological issues. The paper illustrates the use of the SKM approach, and the recently developed Conveyance Estimation System (CES), which is largely based on the SKM, through a number of case studies. The CES features in the Conveyance and Afflux Estimation System (CES-AES) software, which is freely available at www.river-conveyance.net. The paper concludes with a discussion on the limitations and the future potential of the methodology, including the possibility of analyzing steady flows in non-prismatic channels and determining kinematic wave speed versus discharge relationships in unsteady flow.

Keywords: Floods, Stage-discharge relationships, Velocity distributions

1 INTRODUCTION

1.1 *Why bother with such a simple model?*

It is worth noting that higher dimensionality of model does not necessarily lead to better accuracy in the results. In certain cases, the opposite may be true. The principle of Occam's razor should always be applied – that of starting with the simplest by assuming the least. (*Pluralitas non est ponenda sine necessitate*; “Plurality should not be posited without necessity”, William Occam, 1285–1347). The principle gives precedence to simplicity; of two competing theories, the simplest explanation of an entity is to be preferred. In the context of modelling flows in rivers, this suggests caution before embarking on 3-D modelling for solving every type of river engineering problem.

1.2 *Do all the assumptions limit the model too much?*

In technical terms, a lateral distribution model (LDM) might be considered to be too simplistic

since it relies on steady flow conditions, the channel cross section being prismatic and only gives the lateral distributions of depth averaged velocity and boundary shear stress. However, it should be remembered that very often simple tools are still used to craft works of art and beauty. Just consider what can be done with a paintbrush, chisel and tape measure. It is the skill to which they are put that portrays the ‘usefulness’ of the tools.

So when dealing with practical river issues, it is worth re-iterating that very often steady flows do need to be analyzed (e.g. when estimating conveyance capacity for drainage systems or in extending stage-discharge relationships), that river reaches or channels are often treated as being prismatic (e.g. reaches near gauging stations), that knowing the distribution of depth-averaged velocity across a channel is useful (e.g. in vegetation studies, and for checking ADCP {acoustic Doppler current profile} or propeller gaugings at specific river gauging sites) and knowing about boundary shear stresses is important in most sediment studies.

2 MODELLING ISSUES

2.1 What are the dominant physical processes?

It should be emphasized that all models are but tools that reflect the concepts and physical processes that are deemed to be of particular importance, and are usually based on some physical data from natural phenomena. The process of obtaining the most appropriate concepts, through to model building and then finally on to calibration is considered elsewhere by Nakato & Ettema (1996), Knight (2006a, 2008) and Mc Gahey *et al.* (2008).

The lateral distribution model used herein is based on the Shiono & Knight method (SKM), described fully in several other places, for example Shiono & Knight (1991), Knight & Shiono (1996), Abril & Knight (2004) and Knight *et al.* (2010). The model attempts to incorporate the key physical aspects illustrated in Figure 1.

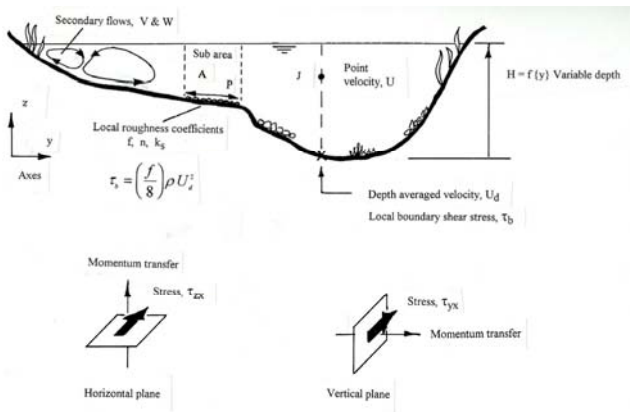


Figure 1. Flow in a natural channel (after Knight & Shiono, 1996)

Given the 3-D nature of the flow in most rivers, it is customary to discretize the cross-section into either slices for a 2-D model or elements for a 3-D model. The discharge, Q , is then obtained by the integration of local velocities with corresponding local areas, using Eq. (1) for a 3-D model, or Eq. (2) for a 2-D model and river gauging.

$$Q = \int_A u dA = \int_0^H \int_0^B u dy dz \quad (1)$$

$$Q = \int_0^B U_d dy \quad (2)$$

where

$$U_d = \frac{1}{H} \int_0^H u dz \quad (3)$$

Since the conveyance, K , is related to a longitudinal slope, S (as yet undefined), by

$$Q = KS^{1/2} \quad (4)$$

where K involves geometric and roughness parameters, it is also possible to deduce the discharge from Eq. (4), provided K and S are known. Furthermore, if K values are calculable for a range of depths under steady flow conditions, then they may be used in an unsteady flow model, such as one based on the St Venant equations, to estimate the discharges at given river stages for both in-bank and overbank flows in unsteady flow.

