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Numerical Simulation of Fluid Mud Dynamics – The isopycnal Model MudSim

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Zusammenfassung

In den letzten Jahrzehnten hat der fortschreitende Ausbau von Seeschifffahrtsstraßen zu einer Zunahme der Verschlickung und Entstehung von Flüssigschlick in Bereichen der ästuarinen Schifffahrtstraßen, Häfen und Hafeneinfahrten geführt. Der Bedarf an fundierten Kenntnissen über die Flüssigschlickdynamik wächst, um neue Unterhaltungsstrategien und Renaturierungsmaßnahmen in Ästuaren zu entwickeln und bestehende zu optimieren. Numerische Modelle dienen als Werkzeug zur Beurteilung dieser Strategien und Maßnahmen. Ziel des MudSim-Projektes (03KIS67) ist daher, die numerische Simulation der Dynamik von Flüssigschlick zu ermöglichen. Flüssigschlick entsteht in Bereichen erhöhter Akkumulation von kohäsiven Sedimenten. Diese bilden Aggregate und führen zum Aufbau einer inneren Struktur, mit der sich das Fließverhalten der hochkonzentrierten Schlicksuspension von Newtonschen zu nicht-Newtonschen Verhalten verändert. Die derzeit etablierten hydrodynamischen Modelle lösen die Flachwassergleichungen unter der Annahme eines Newtonschen Fluids. Es wird daher ein herkömmliches numerisches Verfahren für die Reynolds-gemittelten Navier-Stokes Gleichungen für die Simulation von nicht-Newtonschen Verhalten erweitert. Die Entwicklungen für die Simulation der Flüssigschlickdynamik bauen auf einem bestehenden isopyknischen numerischen Modell auf. Eine vertikale Auflösung durch Isopyknen - Schichten gleicher Dichte - ist eingesetzt worden, um stark geschichtete Strömungen in Gewässern mit hochkonzentrierten Schlicksuspensionen zu realisieren. Insbesondere die Grenzschicht zwischen Flüssigschlick- und Wasserkörper ist durch einen ausgeprägten Dichtesprung gekennzeichnet. Das rheologische Verhalten von Flüssigschlick wird im numerischen Modell durch eine zeit- und ortsabhängige rheologische Viskosität umgesetzt. Anwendungen auf schematische und realistische Modellgebiete verdeutlichen die Möglichkeiten und Leistungsfähigkeit des weiterentwickelten isopyknischen numerischen Modells für die Simulation der Flüssigschlickdynamik. Weiterhin zeigen die Modellanwendungen, dass die Entwicklung von Flüssigschlick im tidebeeinflussten System mit diesem Modellverfahren simuliert werden kann.

Schlagwörter

Flüssigschlick, 3D numerisches Modell, kohäsive Sedimentsuspension, isopyknisches Modell, Rheologie

Summary

The progressive extension and development of coastal waterways has led to an increase in siltation and the formation of fluid mud in sections of estuarine shipping channels, ports and port approaches over the past *decades. Due to the fact that the need for a better understanding and profound knowledge of fluid mud dynamics has increased, it is necessary to develop new maintenance strategies and renaturation measures in estuaries as well as to optimize existing ones. Numerical simulations contribute to the evaluation of such strategies. For this reason, the aim of the MudSim project (03KIS67) is to permit the numerical simulation of fluid mud dynamics. Fluid mud forms by the buildup of a structure of aggregates in regions in which there is an increasing accumulation of cohesive sediments. Although the water content of the high-concentration suspension can be very high, the flow behavior changes from Newtonian to non-Newtonian. However, most of the current established hydrodynamic numerical models solve the shallow water equations based on a Newtonian assumption. A standard numerical model approach for the Reynolds-averaged Navier-Stokes equations is therefore extended to cover the simulation of non-Newtonian behavior. These developments are based on an existing numerical model in isopycnal coordinates. A vertical resolution using isopycnal layers - layers of constant density - is pursued, as the flow can be strongly stratified in systems with high-concentration suspensions. In addition, sharp density gradients characterize the transition zone between fluid mud and the water body. The isopycnal discretization permits a vertically-resolved simulation of the velocity and density distribution within the fluid mud body. A time- and space-dependent rheological viscosity is therefore introduced for simulating the rheological behavior of fluid mud. Applications to schematic and realistic model domains demonstrate the capabilities and performance of the extended isopycnal numerical model for simulating fluid mud dynamics such as the simulation of fluid mud under the influence of tidal currents.*

Keywords

fluid mud, three-dimensional numerical model, cohesive sediment suspension, rheology, isopycnal model

Contents

1 Introduction

Fluid mud (high-concentration mud suspension) is a suspension consisting of mineral particles, organic substances, water, and in some cases, small amounts of gas. The fraction of clay particles is accountable for the specific flow behavior of fluid mud because of the cohesive properties of the clay particles. Cohesive sediments in water are transported by turbulent currents, whereas in regions of quiescent flow or during periods of low currents, e.g. during slack water in tidal currents, the particles settle and accumulate on the bed. Fluid mud then forms where there is an adequate supply of suspended matter. Fluid mud describes a state in which mud is capable of flowing in spite of very high concentrations of suspended matter in the range of several 10 g/L (see Table 1). The flow behavior of fluid mud depends on the shear state and can be described as viscoelastic with a yield stress. By comparison, water is characterized as an ideal viscous Newtonian fluid. Fluid mud, being a non-Newtonian fluid, is therefore governed by a different rheology than clear water.

Naturally-occurring mud provides nutrients for aquatic organisms, as mud exhibits a relatively high content of organic substances. However, mud becomes an unwanted material when it accumulates, deposits and consolidates. In many estuarine waters and harbors in particular, the mud budget has been greatly affected by infrastructure projects over the past few decades. The increasing siltation of harbor basins, harbor access channels and parts of shipping channels leads to an increase in the level of maintenance requirements and, as a consequence, to higher costs. Another issue is the determination of the nautical depth, whereby the presence of fluid mud in waterways needs to be considered. Hence, an almost stationary fluid mud layer may be navigable in spite of a high concentration of solids if the vessel overcomes its yield stress (WURPTS 2005).

A profound understanding of the process of the formation, development and transport of fluid mud and the description of its rheological behavior is needed in order to enable construction work, maintenance work and activities aimed at reducing siltation to be evaluated, planned and optimized. Today, the required detailed investigations and prognoses of the behavior and reaction of water systems are supported by numerical modeling.

The numerical modeling of estuaries is carried out by means of three-dimensional models which take account of physical processes such as suspended sediment transport, salt transport, density-induced currents, and turbulence. These conventional models are based on the assumption of a Newtonian fluid. However, high-concentration mud suspensions exhibit a distinctly non-Newtonian behavior. Therefore, a module for the simulation and prediction of the dynamics of fluid mud is developed. An existing numerical method has been extended to include an approximation of the internal stresses in a non-Newtonian fluid under consideration of a parameterized approach for the description of the specific rheological behavior of fluid mud. In addition, major subprocesses of fluid mud transport are taken into account by parameterizations in the model.

There is usually a strong density gradient in the transition zone between a fluid mud layer and the body of water above it. This transition zone is known as a lutocline. The two fluid layers exhibit very different flow behaviors and interact by virtue of the shear forces acting in the transition zone. A common approach is therefore to model the fluid mud as a two-dimensional, depth-averaged layer. Processes such as the formation and resuspension of fluid mud lead to changes in the density gradient and to the development of a system with multiple layers. An isopycnal approach, in which the mud suspensions are resolved three-dimensionally by means of layers of constant density, has been chosen in this research work to improve the resolution of such mechanisms. An existing numerical method is extended to model the dynamics of fluid mud. The fundamental properties and flow behavior of fluid mud are described in Sections 2 and 4.2, respectively, while the main processes governing the dynamics of fluid mud are discussed in Section 3. The conceptual principles of the numerical model are described in Section 4, followed by a presentation of the isopycnal numerical method. The application of sectional models of the Ems and Weser estuaries presented in Section 5 illustrates how the method can be applied to more complex estuarine systems. The paper concludes with a discussion of the results and the identification of possible future developments and additional aspects requiring further research.

This paper presents the results of the MudSim-B-project (03KIS76). The elaboration of this paper is based on the thesis of WEHR (2012).

2 Properties of High-Concentration Mud Suspensions

Natural mud suspensions or fluid mud basically consist of water and mineral grains with a mean diameter ranging from 1 µm to 10 µm. These also contain small concentrations of organic components, which are subject to high seasonal fluctuations, and gas. These two minor components are not considered in this work.

The solid particles are mostly clay (particle size < 2 µm) and silt (particle size < 63 µm). Additionally, colloids with a particle diameter of about 0.1 µm form a subfraction of the clay fraction. The inorganic particles consist of different types of mineral (clay minerals, quartz, silicates), and their distribution is site-specific. Other comparable suspensions such as industrial water-debris mixtures, cement and bentonite often contain coarser grain sizes than colloidal clay suspensions. Estuarine mud suspensions have a high clay content which significantly influences their rheological behavior.

COUSSOT (1997) indicates three fundamental physical parameters which have an effect on the rheology of mud suspensions: the concentration described by the solid volume concentration, the grain-size distribution and the ion concentration (natural clay suspensions are cation-saturated and have a pH of about 7).

The solid volume concentration is defined by the relationship between the volumes of the two components, water (index w) and solids content (index s): $\phi_s = V_s / \left(V_s + V_w\right)$.

The relation between the solid volume concentration ϕ_s and the solid mass concentration c_s is $c_s = \phi_s \rho_s$, and in terms of density, the bulk density is defined by

$$\rho = \rho_w + \left(1 - \frac{\rho_w}{\rho_s}\right) c_s = \phi_s \rho_s + \left(1 - \phi_s\right) \rho_w \tag{1}$$

where ρ_s is the particle density and ρ_w , the water density.

In mud suspensions, a distinction can generally be made between the different interactions of the constituents (COUSSOT 1997):

- water molecule interactions
- colloidal interactions of particles <10 μm such as van der Waals attraction, double-layer interaction, Born repulsion
- friction or collision between particles >10 μm

The weight of clay particles is small enough to ensure that even Brownian motion keeps them in suspension. Moreover, clay particles have a negative charge on their surface so that they repel each other. These electrical repulsive forces are neutralized by ambient water ions or by organic polymers. Colliding clay particles stick to each other, forming aggregates or flocs. This is the cohesive property of clay particles. Cohesion is the most important mechanism governing the behavior of mud suspensions. The flocs that have formed can now settle under the action of gravity. They may be disrupted under shear impact and then re-aggregate with decreasing shear if the attractive forces are stronger than the repulsive forces. This process of the break-up and aggregation of flocs is known as flocculation. A more detailed description of these mechanisms is given by MCANALLY and MEHTA (2001) and DANKERS (2006) amongst others.

Fluid mud is formed by aggregates which hinder each other in settling as their concentration increases. Under quiescent conditions the aggregates form a granular structure, also known as a gel. A high density gradient develops between the highly-concentrated mobile or static mud at the bottom and the water body above. This gradient is known as the lutocline. An additional characteristic is that below the lutocline, the flow behavior is non-Newtonian and the fluid mud behaves in a laminar manner. ROSS and MEHTA (1989) indicate that lutoclines form if the concentration exceeds 10 kg/m^3 (ρ =1006.2 kg/m^3).

However, it is difficult to define a characteristic concentration or bulk density of fluid mud. The concentration is dependent on several constituents of the aggregates and can be site-specific. Different concentration ranges can be found in the literature. Some of these are shown in Table 1. A more detailed description of estuarine muds is given by WINTERWERP and VAN KESTEREN (2004), COUSSOT (1997) and ROSS (1988).

Table 1: Overview of mud suspension concentrations.

Soulsby (2000) in Whitehouse et al. [2000]		flocculated suspension, 0.01–3 kg/m^3	fluid mud, 3–100 kg/m^3	consolidating settled bed, 50–100 kg/m^3
Winterwerp [1999]	low-concentration mud suspension, several 0.01 kg/m^3 to a few 0.1 kg/m^3	high-concentration mud suspension, few 0.1 kg/m^3 to few 1 kg/m^3	fluid mud, several 10 kg/m^3 to 100 kg/m^3	
van Rijn [2005] revised from Bruens (2003)	dilute mud suspension, 0–10 kg/m^3		fluid mud, 10–300 kg/m^3	consolidated mud, >300 kg/m^3

2.1 Fluid Flow Behavior

A fluid may be characterized according to its behavior under the action of external pressure or shear stress. The first type of behavior distinguishes between compressibility and incompressibility depending on whether a fluid element reacts to the applied pressure or not.

For most intents and purposes, fluids may be considered to be incompressible. Gases, on the other hand, are treated as being compressible. This assumption is used to set up the continuity equation for a fluid element. The influence of shear on a continuous fluid element is more important. The shear stress can be expressed by different rheological constitutive laws. These describe the fluid behavior according to flow curves (shear stress τ_{ij} versus shear rate $\dot{\gamma}_{ij} = \partial u_i / \partial x_j$), as illustrated in Fig. 1. There are two elementary fluid behaviors, known as the Newtonian and the non-Newtonian fluid behavior respectively. A Newtonian fluid is defined by the linear dependence of the two parameters shear rate and shear stress, with viscosity as the constant of proportionality. Newtonian fluids, e.g. water, are homogeneous and isotropic. The viscosity of materials in simple shear (one-dimensional shear) is defined as $\mu = \tau/\dot{\gamma}$. The viscosity of non-Newtonian fluids is defined as a function of the shear rate $\mu(\dot{\gamma}) = \tau(\dot{\gamma})/\dot{\gamma}$ or shear stress $\mu(\tau) = \tau/\dot{\gamma}(\tau)$ for a specific state of strain (DIN 1342: 2003, CHHABRA and RICHARDSON 1999; CHHABRA and RICHARDSON 2008).

In the following, this viscosity is also referred to as rheological viscosity μ_r, in contrast to the turbulent viscosity μ_t encountered in hydrodynamics. Non-Newtonian fluids have a non-linear flow curve and/or can have a yield stress τ_y. The reaction of these complex fluids to shear impact may be time-dependent. Not only may they exhibit viscous behavior but, as in the case of solids, may additionally have elastic characteristics. These are referred to as viscoelastic fluids.

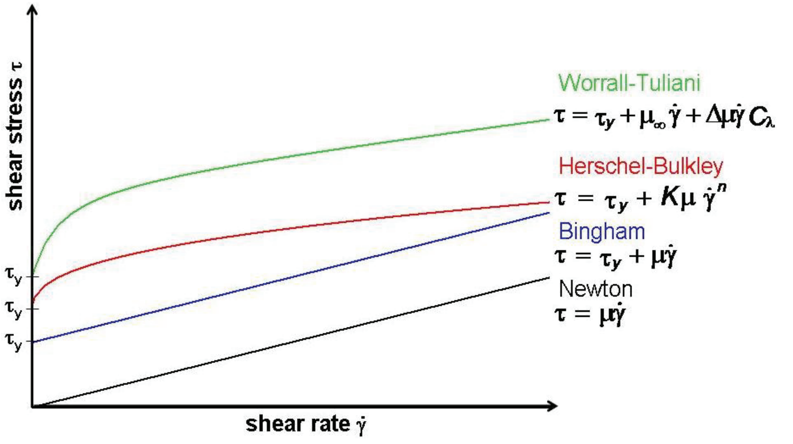

Figure 1: Rheological constitutive laws. These describe the behavior of a material under the influence of shear impact. The simplest constitutive law is that of a Newtonian fluid with a constant viscosity. Some materials have to overcome a specific stress, the yield stress τ_y, before they deform. The yield stress and other parameters of the constitutive laws can be determined by rheological measurements.

2.2 Rheological Behavior of Fluid Mud

Fluid mud contains a considerably large amount of clay. Therefore, the cohesive properties of clay dominate the rheological behavior of mud suspensions.

High-concentration mud suspensions and fluid mud may be characterized as shear-thinning, thixotropic, viscoelastic, yield stress fluids. However, these characteristics do not necessarily affect the behavior of fluid mud under all flow conditions and for every mud consistency. An understanding of the rheological behavior of fluid mud is essential for simulating its (laminar) flow and transport.

In mud suspensions in which clay particles predominate, shear-thinning is mainly induced by the break-up of flocs and/or the orientation of particles or aggregates. WORRALL and TULIANI (1964) derived a constitutive approach with a structural parameter to account for the degree of aggregation and the point at which break-up of the flocs occurs. TOORMAN (1997) and WURPTS (2005) analyzed the Worrall-Tuliani model and found that the flow curves of colloidal soils and cohesive suspended sediments match observations very well as long as they contain a large proportion of clay and only a small amount of organic substances. Further details of the constitutive law and parameterization of the model are given in Section 4.2.

High-concentration mud suspensions behave elastically at low deformations (low shear impact) below the yield stress. Rheological measurements require different methods to analyze their behavior. In the case of viscoplastic behavior the fluid deformation is measured under a permanent shear impact with increasing shear stress, whereas for elastic behavior, the deformation is measured under oscillating shear.

Further elaboration of rheological measurements and the specific flow behavior of fluid mud is given by MALCHEREK (2010) and MALCHEREK and CHA (2011). In this work, fluid mud is assumed to be a viscoplastic shear-thinning fluid (yield stress fluid).

3 Fluid Mud Dynamics

The most important fluid mud transport processes and fluid mud dynamics described in the following are based on a review of the available literature. Further details may be found in MCANALLY et al. (2007a), MCANALLY et al. (2007b), MEHTA et al. (1989), WHITEHOUSE et al. (2000), WAN and WANG (1994), WINTERWERP and VAN KESTEREN (2004) and VAN KESSEL (1997).

3.1 Formation of Fluid Mud

Fluid mud formation is related to the amount of cohesive material available in the water body. Cohesive material is transported into the system in different ways, for example by land erosion, elutriation and shore erosion. Fine material is transported downstream in rivers.

Fluid mud occurs in coastal regions in the maximum turbidity zone of estuaries, on shores, mudflats and in harbors. Fluid mud forms layers ranging from a few decimeters to several meters in thickness.

The formation of fluid mud occurs by a combination of the settling and flocculation of suspended cohesive material and the fluidization of mud deposits by waves. Additionally, the erosion of consolidated mud enriches the cohesive suspended load in the water system. The formation and transport processes are illustrated in Fig. 2. The transport processes are further described in Sections 3.2 and 3.3. Once fluid mud is formed, it is mainly transported by advection due to currents or waves, shear flow and gravity-driven currents. Vertical transport occurs due to the entrainment of fluid mud into the overlying water body. This results in resuspension of the fluid mud.

Furthermore, fluid mud settles during decelerating or slack currents with a reduced capacity to carry particles or flocs. This is often a temporary mechanism, as in tidal currents. If decelerated currents continue in the long term, the fluid mud consolidates. This situation is observed in harbor basins and on river banks.

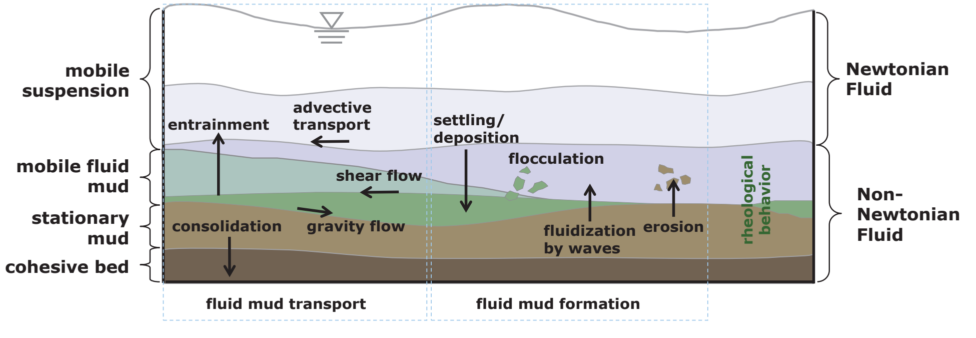

Figure 2: Significant physical processes governing fluid mud dynamics. In this scheme, the cohesive sediment concentration increases from surface to bottom. The left-hand side shows transport processes of mud suspensions. The right-hand side shows processes which contribute to the formation of fluid mud. The physical processes are dependent on rheological behavior.

3.1.1 Flocculation

Flocculation covers the processes of aggregation and the break-up of flocs. Aggregates or flocs consist mainly of cohesive material and smaller amounts of organic material, other sediment particles, nutrients and a large amount of water. Owing to their high water content, the density of flocs may be only slightly higher than the density of water, resulting in very low settling velocities.

The state of dispersion and aggregation of flocs depends on the balance between attractive and repulsive forces. Aggregates form due to cohesive forces, collision and polymeric bonding of the solid particles. Collision favors aggregation and is induced by turbulent flow and the increasing concentration of suspended matter. Brownian motion also leads to collision. Another mechanism for the build-up of aggregates results from particles with higher settling velocities overtaking those with lower settling velocities.

Although turbulent flow leads to aggregation, strong turbulent flow causes a break-up of the aggregates as the repulsive forces overcome the attractive forces.

Turbulence affects the flocculation process while the fluid mud formation process influences turbulence. The generated aggregates settle to the bottom under conditions of hindered settling and the vertical concentration profile increases downwards. This stratification attenuates the turbulence in regions of high concentrations, allowing fluid mud to form.

In tidal currents, the flocculation process and the resulting suspended sediment transport is strongly coupled to the intensity of turbulence, buoyancy destruction and the vertical suspended sediment concentration profile (WINTERWERP 2011).

3.2 Horizontal Transport Processes of Fluid Mud

3.2.1 Shear Flow

The advective flow of fluid mud can be caused by currents and waves. Currents above a fluid mud layer can force the fluid mud to flow owing to interfacial friction (shear flow), while stronger boundary-layer flow leads to entrainment of fluid mud into the water body (by exceeding the fluid resistance). If the oscillating currents of wind-driven waves impact the mobile mud layer, these induce a movement of the fluid mud layer parallel to the direction of wave propagation. This type of transport occurs e.g. in shallow waters in shelf regions.

3.2.2 Gravity Flow

Density-driven currents are referred to as density flow, gravity flow or turbidity flow. In general, density flows are currents caused by gravitational forces which have an effect on any density differences in a fluid. Gravity flow describes the down-slope movement of a suspension due to the impact of gravity. Mobile mud can be transported by gravity flow whereas turbidity currents describe the density currents of suspensions containing solid particles. Such suspensions are not necessarily fluid mud.

In this case, McAnally et al. (2007b) distinguishes between three kinds of gravity flow:

1. non-turbulent, laminar down-slope fluid mud flow
2. turbulent down-slope fluid mud flow in which turbulence is induced by the mud suspension itself
3. gravity-flow induced by the flow of the ambient suspension or waves.

The internal shear strength of the fluid mud must be exceeded in order for gravity flow to be initiated and is characterized by the yield stress.