2.2 Governing equations

The SKM is now outlined very briefly. Some of the constants and parameters are therefore undefined herein and further details should be obtained via the references. The governing equation for the depth-averaged velocity in a prismatic channel is assumed to be given by

$$\rho \left[\frac{\partial H(UV)_d}{\partial y} \right] = \rho g H S_o + \frac{\partial H \bar{\tau}_{yx}}{\partial y} - \tau_b \sqrt{1 + \frac{1}{s^2}} - \frac{1}{2\delta} \rho (C_D \beta A_v) H U_d^2 \quad (5)$$

where the overbar or the subscript d refers to a depth-averaged value, $\{U, V, W\}$ are velocity components in the $\{x, y, z\}$ directions, as illustrated in Figure 1, with x = the streamwise direction parallel to the channel bed, y = lateral direction and z = direction normal to the bed, H = depth of flow, ρ = fluid density, g = acceleration due to gravity, S_o = bed slope, τ_{yx} = Reynolds stress on plane perpendicular to the y direction, τ_b = boundary shear stress, C_D = drag coefficient due to vegetation, β = shape factor for the type of vegetation, δ = porosity and A_v = projected area of vegetation in the streamwise direction per unit volume.

For flow over a flat bed in a vegetated channel, the analytical solution for U_d from Eq. (5) is

$$U_d = [A_1 e^{\gamma y} + A_2 e^{-\gamma y} + k]^{1/2} \quad (6)$$

where A_1 and A_2 are, as yet, unknown constants for each panel, but obtained by applying appropriate boundary conditions. The constants γ and k are given by

$$\gamma = \sqrt{\frac{2}{\lambda} \left(\frac{8}{f} \right)^{1/4}} \frac{1}{H} \sqrt{\frac{f}{8} + \left(\frac{H}{2\delta} \right) C_D \beta A_v} \quad (7)$$

$$k = \frac{g S_o H - \Gamma / \rho}{f / 8 + (H / (2\delta)) C_D \beta A_v} \quad (8)$$

For flow over a linearly sloping bed without vegetation, U_d is given by

$$U_d = [A_3 \xi^\alpha + A_4 \xi^{-(\alpha+1)} + \omega \xi + \eta]^{1/2} \quad (9)$$

where the constants α , ω and η are given by

$$\alpha = -\frac{1}{2} + \frac{1}{2} \left\{ 1 + \frac{s(1+s^2)^{1/2}}{\lambda} (8f)^{1/2} \right\} \quad (10)$$

$$\omega = \frac{gS_0}{\frac{(1+s^2)^{1/2}}{s} \left(\frac{f}{8}\right) - \frac{\lambda}{s^2} \left(\frac{f}{8}\right)^{1/2}} \quad (11)$$

$$\eta = \frac{-\Gamma}{\rho \left(\frac{f}{8}\right) \left(1 + \frac{1}{s^2}\right)^{1/2}} \quad (12)$$

Similar to the flat bed case, A_3 and A_4 are unknown constants for each panel, but are obtained by applying appropriate boundary conditions. See Knight *et al.* (2004 & 2007) and Shiono *et al.* (2009) for further details.

As shown by Eqs (7)-(8) or Eqs (10)-(12), each panel requires three calibration parameters (f , λ and Γ) to be known for each part of the flow, where f = Darcy-Weisbach friction factor; λ = dimensionless eddy viscosity; s = the channel side slope of the banks (1:s, vertical: horizontal) and Γ = lateral gradient of the advective term in Eq. (5).