If neither currents nor waves act on a fluid mud layer (Case 1 or 2), the bottom slope must be steep enough to enable the gravitational force to overcome the yield strength. The flow regime changes from laminar to turbulent flow with increasing slope and increasing internal shear. McAnally et al. (2007a) concluded that a slope of less than one degree leads to laminar gravity currents in fluid mud.

If the down-slope turbidity current of a fluid mud layer exceeds the critical shear stress for erosion, the mud layer is enriched with additional sediment load from the bottom, which in turn accelerates the velocity of the layer. This is referred to as auto-suspending turbidity flow (Scully et al. 2002, Wright et al. 2001). However, dissipative turbidity flow decelerates fluid mud movement due to mixing of the denser suspension with the less-concentrated suspension at the interface. This interfacial mixing leads to a decrease in the concentration of the fluid mud layer. In nature, gravity flow is characterized by the mechanisms of auto-suspending and dissipative turbidity concurrently.

The presence of high ambient turbulent currents (Case 3) leads to an increase in the internal resistance of the fluid mud layer and a deceleration of the gravity current. At the same time, however, turbulent shear forces ensure that the fluid mud remains mobile.

3.3 Vertical Transport Processes of Fluid Mud

The vertical processes governing mud suspensions and stationary mud depend on sediment concentration and impact on the mud. These describe the transition from dilute to highly-concentrated suspensions and then to mud beds. In terms of mobility, these processes transform mud suspensions from the mobile to the stationary condition and vice versa. These mechanisms are illustrated in Fig. 3.

3.3.1 Settling

Settling is a process influenced by the gravititational force acting on the particles or aggregates, the viscous drag of the ambient fluid and the interaction between the aggregates (Mehta et al. (1989)). Therefore, the settling velocity of particles and aggregates depends on their density, size, shape and the properties of the ambient fluid.

The settling velocity and formation of fluid mud itself depend on the size, density, shape and strength of the flocs. Formulations for the settling velocity which consider these aspects are often based on the approximation that flocs are self-similar entities. The settling velocity of aggregates ranges from about 10^{-5} m/s to 10^{-2} m/s (McAnally et al. 2007a).

In general, the settling velocity increases with increasing concentration. At very high suspended matter concentrations the settling velocity decreases due to inhibiting aggregates (hindered settling). This begins at concentrations of around 5-10 kg/m^3 (MEHTA et al. 1989), which also corresponds to the concentration range in which fluid mud is generated. With increasing contact between the aggregates, the settling process is increasingly replaced by consolidation. The concentration at this point is defined as the gelling concentration.

3.3.2 Entrainment

Entrainment describes the transition from a highly-concentrated suspension to a suspension with a lower concentration as a result of turbulent mixing (mobilization of fluid mud).

There are two different entrainment cases (MCANALLY et al. 2007a; KRANENBURG 1994):

1. Considering a turbulent mixed water layer above a nearly quiescent fluid mud layer, turbulent eddies will cause the fluid mud to mix with the water layer.
2. The water layer is assumed to be static, while the fluid mud layer is turbulent. The fluid mud layer moves between a lower rigid bed layer and a water layer. If the mud suspension permits sufficiently high Reynolds numbers, the shear stress along these boundaries will cause turbulence and water will be entrained into the fluid mud layer.

The first case is the most frequently observed case, whereas the second case is a phenomenon commonly observed in estuaries. During slack water, the water layer may move far slower than the inertially-flowing mud suspension layer or even be static. In both entrainment cases, the thickness of the water layer decreases while the thickness of the mud layer increases with simultaneously decreasing concentration. The turbulent mixing involves turbulence damping due to stratification effects. The stratification of a fluid and the turbulence structure of the flow can be characterized by the gradient Richardson number Ri

$$\mathrm{Ri} = -\frac{g\,\partial\rho/\partial z}{\rho\left(\partial u/\partial z\right)^2} \tag{2}$$

where z is the height above the bottom, ρ is the suspension density, and u is the current velocity at depth z. As the density gradient increases, the turbulence is damped and the flow becomes laminar. At a Ri value between 0.1 and 0.3, the turbulence is totally damped by stratification (WHITEHOUSE et al. 2000).

Additionally, the Ri number may be an indicator of interfacial mixing (initiation of entrainment). WHITEHOUSE et al. (2000) report, for example, that entrainment occurs at Ri numbers lower than about 10. In this case, the bulk Richardson number, which is a discretized form of the gradient Ri number, is applied

$$\mathrm{Ri} = -\frac{\left(\rho_w - \rho_{mud}\right)}{\rho_{mud}}\frac{gH_{mud}}{\left(u_w - u_{mud}\right)^2} \tag{3}$$

This equation considers a two-layer system comprising a water layer (index w) and a fluid mud layer (index mud). The degree of stratification may be classified as being:

$Ri < 1/4$ instable stratification
$Ri > 1/4$ stable stratification

Entrainment approaches for the determination of entrainment rates can be realized according to WHITEHOUSE et al. (2000) and KRANENBURG and WINTERWERP (1997).

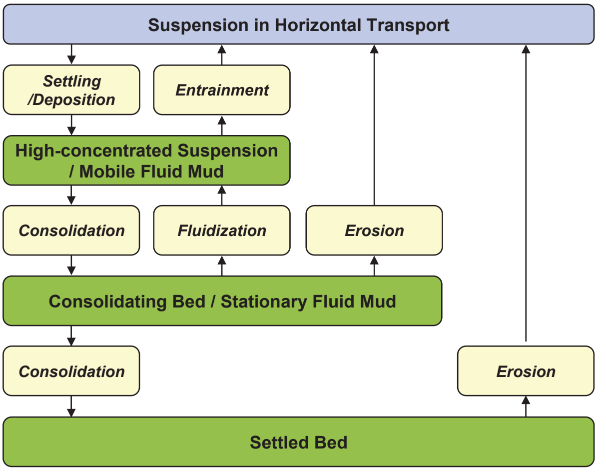

Figure 3: Vertical transport processes for cohesive sediments (modified according to MEHTA et al. 1989). The vertical transport processes (yellow panels) depend on rheological conditions and sediment concentration. These transform a dilute suspension of cohesive material in horizontal transport (blue panel) to a high-concentration suspension/mobile fluid mud followed by a transition to stationary fluid mud and a consolidated mud bed (green panel) and vice versa.

3.3.3 Fluidization of Mud Deposits by Waves

Fluidization is the process of transition from a consolidated cohesive bed to fluid mud under the impact of waves. As shown in Fig. 4, it is first necessary to study the distribution of the resultant stresses in suspensions and mud beds. Fluid mud deposits or consolidated mud beds are regarded as solids in which the aggregates are in contact with each other, thereby forming a soil matrix with fluid-filled pores. The total load is sustained by the soil matrix and the pore water, as described by the total normal stress σ. The effective normal stress σ' represents the load sustained by the granular structure, whereby u_p is the pore water pressure. These are related according to $\sigma' = \sigma - u_p$. The pore pressure is the sum of the hydrostatic pressure p_h and the excess pore pressure $\Delta u_p : u_p = \Delta u_p + p_h$. In mobile suspensions or fluid mud there is no permanent contact between the aggregates, and the entire load is sustained by the fluid phase. The effective normal stress is then zero, and $\sigma = p_h$. Both the aggregates and the pore water sustain the load as the contact between the aggregates increases and a granular structure is formed. The effective normal stress then increases.

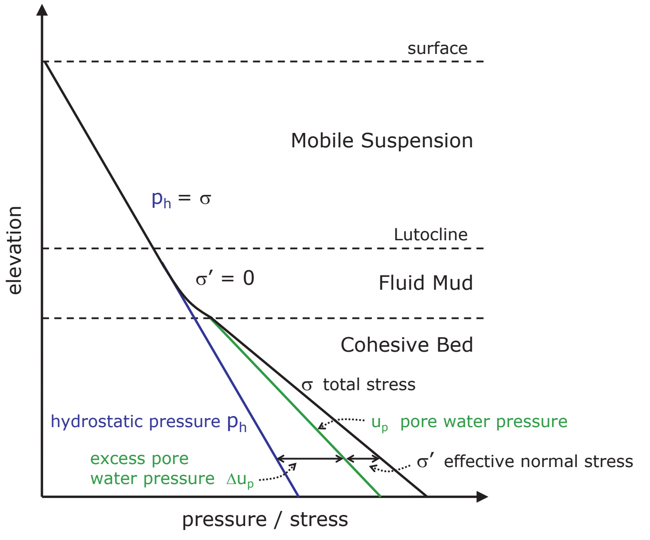

Figure 4: Schematic representation of the vertical stress profile from a mobile suspension to fluid mud and then to a cohesive bed (modified from ROSS and MEHTA (1990).

Fluidization occurs in freshly-consolidated mud beds whereas material is increasingly eroded as consolidation progresses. Additionally, a low permeability of the mud bed favors fluidization.

Waves cause oscillating pressure gradients in the pore water, progressively weakening the grain structure. As a result, the pore water starts to flow. If the upward velocity is sufficiently high, the aggregate bonds will break up. The excess pore water pressure increases as the effective stress decreases. If the effective stress approaches zero, the mud bed is transformed from the solid state to a fluid with a specific viscosity. This represents the fluidization process. The reaction of the mud bed is both elastic and viscous. The elasticity restores the initial condition of the mud bed after impact whereas the viscous behavior responds in a dissipative manner. This attenuates wave action and the amplitude of the waves propagating over the fluid mud. A description of viscoelastic behavior must additionally be included when modeling this process. Once the mud bed is fluidized, it can easily be entrained or transported due to shear and gravitational forces.

Research aimed at a more precise understanding, description and modeling of wave attenuation over a fluid mud bed including fluidization due to wave action is being carried out by several research groups e.g. MEHTA (1996), JAIN and MEHTA (2009), FODA et al. (1993) and SOLTANPOUR and HAGHSHENAS (2009).

3.3.4 Consolidation

As the suspended matter concentration increases, the aggregates hinder each other in settling. This mechanism leads to the generation of fluid mud. The gelling concentration then reached marks the point at which the aggregates come into contact with each other. The aggregates now form a granular structure with water-filled pores. Consolidation of this soil structure begins if the fluid mud becomes stationary. The weight of the overlying water column is sustained by the granular matrix and the pore water. The hydrostatic load causes the pore water to escape and the soil matrix to densify. This results in a reduction of the volume of the cohesive bed, thereby leading to primary consolidation. Secondary consolidation then begins with full dewatering of the pores. Sorting of the aggregates then leads to a further reduction of the interstitial space. Moreover, the elevation of the lutocline decreases during the consolidation process. Consolidation of cohesive material is a relatively slow process compared with the other transport processes that have been mentioned and takes place on a time scale ranging from hours to years. By contrast, consolidation in sandy beds occurs immediately; the pore water escapes and the grains are rearranged. Owing to the slowness of this process, fresh deposits are easily entrained or eroded again. Accordingly, the degree of consolidation provides information on the erodibility of the cohesive bed (LICK and MCNEIL 2001). The evolution of strength in relation to the effective stress of the cohesive bed has been studied by MERCKELBACH (2000). The yield stress of the consolidated cohesive bed serves as an indicator of the strength or resistance against erosion.

In numerical modeling, consolidation rates may be treated as a settling velocity. A combined formulation for consolidation and hindered settling by way of the density is described by TOORMAN and BERLAMONT (1993), for example. Parameterizations are obtained from settling column experiments.

3.4 Fluid Mud Dynamics under Tidal Flow

The formation and transport of fluid mud in estuaries and coastal regions is forced by tidal conditions. Estuarine fluid mud most commonly develops in the maximum turbidity zone. Depending on the tidal phase, the mud settles during slack water and is mobilized or entrained by highly turbulent ebb and flood currents. Fluid mud is only present during certain hydrological events or during low current tidal phases, as determined by the hydrological situation and the availability of mud in the estuary. This is the case in the Weser estuary, for example, or in wide areas during all tidal phases, as in the Ems estuary. SCHROTTKE et al. (2006) observed that in the Weser, fluid mud may occur in the troughs of dunes in the turbidity zone. In the Ems estuary, fluid mud layers of several meters in thickness appear in the maximum turbidity zone, especially during the ebb tide on account of tidal asymmetry. For example, this was observed during a field survey in July 2009 (see Fig. 5). Fig. 6 shows a lutocline detected during a flood tide in which internal waves are observed.

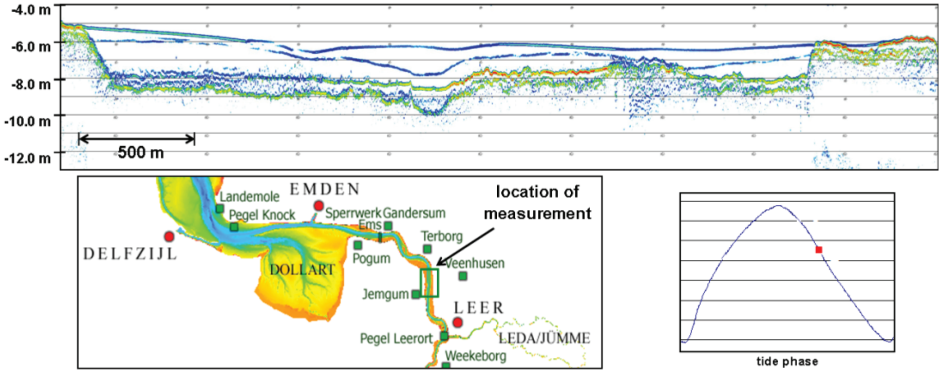

Figure 5: Multi-layer system of fluid mud detected by means of sediment echo sounding measurements (parametric sub-bottom profiler for shallow water) during an ebb tide. The longitudinal section is located between Terborg and Leer in the river Ems. The blue lines indicate strong density gradients and the horizon in red to yellow indicates the sediment bed. The field survey was carried out in July 2009 by the Federal Waterways Engineering and Research Institute (BAW).

Mud suspensions settle and are deposited on the bed during slack water. As flood or ebb currents increase, the mud deposits are then eroded and the fluid mud or mobile mud is entrained into the water body (see Fig. 7). Depending on the intensity of the currents, the fluid mud may become totally mixed with the water body. During phases of moderately turbulent currents, fluid mud is generated owing to flocculation processes and hindered settling of the flocs. This results in the formation of a sharp lutocline. The mud concentration is so high below the lutocline that the flow behavior differs from that in the water column. Vertical fluid mud transport processes during a tide period are dominated by the settling and formation of fluid mud as well as by entrainment, particularly in deep channels.

A typical phenomenon observed in estuaries is the decoupled flow of the water body and fluid mud layer. The fluid mud has a more inertial flow than water. The currents in the water body are almost zero at the beginning of slack water whereas the fluid mud is still in motion. However, once the mud movement stagnates, the shear forces of the main flow have to overcome the resistance of the mud to force the fluid mud to flow.

The estuarine system may be largely influenced hydrodynamically by fluid mud when the fluid mud layers attain a certain thickness and the fluid mud covers wide areas. The strong density stratification results in turbulence damping in the region of the water-fluid mud interface and finally leads to reduced bottom friction.

Fluid mud deposits on river banks or mudflats can move down slope due to gravitational forcing. Gravity flow may transport fluid mud to the deepest parts of an estuary such as the shipping channels as well as over great distances in the longitudinal direction of estuaries. The near-bed transport of cohesive sediments in the form of fluid mud layers is associated with significantly higher transport rates than the transport in suspension in the water body above. Knowledge of the gravity flow of fluid mud can be applied most effectively for maintenance purposes in harbor basins (WURPTS 2005).

In consolidated mud, the cohesive forces and the density of the granular structure increase. This means that high impacts, for example wind-induced or ship-induced waves, are required for re-mobilization. Ship-induced waves are more likely to cause erosion on the river banks. Impacts by waves require wide areas such as tidal flats or shorelines where the waves have enough fetch to develop. The oscillatory currents acting on the mud deposits cause them to become fluidized and softened owing to their viscoelastic behavior.

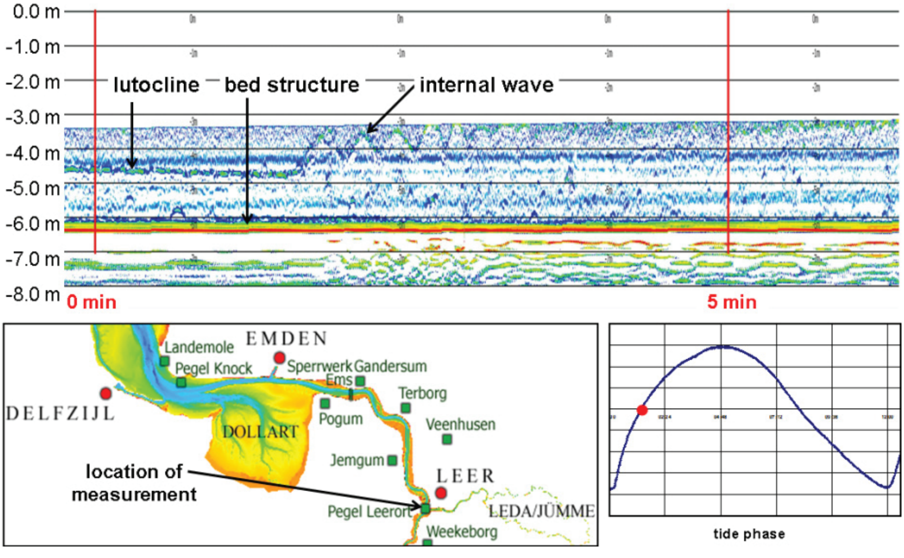

Figure 6: Fluid mud detected by means of sediment echo-sounding measurements (parametric sub-bottom profiler for shallow water) in the Ems estuary during a flood tide. The time series at a position near Leerort is shown where the measured data begins at approximately -3.0 m below the water surface. The lutocline is initially located at a water depth of about -4.5 m prior to the generation of an internal wave. The field survey was carried out in June 2011 by the Federal Waterways Engineering and Research Institute (BAW).

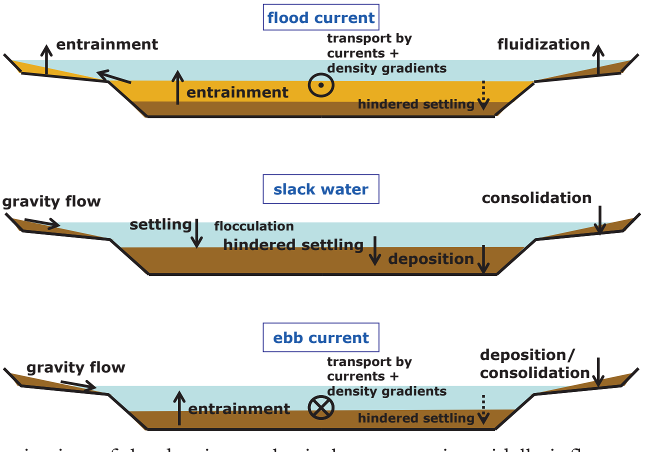

Figure 7: Schematization of the dominant physical processes in a tidally-influenced cross-section.

4 Basic Concept and Properties of the Model

Fluid mud and dilute suspensions differ fundamentally in their specific rheological behavior (see Section 2.1 and in detail in MALCHEREK (2010)). Hydrodynamic numerical simulations consider the flow behavior to depend on the stress terms of the momentum equations.

The momentum conservation of every viscoelastic material can be described by Cauchy's equation of motion given by

$$\frac{du_i}{dt} = \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j} + f_i \tag{4}$$

The rheological behavior is characterized by the first term on the right-hand side containing the stress tensor σ_{ij}. This term is equal to $-\partial p_i / \partial x_i + \partial \tau_{ji} / \partial x_j$ for incompressible fluids. For Newtonian fluids, the internal stresses are defined by

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

Therefore, the Navier-Stokes equations used for hydrodynamic simulations are a special case of the Cauchy equations. The same applies to the Reynolds-averaged Navier-Stokes equations, which are obtained by choosing the shear stress tensor as

$$\tau_{ij} = \left(\mu_t + \mu_{mol}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

where μ_t and μ_{mol} are the turbulent and molecular dynamic viscosity, respectively. A possible realization of the general rheological fluid behavior in a numerical model is by introducing the rheological viscosity μ_r, which can be determined according to different constitutive laws (see Section 2.1). Thus, the form of the shear stress tensor of the Navier-Stokes or the Reynolds-averaged Navier-Stokes equations does not change:

$$\tau_{ij} = \mu_r\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{7}$$

It is therefore possible to also implement conventional 3-D codes applied in river or coastal engineering for the simulation of fluid mud dynamics. Based on a dimensional analysis, the important components of the internal stress tensor are identified in WEHR (2012). This analysis demonstrates a suitable approximation of the non-Newtonian stresses for fluid mud. This is adapted at a later stage to the numerical model presented in Section 4.3 and described in detail in WEHR (2012).

Consequently, a rheological approach has to be identified to describe the flow behavior in a mud-water system both qualitatively and quantitatively. According to MEHTA (1991), BERLAMONT et al. (1993), and COUSSOT (1997), the rheological behavior in terms of the rheological viscosity μ_r might be dependent on:

- suspended matter concentration starting from clear water up to the sediment bed
- flow shear rates ranging from zero to values for highly turbulent motion
- size distribution of the suspended matter

- temperature and salinity of the water body
- biochemical behavior of the suspended material (flocculation, organic polymer formation)

In this work, it is assumed that the rheological approach is sufficiently described by two indicators: the bulk density, which is proportional to the suspended matter concentration, and the flow shear rate, whose formulation indicates a Newtonian or a non-Newtonian fluid. The applied rheological model and corresponding parameterizations are described in MALCHEREK (2010) and in KNOCH and MALCHEREK (2011).

The suspended matter concentration is not only an important parameter for determining the rheology, but is also used for the numerical discretization scheme. A high-concentration benthic layer often has a sharp density gradient at the transition to a layer of lower concentration, known as the lutocline. Therefore, the numerical model should be able to reproduce a highly-stratified flow. Conventional three-dimensional hydrodynamic models require a very fine vertical resolution in order to reproduce sharp density gradients and their movement. This can result in high computational effort because the entire domain is modeled using the same vertical grid spacing. Although other methods of domain decomposition or dynamic grid refinement exist, they are not necessarily more efficient.

A fluid mud body and the overlying water body behave very differently and interact at their interface. Some numerical approaches thus simulate the fluid mud as a single layer coupled with a hydrodynamic model, as for example in WINTERWERP et al. (2002) and CRAPPER and ALI (1997). However, fluid mud is not stable all of the time, especially under the action of tidal currents. The fluid mud dynamics mostly results in a change of the solid concentration, which can hardly be reproduced by a single- layer model with one specific density and rheological state.

In the research project MudSim an isopycnal numerical approach is pursued in which discretization follows the physical parameter bulk density, which directly relates to the suspended matter concentration. A change in stratification is thereby accompanied by a change in the discretization. The vertical discretization associated with isopycnals or layers of constant density is described in the next section (Section 4.1).