2.3 Solution methodology

The essence of the SKM is that any prismatic cross-section may be discretized by a series of linear elements, thus producing panels with either a flat or a sloping bed. A set of linear simultaneous equations, based on Eqs (6) and (9), are then solved to obtain the two A_i coefficients required for each panel. Depending on the number of panels adopted, the equations may be solved either algebraically by hand or numerically, using standard procedures. A free software program may be downloaded from www.river-conveyance.net that will handle any reasonable number of panels per cross-section. Alternatively the algebraic equations may be solved using a standard matrix solver, as in Microsoft Excel.

3 EXAMPLES

3.1 Obtaining lateral distributions of velocity and boundary shear stress

Figure 2 illustrates the solution for the simplest possible case, that of flow in a rectangular channel using just one panel. The overall width of the channel is 20 m and the water flows 5.0 m deep at

a bed slope of 0.001, with $f = 0.02$, and constant values for λ and Γ taken to be 0.07 and 0.15 respectively.

In this particular case the roughness (f), eddy viscosity (λ) and secondary flow (Γ) values are chosen so that the flow is symmetric about the centerline, although this need not necessarily be the case if the channel has a slight bend or the roughness varies across the channel. Under these circumstances, more panels would be required to simulate the flow. Figure 3 shows the corresponding boundary shear stress distribution.

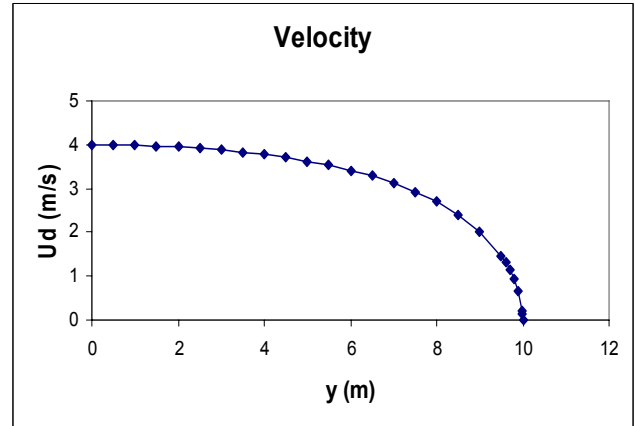


Figure 2. Flow in 20 m wide rectangular channel (single panel results, with $y = 0$ at centreline).

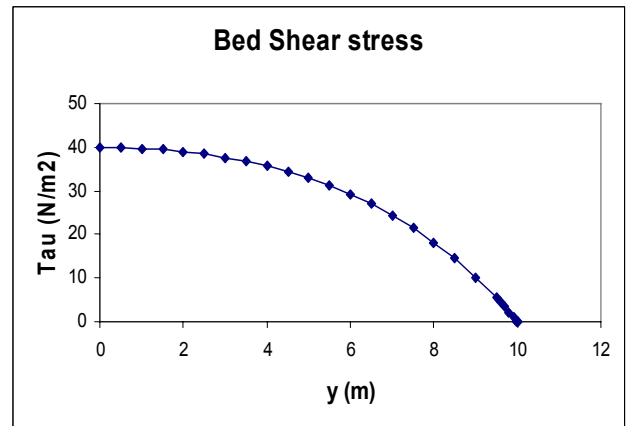


Figure 3. Boundary shear stress distribution in a 20 m wide rectangular channel (single panel results, with $y = 0$ at centreline).

The effect of varying the three calibration parameters, as well as the number of panels that constitute the cross section has been extensively studied, using high quality laboratory data, available at www.flowdata.bham.ac.uk. See Chlebek & Knight (2006), Knight (2008), Sharifi & Sterling (2009) and Tang & Knight (2009) for details.