A 3-D hydrodynamic isopycnal model of this type has been adapted for the simulation of fluid mud dynamics in this research work. A detailed description of the numerical model and extensions of the model is given in WEHR (2012). The advection-diffusion equation for suspended-load transport is not solved here; the classic hydrodynamic momentum equations in an isopycnal discretization scheme are solved instead. Every density layer represents a homogeneous suspension with a specific rheological behavior. The numerical model covers the entire water column from the free surface to the stationary bed. Mud suspension transport is realized according to changing thicknesses of the density layers. The rheological approach is applied to the entire water column. Therefore, the resulting viscosity in the numerical model is given by the sum of the rheological and turbulent viscosities. An advanced turbulence model with damping due to stratification effects is not implemented in the numerical model, as turbulence modeling is not an objective of this work. Thus, the turbulent viscosity is kept simple here and set to a constant value for each density layer.

In addition to a fundamental determination of the flow behavior, the fluid mud dynamics are described by transport processes (Fig. 2). Gravity flow is solved according to the pressure term of the momentum equation, which considers density differences. Shear flow is caused by vertical, interfacial shear stresses at the isopycnal interface (interfacial momentum transfer). The vertical transport processes induce mass fluxes at the isopycnal interfaces, thus changing the state of stratification. An approach for the vertical mass transfer in an isopycnal system is derived in WEHR (2012). This approach enables density layers to vary in thickness by applying settling fluxes or mixing fluxes. A settling velocity approach with hindered settling is implemented for the formation of fluid mud according to WINTERWERP and VAN KESTEREN (2004). Consolidation is not considered owing to the fact that the applications presented later only cover a few days, which is far below the time scale of mud consolidation. Entrainment is introduced for modeling the mobilization and mixing of fluid mud due to shear impact. Fluidization due to wave impact is not considered here, as coupling with a wave model is not the aim of this work.

Most of these processes are transformation or exchange processes from dilute suspensions to high-concentration layers and from mobile fluid mud layers to stationary layers or vice versa, which can be realized by means of interfacial mass fluxes. The basis for including these transport processes is established by developing and implementing a mathematical approach for diapycnal mass transfer (WEHR 2012). Additional mechanisms can be introduced into the isopycnal numerical model in the same way as in the settling and entrainment approach.

4.1 Vertical Resolution by Isopycnals

The discretization scheme must be capable of reproducing sharp density gradients as well as the formation of three-dimensional concentration profiles. A z-layer-based model requires a very high resolution near the river bed in order to represent sharp density gradients. The bottom depth can vary considerably with an increasing model domain, which leads to a vertical resolution in the range of centimeters or decimeters over large areas. This increases the computational effort significantly. By comparison, σ-layers have the advantage of following the topology of the model domain and thus guarantee a high near-bed resolution. However, similar to z-layered discretization, the high resolution covers the entire model domain and hence also increases the computational effort. A discretization using ρ-layers permits an adaptation of the resolution according to physical phenomena rather than to bathymetric conditions. This kind of discretization is defined by layers of constant density - the isopycnal layers - whose thickness changes according to physical processes. The interfaces of these layers always define a density gradient.

Because the density is related to the concentration of a suspension, changes in the sediment transport regime automatically have an impact on the discretization. An adaptive discretization concept of this kind is applied in this work.

Sharp density gradients can be resolved by a few layers based only on the predefined density differences between the layers. Thus, the smaller the density differences, the thinner are the layers for a specific density gradient (see Fig. 8). The maximum number of isopycnal layers in the model domain and the density classes of the layers are predefined. Isopycnal approaches are often used for oceanographic problems where density-driven currents are dominant.

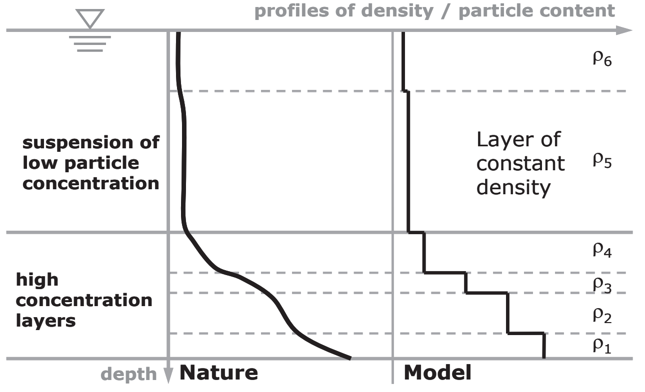

Figure 8: Schematization of the isopycnal approach. In an isopycnal model the vertical density profile is described by layers of constant density. In this case, each layer represents a suspension with a specific sediment concentration. Their thickness varies according to physical processes.

According to the isopycnal principle, the constraint $\partial\rho_m / \partial t = 0$ has to be satisfied, where ρ_m is the density of the m-th isopycnal layer. It is also necessary to guarantee stable stratification. The bulk density is given by

$$\rho = \rho_w + \left(1 - \frac{\rho_w}{\rho_s}\right) c_s \tag{8}$$

whereby the dry density of the sediments is denoted by ρ_s, the water density by ρ_w and the volumetric sediment concentration by c_s.

The isopycnal layers interact due to momentum transfer, mass transfer and interfacial shear stresses. Advective and gravitational transport as well as mixing and settling of cohesive sediment suspensions are realized by changes in the isopycnal layer thicknesses, as each layer represents a suspension with a specific sediment concentration. The vertical transport rates are determined by parameterizations of transport subprocesses, leading to variations in the layer thicknesses.

4.2 Rheological Approach for Mud Suspensions

The velocity distribution of highly-concentrated mud suspensions is inevitably subject to the modeling of rheological behavior. A brief introduction to the rheology of mud suspensions is given in Sections 2.1 and 2.2. The approach adopted for realizing the parameterized rheological model presented in this section is based on the investigations of MALCHEREK (2010) and MALCHEREK and CHA (2011). The rheological model implemented in the numerical model is a parameterized form of the Worrall-Tuliani model outlined by WORRALL and TULIANI (1964)

$$\tau = \tau_y + \mu_\infty \dot{\gamma} + \Delta\mu\dot{\gamma}c_\lambda \tag{9}$$

which is discussed in detail by TOORMAN (1994) and TOORMAN and HUYSENTRUYT (1997) for cohesive suspensions. The viscosity μ_∞ is the asymptotic value for $\dot{\gamma} \to \infty$ at the structural state of full break-down of the structure and $\Delta\mu$ is the viscosity at a specific degree of structure. The first two terms of this model take the form of the Bingham model with a yield stress τ_y and a linear dependence between shear rate and viscosity (viscoplastic behavior). The third term accounts for time-dependent changes in the structure according to the structural parameter c_λ. Depending on the shear impact, aggregates can break-up and recover in the mud suspension. Such mechanisms occur gradually and not immediately, however (thixotropic behavior). The structural parameter ranges between one and zero. The rate of change under shear impact is given by

$$\frac{dc_\lambda}{dt} = c_{aggr}\left(1 - c_\lambda\right) - c_{break}\,\dot{\gamma}c_\lambda \tag{10}$$

The parameters c_{break} and c_{aggr} denote the empirical constants for the break-up and (re-)aggregation of the flocs, respectively. The first term is not dependent on the shear impact whereas the break-down of aggregates (second term) increases with increasing shear rate. One solution of the differential equation may be obtained from the equilibrium state in which break-up and recovery of aggregates are in equilibrium (no change with time $dc_\lambda / dt = 0$). A formulation for c_λ then results in

$$c_\lambda = \frac{c_{aggr}}{c_{break}\,\dot{\gamma} + c_{aggr}} \tag{11}$$

This leads to the first order shear stress formulation for the equilibrium structural state

$$\tau = \tau_y + \mu_\infty\dot{\gamma} + \Delta\mu\dot{\gamma}\frac{1}{\left(c_{break}/c_{aggr}\right)\dot{\gamma} + 1} \tag{12}$$

where the yield stress is independent of the structural parameter (TOORMAN 1997). In the case of rheological measurements, this implies that the shear stress has to be increased slowly enough and continuously to ensure that the aggregate bonds are in equilibrium with the applied shear rate. Consequently, rheological measurements are required to determine the four parameters τ_y, μ_∞, $\Delta\mu$ and the ratio c_{break} / c_{aggr} in relation to the solid volume concentration. MALCHEREK and CHA (2011) analyzed the rheological behavior and developed parameterizations for the above-mentioned parameters of the Worrall-Tuliani model.

The parameterized formulation according to Worrall-Tuliani adopted in the numerical model is as follows

$$\tau = 7021\text{Pa}\phi_s^{4.245} + \mu_0 \exp\left(14.69\phi_s\right)\dot{\gamma} + \frac{0.8358\text{Pa s}\phi_s\dot{\gamma}}{0.02193\text{s}\phi_s^{-0.5808}\dot{\gamma} + 1} \tag{13}$$

Accordingly, the rheological viscosity has the following form

$$\mu_r = \frac{7021\text{Pa}\phi_s^{4.245}}{\dot{\gamma}} + \mu_0 \exp\left(14.69\phi_s\right) + \frac{0.8358\text{Pa s}\phi_s}{0.02193\text{s}\phi_s^{-0.5808}\dot{\gamma} + 1} \tag{14}$$

The parameterized rheological model describes the shear-thinning behavior and viscoplastic behavior of a mud suspension qualitatively and quantitatively as a function of the solid volume concentration or the bulk density, respectively. This approach represents a continuous formulation from clear water to a high-concentration suspension. For a particle concentration equal to zero, the rheological viscosity reduces to the viscosity μ_0 of clear water and behaves as a Newtonian fluid (Fig. 11 for $\rho = 1,000 \mathrm{kg/m^3}$).

In the following, the parameterized quantities used to describe the behavior of mud suspensions are shown as a function of the bulk density, as adopted in the isopycnal numerical model. The yield stress increases exponentially with increasing bulk density of the mud suspension, as shown in Fig. 9. The next diagram shows flow curves for different bulk densities (Fig. 10). In this case, the shear rate (intensity) is chosen to range from 0 to 100 s^{-1}, as recommended by BERLAMONT et al. (1993) for sedimentological investigations. Shear rate intensities occur in the range of 0-10 s^{-1} within the fluid mud body, whereas in turbulent flow, the shear rate intensity may become much higher. In the former case, a more rapid increase in shear stress in the low shear rate range than in the higher shear rate range is observed. Although the material structure is able to withstand higher stresses initially, this capability decreases with the break-up of the structure due to increasing shear impact. The rheological viscosity is shown in Fig. 11 as a function of the shear rate for different bulk densities. The shear-thinning behavior of fluid mud is represented accordingly by a decrease in rheological viscosity with increasing shear rates. Again, the rheological viscosity decreases rapidly in the low shear rate range with increasing shear. The internal structure breaks up during the shear period in which the viscosity decreases rapidly. With the complete break-up of the aggregates, the viscosity curve progresses asymptotically to a specific viscosity value.

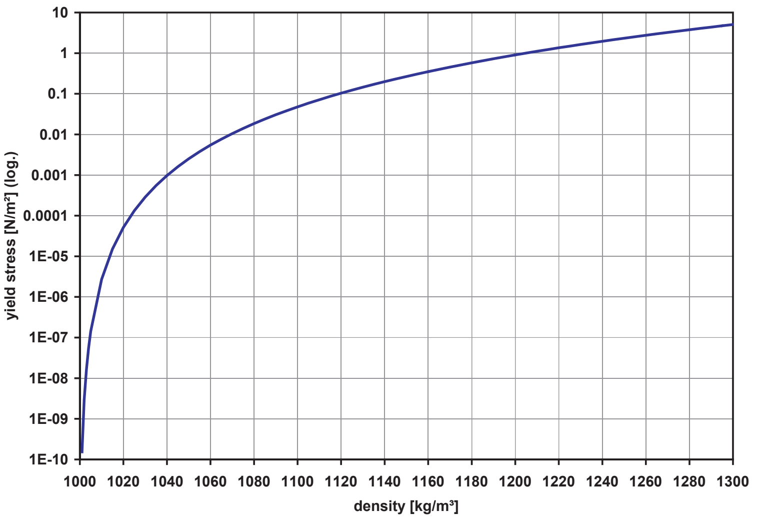

Figure 9: Yield stress as a function of the bulk density according to the parameterized Worrall-Tuliani model (τ_y = 7020 Pa $\phi_c^{4.245}$).

The rheological constitutive laws described in Section 2.1 as well as by Equation (13) consider a medium under simple shear such as in the laminar Couette flow in the x-direction. In this case, the velocity u decreases linearly with depth while the shear stress tensor reduces to the component τ_{xz} and the deformation rate tensor $\mathbf{D}$ to $\dot{\gamma}_{xz}$, respectively. The rheological viscosity is described by the ratio between the shear stress intensity and the shear rate intensity. The rheological viscosity is then given by

$$\mu_r = \frac{|\tau_{xz}|}{|\dot{\gamma}_{xz}|} \tag{15}$$

where the shear rate component $\dot{\gamma}_{xz}$ is equal to $\partial u / \partial z$. All three quantities are scalar values. The shear rate depends on the stress state. In three dimensions, however, both are described by a tensor. An isotropic and homogeneous fluid is assumed. The viscosity of an infinitely small volume is therefore the same in all spatial directions and is represented by a scalar value. In other words, the viscosity does not depend on the direction of shear, but on its magnitude or shear intensity. Thus, in three-dimensions the scalar viscosity has to be a function of the magnitudes of the shear stress and shear rate tensor.

In accordance with MALVERN (1969), ROBERTSON (2008), and GRAEBEL (2007) a general tensor formulation of the shear stress for non-Newtonian fluids may be written as

$$\boldsymbol{\tau} = \mu_r(\mathbf{II}_D)\mathbf{D} \tag{16}$$

whereby the shear rate intensity is

$$\mathbf{II}_D = -2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 \tag{17}$$

The flow regime of a river or estuary is dominated by vertical gradients of the horizontal velocity. Thus, a sufficient approximation for $\mathbf{II}_D$ is obtained by neglecting the derivatives of the vertical velocity component because the horizontal velocity gradients are much greater than the vertical velocity gradients. In addition, the horizontal derivatives of the horizontal velocity components are very small. Based on these assumptions, the shear intensity expression reduces to

$$\mathbf{II}_D = -\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \tag{18}$$

Accordingly, the rheological viscosity of a Worrall-Tuliani fluid in three-dimensional flow results in

$$\mu_r = \frac{\tau_y}{\sqrt{\left|\left(\partial u/\partial z\right)^2 + \left(\partial v/\partial z\right)^2\right|}} + 2\mu_\infty + \frac{\Delta\mu c_{aggr}}{c_{break}\sqrt{\left|\left(\partial u/\partial z\right)^2 + \left(\partial v/\partial z\right)^2\right|} + c_{aggr}} \tag{19}$$

A more detailed derivation of this equation is given in WEHR (2012).

This rheological approach is comparable to the approach adopted in the hydrodynamic modeling of the effects of turbulence by the turbulent viscosity. For example, the mixing-length model describes the turbulent viscosity by

$$\mu_t = \frac{l_m}{\rho}\sqrt{|\mathbf{II}_D|} = \frac{l_m}{\rho}\sqrt{\left|\left(\partial u/\partial z\right)^2 + \left(\partial v/\partial z\right)^2\right|} \tag{20}$$

as a function of the shear rate intensity and the mixing length l_m (MALCHEREK 2001; SCHLICHTING and GERSTEN 1997). Similarly, the shear rate intensity is approximated by vertical gradients of horizontal velocity, as these express maximum shearing.

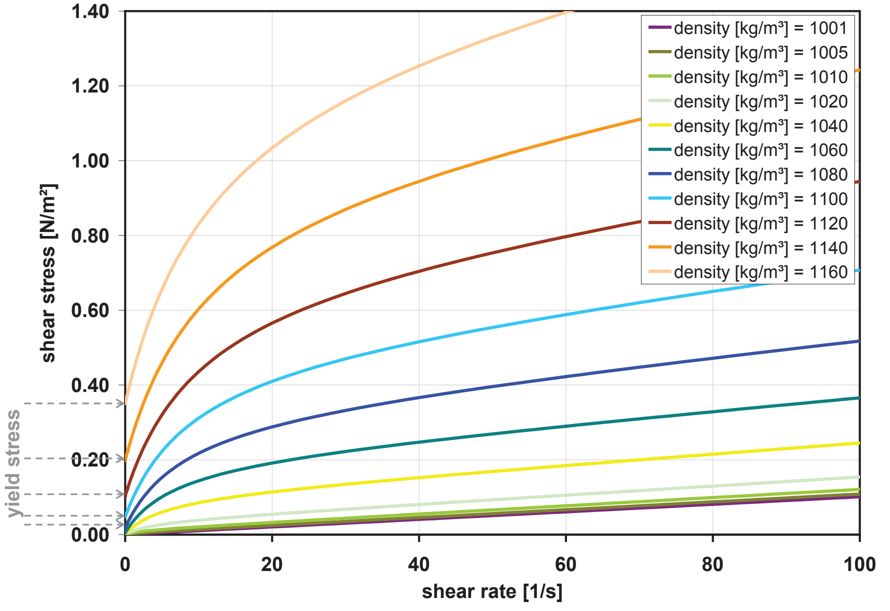

Figure 10: Flow curves for different bulk densities according to the parameterized Worrall-Tuliani model. The shear stress increases with increasing bulk density and shear rate. The initiation of deformation is described by the yield stress, which increases with bulk density.

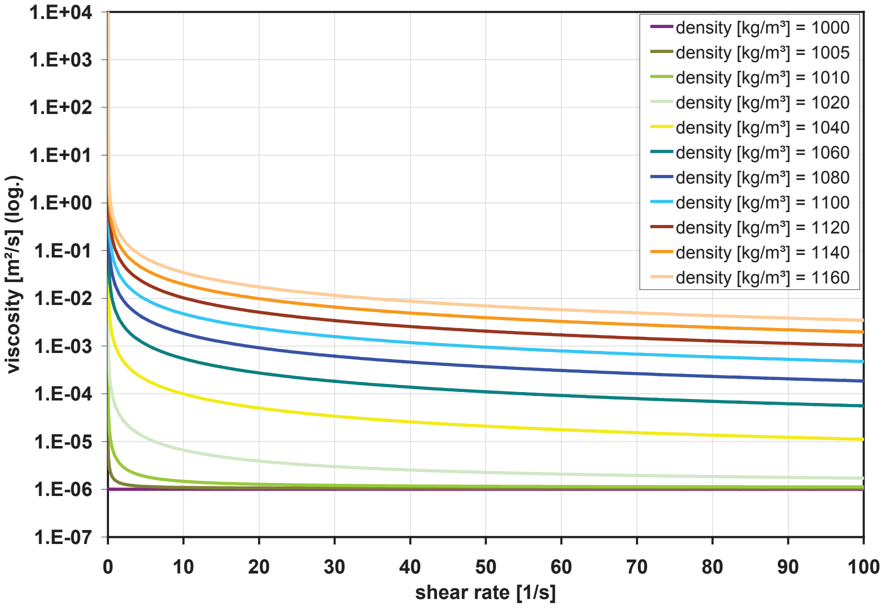

Figure 11: Rheological viscosity-shear rate relation for different bulk densities according to the parameterized Worrall-Tuliani model. The viscosity decreases with break-up of the internal structure caused by increasing shear rate. The viscosity of clear water remains constant, thereby indicating Newtonian behavior.

Therefore, the effect of increasing turbulence or increasing rheological viscosity is the same as in the Navier-Stokes equations, i.e. damping of the current velocity. With regard to the diffusion of suspended particles, however, the level of mixing is increased by turbulence but reduced by an increase in rheological viscosity.

4.3 The Three-dimensional Isopycnal Numerical Model

The numerical realization of the simulation of fluid mud dynamics is achieved by an isopycnal numerical method. A three-dimensional isopycnal model for the simulation of stably stratified baroclinic circulation was developed by CASULLI (1997). The fundamental property of isopycnal models is a vertical discretization by layers of constant density, i.e. isopycnals. An implementation of this 3-D isopycnal model approach for unstructured grids was provided by Prof. V. Casulli of the University of Trento, Italy.

High density gradients due to suspended sediment accumulations and fluid mud formations near the bottom can be resolved by means of isopycnal layers. In this high-density region, the isopycnal layers can become very thin, thereby permitting high resolution of the stratification.

The governing equations and the basic principles of the three-dimensional isopycnal model are described in the next section (Section 4.3.1). It is shown how fluid mud dynamics are simulated using such a model and the extensions required for this purpose.

The 3-D model approach and implementation are extended to include the simulation of shear-dependent viscosity following a non-Newtonian approach (Section 4.3.1) and the vertical mass transfer between isopycnal layers WEHR (2012). Finally, the properties of the numerical method are summarized in Section 4.4.

4.3.1 Governing Equations of the Three-Dimensional Isopycnal Model

The isopycnal circulation model is based on a (x, y, ρ)-coordinate system. This system is illustrated in Fig. 12. Each density layer represents a suspension of constant density corresponding to a specific suspended sediment concentration. The bottom isopycnal layer is referred to as m_0 and the surface layer as M.

Based on the general equations of motion

$$\frac{du}{dt} = \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j} + f_i \tag{21}$$

and the continuity equation, three governing equations result based on two assumptions. Firstly, an incompressible Newtonian fluid is assumed. The density is therefore constant and can be separated from the system of equations. The second assumption is the hydrostatic pressure approximation. Furthermore, the momentum equations are Reynolds-averaged and layer-averaged for each isopycnal layer. The density classes and the maximum number M of isopycnal layers are predefined. The isopycnal layers and their adjacent layers are specified by the following indices:

$m = m_0, \dots, M$	isopycnal layer from bottom to surface
m_b	next active isopycnal layer below the m-th layer
m_t	next active isopycnal layer above the m-th layer

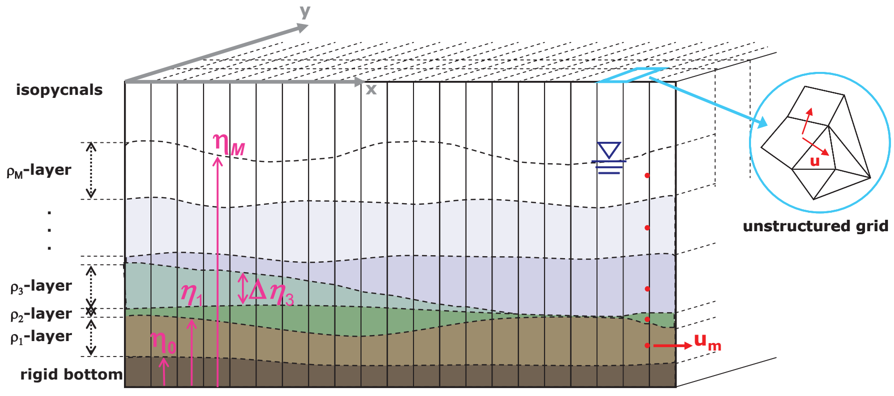

Figure 12: Isopycnal model for three-dimensional flow. The vertical domain is discretized by isopycnal layers (ρ-layer) and the horizontal domain by an unstructured grid. The layered system is stably stratified with ($\rho_1 > \rho_2 > \ldots > \rho_M > 0$). The isopycnal surfaces are denoted by η_m for the m-th isopycnal layer and the rigid bottom by η_0. The velocities u_m are layer-averaged.