Where there are discontinuities in the roughness distribution across the section, it is important to alter the velocity gradient boundary condition between panels, such that Eq. (13) is satisfied, as in these cases $\mu \neq 1.0$. Based on an approximation

of the exact Eq. (6), linearly varying the value of f within each panel, maintaining the mean value,

$$\left(\mu \frac{\partial U_d}{\partial y}\right)^{(i)} = \left(\mu \frac{\partial U_d}{\partial y}\right)^{(i+1)} \quad \text{with } \mu = \lambda \sqrt{f} \quad (13)$$

aids smoothing of the τ_b distributions. Otherwise, τ_b varies in a saw-tooth pattern in direct response to lateral changes in f between panels, since U_d is the same for both panels at the interface. This arises because of the relationship between τ_b and depth-averaged velocity, given by

$$\tau_b = \frac{f}{8} \rho U_d^2 \quad (14)$$

An example of lateral smoothing is illustrated in Figure 4, which shows simulations of boundary shear stress in a smooth trapezoidal channel using 6 panels. Where constant f values are used for individual panels, the saw-tooth pattern results, even though the velocity distribution is smooth (not shown, but readily demonstrated). Furthermore the effect of linearly varying f within a panel smoothes out the distribution of stress, as also observed in data. This is effectively using a 2-D model to mimic a 3-D effect, as shown by data obtained by Tominaga & Nezu (1991). See Omran (2005) and Knight *et al.* (2007) for further details.

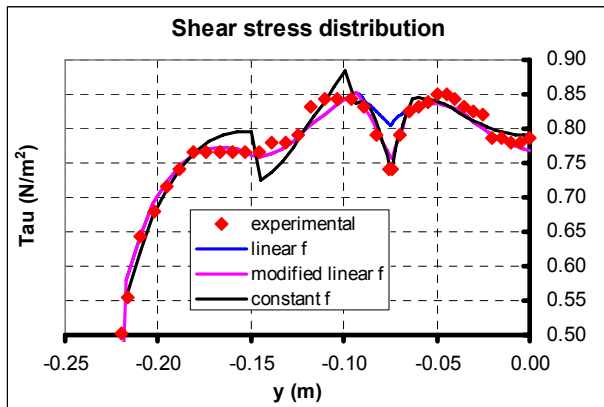


Figure 4. Boundary shear stress simulations of flow in a trapezoidal channel (6 panels, with variable λ , Exp. 16, Yuen), after Omran (2005).

3.2 Obtaining stage-discharge relationships

The velocity distribution may be integrated with respect to y to give the discharge, using Eq. (2). For a simple shape of cross section, this is straightforward, as illustrated in Figure 5.

An alternative procedure to point by point integration is to undertake the calculations analytically, using Eqs (6) and (9) applied to the appropriate element or panels directly, as shown by Liao & Knight (2007a&b). Figure 6 illustrates this for the same case as shown in Figure 5.

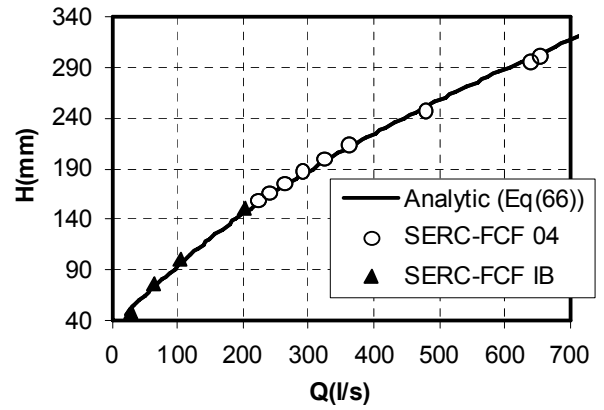


Figure 5. Simulated H v Q for a trapezoidal channel, compared with FCF data, using 2 panels ($S_o=1.027 \times 10^{-3}$, $f_1=0.016$, $f_2=0.018$, $\lambda_1=0.01$, $\lambda_2=0.12$, $\Gamma_1=0.0$, $\Gamma_2=0.0$).

One advantage of this approach is that it is then possible to investigate the influence of a single parameter on a whole range of stage-discharge relationships, as illustrated in Figure 6, in which the eddy viscosity in the flat bed region (panel 1) is varied.

A further example is shown in Figures 7 & 8, where a rectangular compound channel is modelled using just 2 panels, one for the main channel (MC) and the other for the floodplain (FP). Figure 8 shows there is good agreement between the analytically derived stage-discharge relationships and measured data for a range of B/b values (B/b = channel semi width/main channel semi width).