This results in a system of M two-dimensional shallow water equations for each isopycnal layer, ranging from η_{m_b} to η_m with dimensions (x, y, ρ). The momentum equations, which are already discretized, are given by

$$\begin{aligned}
&\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} + v_m \frac{\partial u_m}{\partial y} = -\frac{\partial p_{h,m}}{\partial x} + \frac{\partial}{\partial x}\left(\nu_m^h \frac{\partial u_m}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_m^h \frac{\partial u_m}{\partial y}\right) + \frac{\tau_{x,m+1/2} - \tau_{x,m-1/2}}{\rho_m\left(\eta_m - \eta_{m_b}\right)} \\
&\frac{\partial v_m}{\partial t} + u_m \frac{\partial v_m}{\partial x} + v_m \frac{\partial v_m}{\partial y} = -\frac{\partial p_{h,m}}{\partial y} + \frac{\partial}{\partial x}\left(\nu_m^h \frac{\partial v_m}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_m^h \frac{\partial v_m}{\partial y}\right) + \frac{\tau_{y,m+1/2} - \tau_{y,m-1/2}}{\rho_m\left(\eta_m - \eta_{m_b}\right)}
\end{aligned} \tag{22}$$

The second and third terms on the left-hand side represent the **advection** terms in horizontal direction for the m-th layer. The velocities u_m and v_m are isopycnal layer-averaged quantities. The surface or subsurface elevation of the m-th isopycnal layer is given by η_m, and η_0 is the bathymetric depth. The first term on the right-hand side represents the pressure term. The hydrostatic pressure p_h is normalized using a reference density ρ_r and consists of the barotropic pressure and the atmospheric pressure p_a (pressure per density) at the free surface

$$p_{h,m} = g\left(\sum_{l=m}^{M} \frac{\rho_l - \rho_{l_t}}{\rho}\eta_l\right) + p_a \tag{23}$$

The normalized barotropic pressure considers the pressure from the surface M to the current isopycnal layer m. The density above the water surface is defined by $\rho_{M+1} = 0$. The **gravitational transport** is determined by the pressure term. This indicates increasing gravitational forcing with increasing differences in the density between the isopycnal layers.

The second to fourth terms on the right-hand side characterize the **internal shear stresses**. An approximation for the **non-Newtonian flow behavior** of highly-concentrated mud suspensions is formulated in Section 4 and derived in detail in WEHR (2012). Based on this approximation, the flow behavior is described by the rheological viscosity $\nu_{r,m} = \mu_{r,m} / \rho_m$. This is a function of space x, time t, and density ρ, which corresponds to the suspended sediment concentration and the shear rate intensity $|\mathbf{II}_D|$. The horizontal and vertical viscosity components ν_m^v are functions of the rheological viscosity νr and the turbulent viscosity νt. The detailed interdependence and interaction of the two viscosity components is not yet known and is thus an aspect requiring further investigation in future work. In this study, it is assumed that the horizontal and vertical viscosities can be treated as the sum of both components

$$\nu_m^h = \nu_{r,m} + \nu_{t,m}^h \text{ and } \nu_m^v = \nu_{r,m} + \nu_{t,m}^v \tag{24}$$

As the rheological viscosity has no vectorized components, its horizontal and vertical values are equal. The rheological viscosity is determined by the constitutive formulation of Equation (19). The turbulent viscosity is taken to be constant for the m-th layer. It has to be ensured that the horizontal and vertical viscosities are non-negative, otherwise they will have an accelerating effect on the advective terms.

The interfacial shear between two adjacent isopycnals is described by the vertical shear stress term (last term on the right-hand side of Eqn. 22). These isopycnal interfacial shear stresses for the x- and y-components are determined by

$$\begin{aligned} &\frac{\tau_{x,m+1/2}}{\rho_m} = \nu_{m+1/2}^v \frac{u_{m_t} - u_m}{\eta_{m_t} - \eta_{m_b}}, \qquad \frac{\tau_{x,m-1/2}}{\rho_m} = \nu_{m-1/2}^v \frac{u_m - u_{m_b}}{\eta_{m_t} - \eta_{m_b}}, \\ &\frac{\tau_{y,m+1/2}}{\rho_m} = \nu_{m+1/2}^v \frac{v_{m_t} - v_m}{\eta_{m_t} - \eta_{m_b}} \text{ and } \frac{\tau_{y,m-1/2}}{\rho_m} = \nu_{m-1/2}^v \frac{v_m - v_{m_b}}{\eta_{m_t} - \eta_{m_b}} \end{aligned} \tag{25}$$

The surface boundary condition is given by

$$\frac{\tau_{x,M+1/2}}{\rho_M} = \gamma_a \left(u_a - u_M\right) \text{ and } \frac{\tau_{y,M+1/2}}{\rho_M} = \gamma_a \left(v_a - v_M\right) \tag{26}$$

and the bottom boundary condition is given by

$$\frac{\tau_{x,m_0-1/2}}{\rho_{m_0}} = \gamma_b u_{m_0} \text{ and } \frac{\tau_{y,m_0-1/2}}{\rho_{m_0}} = \gamma_b v_{m_0} \tag{27}$$

where the non-negative friction factors are γ_a for wind friction and γ_b for bottom friction, respectively. The wind velocities are specified as u_a and v_a.

The **vertical velocity** component no longer appears in the momentum equations due to the depth-averaging per isopycnal layer. The vertical movement is represented by the variation of the isopycnal surfaces.

The free surface equation in (x, y, ρ)-coordinates completes the governing equations and is given by

$$\frac{\partial \eta_m}{\partial t} + \frac{\partial}{\partial x}\left(\sum_{l=1}^{m} \left(\eta_l - \eta_{l-1}\right) u_l \right) + \frac{\partial}{\partial y}\left(\sum_{l=1}^{m} \left(\eta_l - \eta_{l-1}\right) v_l \right) = 0 \tag{28}$$

The development of the **isopycnal surface elevation** (surface or sub-surface) is described by the change of the elevation with time and the sum of the horizontal fluxes below the surface η_m. The thickness of the isopycnal layers can vary in time and space. A layer may disappear and reappear if drying and wetting occur.

Vertical transport processes such as settling and mixing change the degree of stratification in a suspension. In an isopycnal model approach, this requires mass transfer between the isopycnal layers. Vertical fluxes are thus applied to the isopycnal interfaces, which are determined according to the parameterizations of transport rates. The layer thicknesses change according to the mass fluxes. The free surface equation then becomes

$$\frac{\partial \eta_m}{\partial t} + \frac{\partial}{\partial x}\left(\sum_{l=1}^{m}(\eta_l - \eta_{l-1})\,u_l\right) + \frac{\partial}{\partial y}\left(\sum_{l=1}^{m}(\eta_l - \eta_{l-1})\,v_l\right) - \Phi_m^{in} + \Phi_m^{out} = 0 \qquad (29)$$

where Φ_m^{in} and Φ_m^{out} are the sum of the inflow and outflow rates through the interfaces of the m-th layer. Fluxes through the rigid bottom and through the surface M are excluded. Moreover, a layer of zero thickness cannot be the origin of a transport flux, but the layer can become active due to transport flux into the layer. The mass transferred from one layer to an adjacent layer is related to the different volumes resulting from the difference in the densities of the two layers. A diapycnal mass transfer approach was therefore developed in WEHR (2012) with regard to volume and mass conservation. The governing equations (22) and (29) lead to the three unknowns u_m, v_m, η_m. Following the approach adopted by CASULLI (1997), a semi-implicit method is applied to the governing equations to obtain a solvable system of equations. The numerical method is demonstrated in detail in WEHR (2012).

4.4 Properties of the Numerical Method

The basic properties of the applied three-dimensional model approach are summarized below:

- calculation on an unstructured grid
- uniform density for each isopycnal layer
- momentum exchange between isopycnal layers
- vertical mass exchange between isopycnal layers
- assurance of stable density stratification
- drying and wetting of isopycnal layers
- vertical discretization using ρ-layers
- a two-dimensional depth-averaged model results if only one isopycnal layer is defined
- shear-dependent viscosity, as calculated by a parameterized rheological approach
- interaction of the isopycnal layers due to interfacial shear

The numerical model approach includes flooding and drying of the isopycnal layers. Layers representing a suspension of a specific concentration such as fluid mud are not necessarily active over the entire model domain.

Vertical transport rates are determined by parameterized formulations for (hindered) settling according to WINTERWERP and VAN KESTEREN (2004) and for entrainment according to WINTERWERP and KRANENBURG (1997), KRANENBURG and WINTERWERP (1997) and WHITEHOUSE et al. (2000).

5 Applications

5.1 Fluid Mud Movement on an Inclined Plane

The three-dimensional isopycnal numerical model was verified on the basis of its ability to reproduce particular processes, phenomena and the behavior of fluids, especially of fluid mud. Several test cases were hence analyzed in WEHR (2012), one of which is presented here. The test case is kept as simple as possible and the model set-up is restricted to the physical process or phenomenon of interest.

Table 2: Simulation overview for the test case "flow on an inclined plane".

simulation number	density of mud layers [kg/m^3]	isopycnal density differences [kg/m^3]	rheological viscosity approach	gravitational effects due to density differences
(1a)	1005/ 1010/ 1030/ 1080	5/ 5/ 20/ 50	const. 10^{-6} m^2/s	no
(1b)			Worrall-Tuliani app.	no
(1c)			const. 10^{-6} m^2/s	yes
(1d)			Worrall-Tuliani app.	yes
(2)	1010/ 1030/ 1080/ 1150	10/ 20/ 50/ 70	Worrall-Tuliani app.	yes
(3)	1005/ 1010/ 1020/ 1030	5/ 5/ 10/ 20	Worrall-Tuliani app.	yes
(4)	1000.1/ 1000.2/ 1000.3/ 1000.4	0.1/ 0.1/ 0.1/ 0.1	Worrall-Tuliani app.	yes

In a channel with an inclined bed it is demonstrated in the following that the horizontal transport phenomena and the horizontal discretization scheme are capable of simulating the flow of fluid mud. The straight channel is set up with a slope of 0.1 %. Four mud suspensions of different concentrations are discharged at the left boundary and flow down the slope. As shown in Table 2, the predefined densities of these isopycnal layers vary in four different model set-ups. Gravity flow and the effects of the fluid mud rheology are studied for different scenarios of simulation (1) as well as for simulations (1) to (4) with different density distributions. These processes are observed with a minimization of other effects. Mass transfer between the isopycnal layers is therefore not considered and the turbulent viscosity is kept constant over the entire domain. The four scenarios of simulation (1) differ by either including or excluding the gravitational forces due to density differences and the effect of the rheological behavior of cohesive mud suspensions. The rheological behavior is described by the parameterized Worrall-Tuliani approach given by Equations (14) and (19). If the rheological behavior of fluid mud is neglected, the rheological viscosity is set to 10^{-6} m²/s for the entire system.