So far, the examples have been based on flow

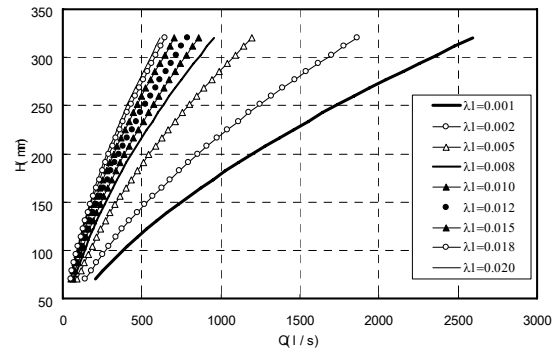


Figure 6. Influence of main channel eddy viscosity (λ_1) on the discharge in a trapezoidal channel ($S_o=1.027 \times 10^{-3}$, $f_1=0.016$, $f_2=0.018$, $\lambda_2=0.12$, $\Gamma_1=0.25$, $\Gamma_2=0.0$).

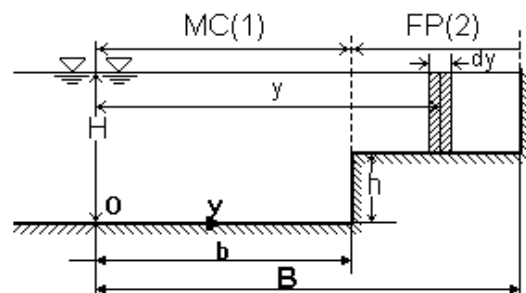


Figure 7. Two-stage rectangular compound channel

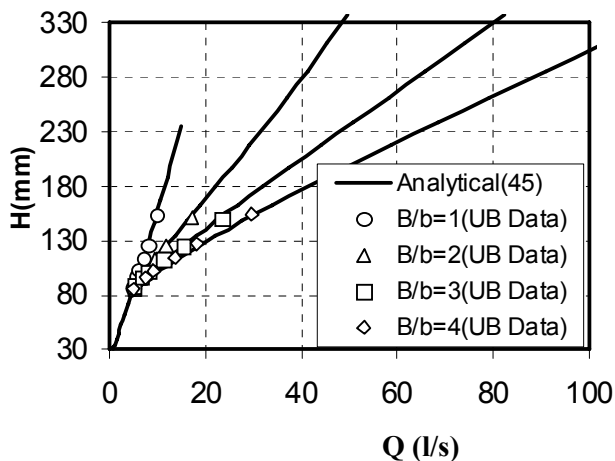


Figure 8. Fig. 13 Stage-discharge relationships for two-stage compound channels with variable B/b ($S_o = 9.66 \times 10^{-4}$, $f_1 = 0.022$, $f_2 = 0.025$, $\lambda_1 = 0.01$, $\lambda_2 = 0.10$, $\Gamma_1 = 0.40$, $\Gamma_2 = -0.15$).

in standard geometric shapes for the cross-section, either rectangular (simple and compound) or trapezoidal. In order to illustrate the use of the SKM for any shape of channel, Figures 9-11 show some simulations based on the CES software, applied to the Ngunguru River at Drugmores Rock, described by Hicks & Mason (1998).

Figure 9 shows the cross-section, Figure 10 a predicted stage-discharge relationship and Figure 11 the back-calculated Manning's n for the reach. In general the agreement with the data is good, with the exception of flows at very low depths when the roughness rises sharply due to the size of boulders. This issue is discussed further by Mc Gahey *et al.* (2009), where alternative roughness laws for mountain rivers are explored.

3.3 Dealing with vegetation issues

The final term in Eq. (5) is included so that emergent and submerged types of vegetation may both be dealt with in a simplified way within the SKM approach. Two cases studies related to the Rivers Avon and Hooke in the UK, where water crowfoot (*Ranunculus pseudofluitans*) predominates, are discussed by Mc Gahey in Knight *et al.* (2010). In general, each panel roughness and drag coefficient may be adjusted to suit the particular type of vegetation in the channel. A Roughness Advisor (RA) is included within the CES software that also acts as a guide to weed cutting/growth patterns for different morphotypes, and the consequent change in roughness with time. This issue is described further in Mc Gahey *et al.* (2009). The general concept of roughness in fluvial hydraulics and its formulation in 1-D, 2-D & 3-D numerical simulation models is discussed by Morvan *et al.* (2008).

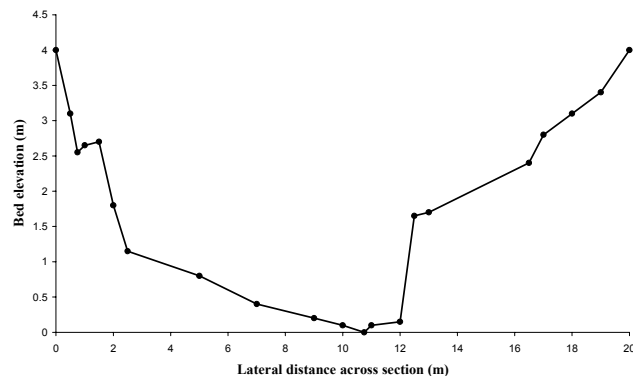


Figure 9. Surveyed cross-section geometry for the River Ngunguru at Drugmores Rock (Hicks & Mason, 1998).

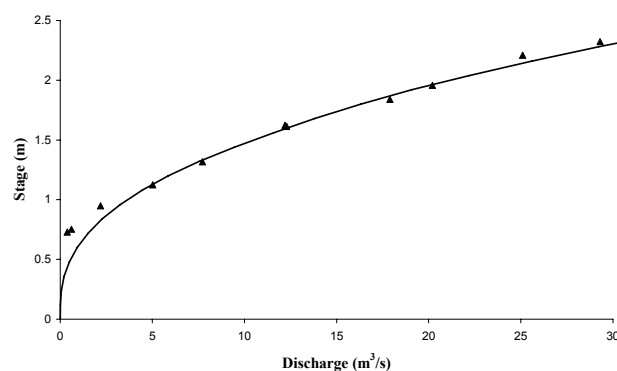


Figure 10. CES stage-discharge prediction and data for the River Ngunguru at Drugmores Rock (Hicks & Mason, 1998; Mc Gahey, 2006).

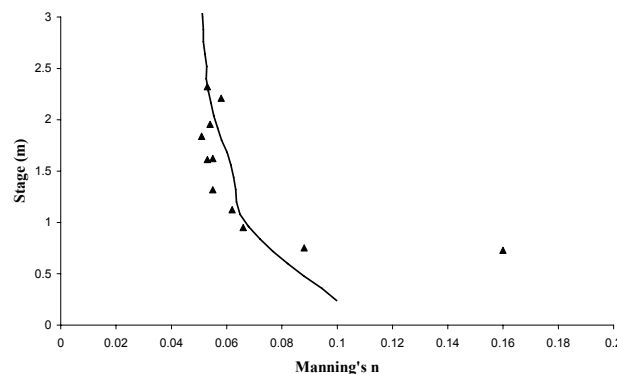


Figure 11. Back-calculated CES Manning n values and measured Manning n values from the River Ngunguru at Drugmores Rock (Hicks & Mason, 1998; Mc Gahey 2006).

Three cases of simulating emergent-type of vegetation in prismatic channels are shown in Figures 12-16. Figure 12 shows inbank flow in a rectangular channel, with the roughness concentrated on the left side. Figure 13 shows the simulation, based on Eq. (6), together with the experimental data. The results indicate that the analytical model simulates the data reasonably well, except for a small region in the middle of the shear zone. Secondary flow effects are seen to be small.

Figures 14-16 show two simulations of over-bank flow with different floodplain roughnesses.

Figure 14 shows the case of uniform roughness

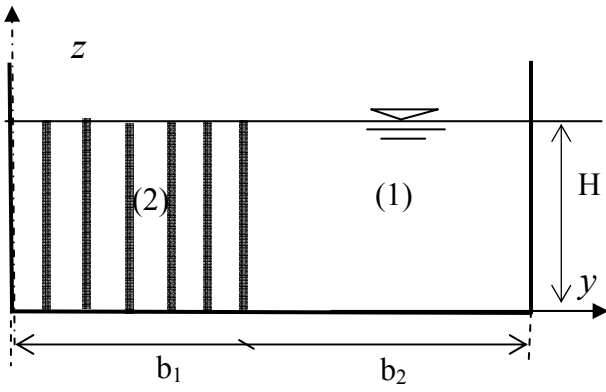


Figure 12. Experimental configuration, with region (2) roughened with simulated vegetation.

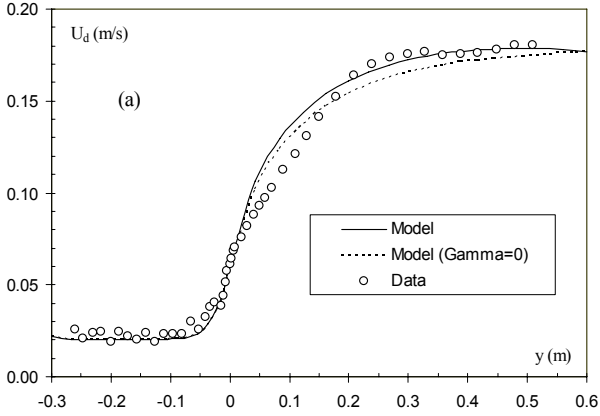


Figure 13. Comparison of predicted U_d distributions with experimental data (Case I), after Tang *et al.* (2010).

spread over the entire floodplain, thereby creating a strong transverse shear layer at the floodplain and main channel interface. The model agrees well with the data obtained by Pasche & Rouve (1985).

Figure 15 shows a commonly occurring non-uniform distribution of roughness on a floodplain, caused by a strip of vegetation at the edge of the floodplain, typically composed of trees or bushes. Figure 16 shows the simulated lateral distribution of U_d , indicating a localized dip at the floodplain edge. The agreement with the experimental data is again reasonable, as discussed by Tang & Knight (2009). Similar simulations have been undertaken by Rameshwaran & Shiono (2007) and Shiono *et al.* (2009), who show how the SKM can be developed further analytically to simulate floodplain and bankside roughness even better.

3.4 Dealing with sediment issues

The SKM lends itself to dealing with sediment issues on account of it being able to predict the distribution of boundary shear stress around the wetted perimeter of any shaped prismatic channel.

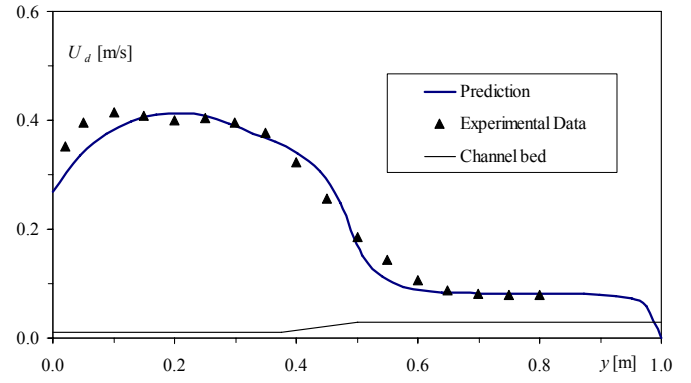


Figure 14. Comparison of modelled U_d distributions with experimental data for $\phi = 1.26\%$ (Pasche & Rouve, 1985).

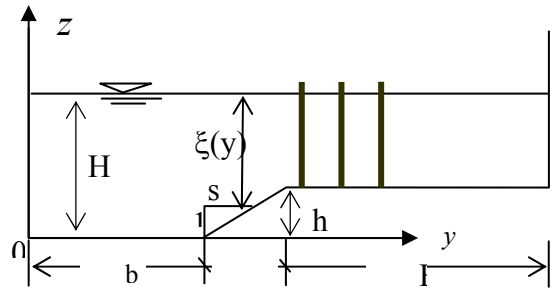


Figure 15. Cross section of partially vegetated compound channel (after Sun & Shiono, 2009).

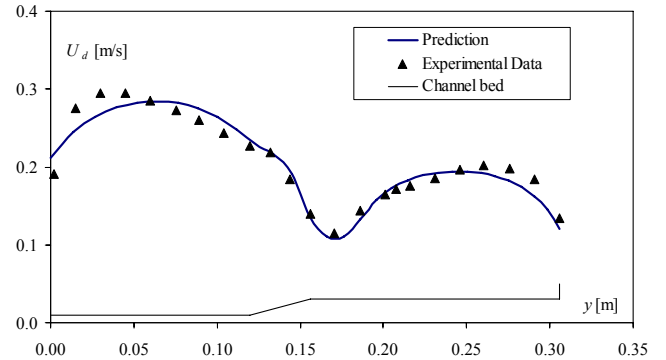


Figure 16. Comparison of predicted U_d with data (Run 2b).

Since the bed shear stress, τ_b , or shear velocity, $U_* (= \sqrt{\tau_b / \rho})$, occurs in many sediment transport equations, this allows for the transport rate to be determined panel by panel across the channel. These values may then be integrated laterally to give the total transport rate, as shown by Knight & Yu (1995). An example of it being used to predict sediment transport rates in a compound channel is given in Knight *et al.* (2010).

The method may also be used to investigate scour and erosion issues as τ_b is one of the key parameters in such phenomena. More usefully, the SKM allows one to determine directly the shear force on any element on the wetted perimeter of a prismatic channel in purely analytic terms.

The apparent shear force on any internal interface within the flow is given by

$$\tau_a = \frac{1}{H} \int_0^y [\rho g H S_o - \rho \frac{f}{8} U_d^2 \sqrt{1 + \frac{1}{s^2}}] dy \quad (15)$$

Thus by inserting Eqs (6) and (9) for the velocity in the appropriate panels into Eq. (15), it is possible to integrate the ensuing expression between any two limits to obtain analytic expressions for the shear force on any element, say SF_i on the i^{th} element, only involving the same constants A_i which will be known through having applied appropriate boundary conditions when solving for the velocity distribution. Furthermore, this provides a link between the zonal discharge in any zone of the channel and the boundary shear on the associated wetted perimeter element. See Knight & Tang (2008) for details and how this also links to the ‘area’ method for analyzing overbank flows.

3.5 Dealing with non-prismatic channels

Having established the methodology for dealing with prismatic channels, it is worth noting that the SKM has been applied to some types of overbank where there is a simplified transitional geometry from one cross-section to another. Typical examples tested are those where the floodplain narrows or widens, maintaining the same type of prismatic channel. See for example work by Bousmar & Zech (2004) and Rezaei & Knight (2009).

3.6 Dealing with unsteady flow issues

One application of SKM for unsteady flow is in the determination of the speed of a flood wave. Since the kinematic wave speed, c , is related to the inverse slope of the H v Q relationship by

$$c = \frac{1}{B} \frac{dQ}{dH} \quad (16)$$

where B = surface width, it is possible to use the predicted H v Q relationship obtained from Section 3.2 to obtain the c v Q relationship, based purely on cross-section geometry. See Knight (2006b) and Tang *et al.* (2001) for examples.

4 LIMITATIONS OF THE METHODOLOGY

4.1 Limitations of the methodology

The simple SKM outlined herein, as well as the CES tool referred to, should be recognized for what they are and what they are not. As always, the modeller has to use the right tool for the right job. The SKM has inherent limitations, such as being only appropriate for analyzing steady flows

in straight prismatic channels. It is deliberately a simple 1-D approach, but has some 3-D features that make it useful for solving certain types of fluvial problems.

4.2 Equifinality and calibration issues

A particular difficulty arises in multi panel models where several parameters are used to simulate certain physical flow mechanisms in each panel. There is usually a lack of sufficiently comprehensive data from which to select such parameters and hence to calibrate the model without ambiguity for practical use. In the SKM approach for example, the choice of the three calibration parameters (f , λ and T) is fraught with difficulties due to lack of measured data for flows in even generic shaped prismatic channels. Until such work is undertaken this will inevitably limit the application of this model. Recent work by Sharifi (2009) and Chlebek & Knight (2006) and Sharifi and Sterling (2009) indicate that it is possible to investigate numerically the physical parameters more thoroughly than hitherto.

4.3 Comparison with 3-D simulations

Although the SKM cannot match the details from a full 3-D flow simulation, the general features of key parameters, such as U_d and τ_b , can be reproduced moderately well for the restricted types of flow outlined above. Some comparisons have been made between the results generated by the SKM and large eddy simulations (LES), as illustrated by Omran *et al.* (2008).

5 CONCLUDING REMARKS

The use and limitations of a simple depth-averaged velocity lateral distribution model have been demonstrated through worked examples covering inbank and overbank flows. Refer to the website www.river-conveyance.net and Knight *et al.* (2010) for further information.

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