The gravitational forcing according to density differences is modeled by the following pressure term $-g\partial/\partial x\left(\sum_{l=m}^{M}\eta_l\left(\rho_l-\rho_{l_t}\right)/\rho_r\right)$ for the m-th isopycnal layer of the momentum equation (x-component) - see Equation (23). Neglecting the density differences influencing the flow, this term reduces to $-g\partial/\partial x\left(\partial\eta_M\right)$. In this case, the action of gravity on the water column is as if only one isopycnal layer has been defined.

The density distribution in the high-concentration layers is in the range of 1005 to 1080 kg/m³ (see Table 2). The water layer has a density of 1000 kg/m³. This applies to all scenarios of the simulation set-up (1). The simulation results are shown in Fig. 13.

In the first scenario (1a), neither density differences nor rheological effects are considered. The discharged fluid mud now propagates very slowly and spreads significantly in the vertical direction. The layers flow above each other with increasing velocities. The velocity is therefore much higher near the surface and at the layer fronts. The bottom friction decelerates the movement when the different layers are in contact with the bottom.

Simulation (1b) includes variable rheological viscosity. This does not influence the Newtonian water layer but the structural viscosity of the fluid mud layers. The rheological viscosity changes according to the density and decreases with increasing shear rate intensities, i.e. the shear-thinning behavior of mud suspensions. Without this structural behavior, each isopycnal layer would have a constant viscosity. The viscosity varies, however, within a certain range in an isopycnal layer according to the implemented rheological approach. In particular, this becomes more evident in Fig. 15 with a different color scale. In addition, the viscosity differs between the isopycnal layers up to an order of magnitude depending on their differences in density and active state of shear.

The horizontal propagation of the two layers with lower concentrations (1005 and 1010 kg/m³) in simulation (1b) is comparable to that of simulation (1a) because their rheological viscosity is only slightly higher. The viscosity of the two layers with higher concentrations is much greater and both move with a similar velocity. The vertical stratification becomes more even and stable. Accordingly, the horizontal movement now dominates over the vertical spread of the two fluid mud layers (1030 and 1080 kg/m³).

The third scenario (1c) includes the gravitational forces due to density differences but neglects the rheological viscosity which describes shear-thinning and structural behavior. Compared with variants (1a) and (1b), the results in this case indicate that gravitational flow is responsible for downslope propagation. Each layer moves with its own velocity due to the differences in the density between the isopycnal layers. The layers cover significantly different distances after an hour of downslope movement. In particular, bed friction decelerates the bottom layer.

The fourth scenario (1d) considers both gravity flow and the rheology of mud suspensions. Again, the rheological viscosity increases with the density of the isopycnal layers, but also varies within a layer due to changes in the shear intensity. The high rheological viscosities decelerate the average downslope flow and the layer fronts are closer together. However, the fluid mud layers now appear more as a compact fluid mud body interacting due to the interfacial shear stresses. This shows that both processes are necessary to reproduce a realistic and plausible high-concentration flow on an inclined plane.

The model set-ups of simulations (1d), (2), (3) and (4) differ in terms of the density distributions of the four mud suspensions (see Table 2). The simulation results are given in Fig. 14. The average density of the mud layers decreases from the top panel (simulation (2)) to the bottom panel (simulation (4)) of Fig. 14. Furthermore, the differences in the density of the isopycnal layers also decrease. The definition of the pressure term (Equation (23)) indicates increasing acceleration of the mud layers with increasing density differences. This can be observed in the varying propagation times of the fluid mud body. Accordingly, the fluid mud front advances the farthest in simulation (2) and slows down further in simulations (1), (3) to (4).

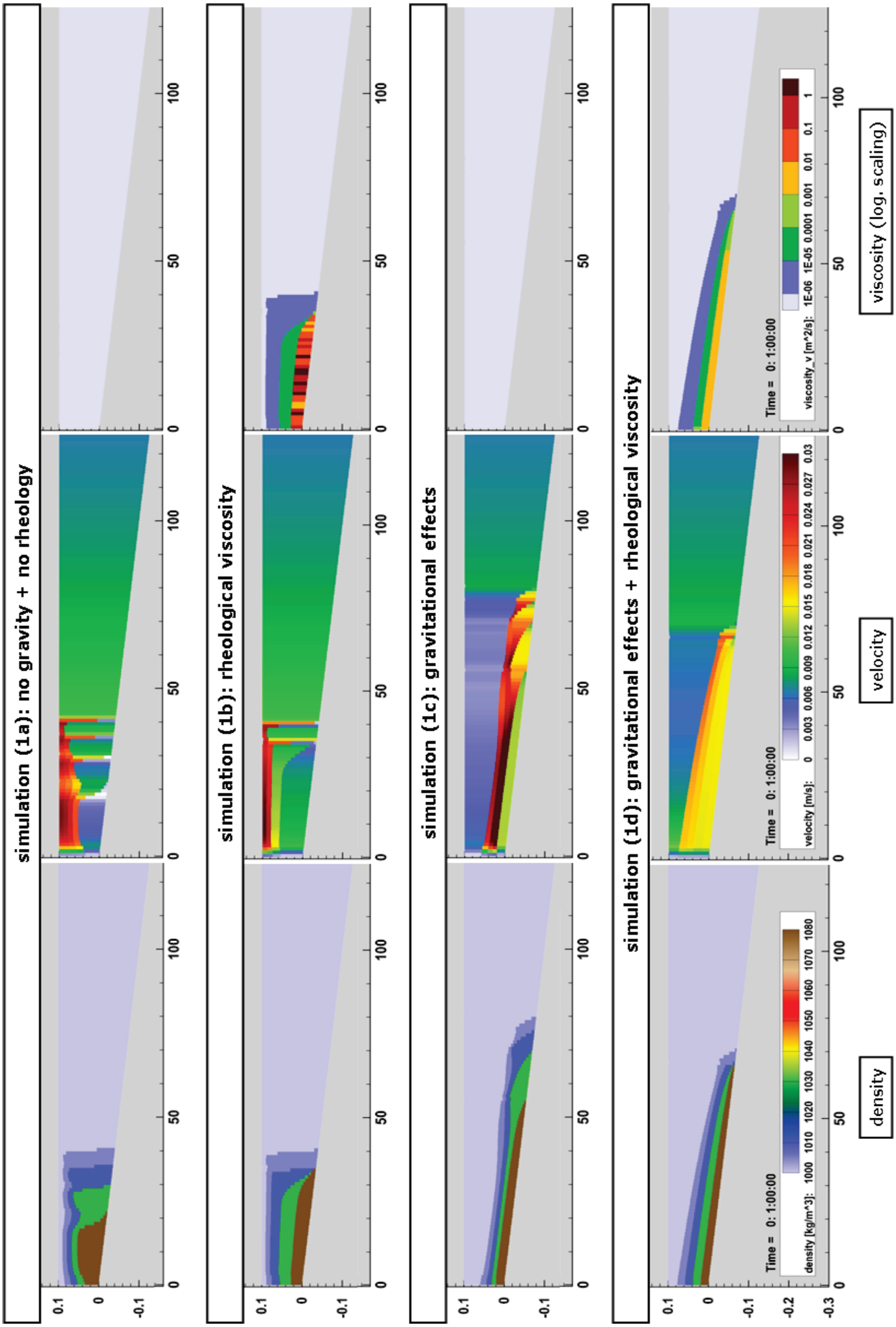

Figure 13: Propagation of high-concentration layers – variations of simulation set-up (1) (first column: density; second column: velocity; third column: viscosity in logarithmic scaling).

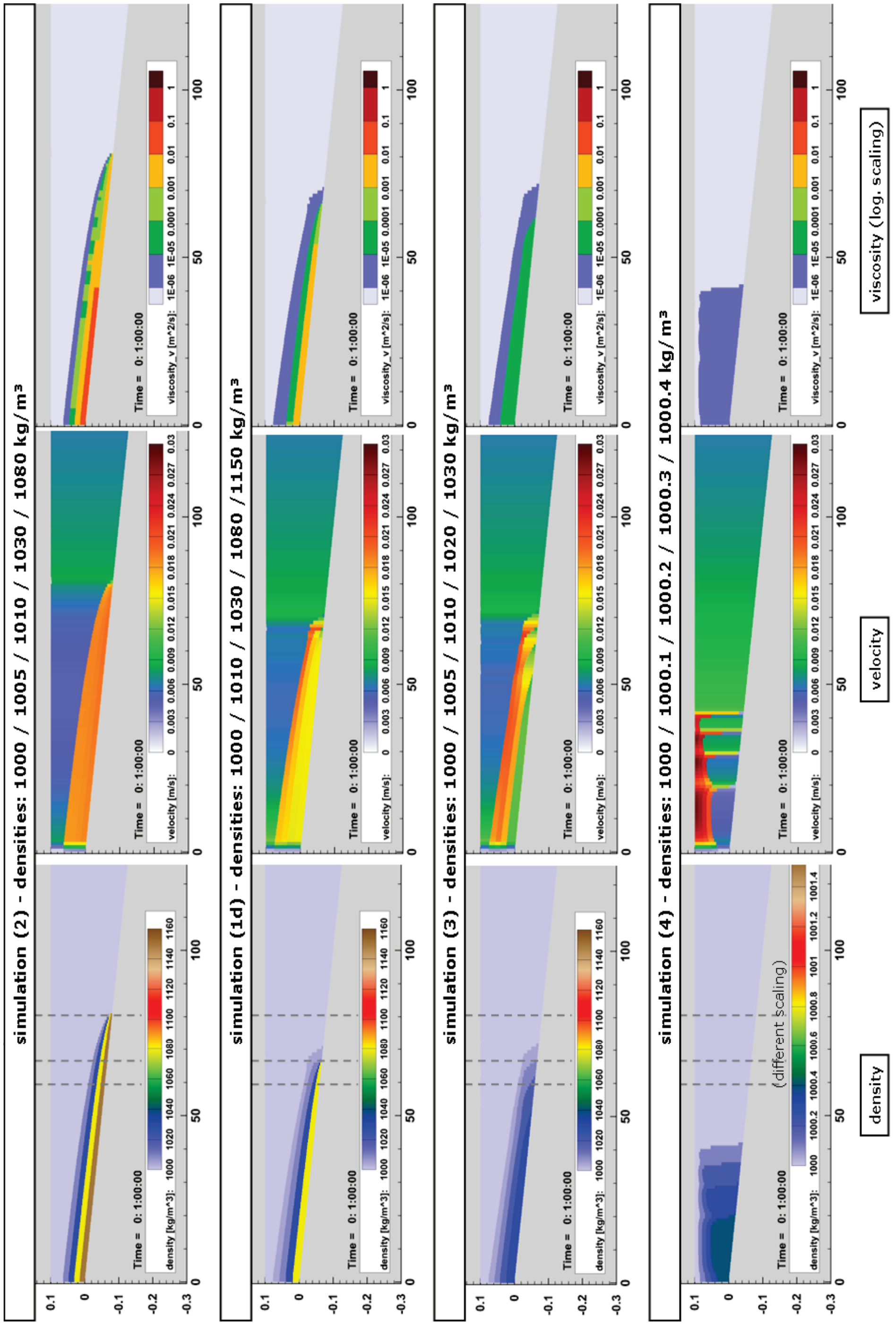

Figure 14: Simulation results of model runs (2), (1d), (3) and (4) (first column: density; second column: velocity; third column: viscosity in logarithmic scaling).

The structural viscosity counteracts the progressive movement of the fluid mud body by deceleration due to increasing viscosities with simultaneously increasing densities. Model run (4) exhibits similar results to run (1a) due to the fact that the density differences of run (4) are fairly small and are omitted from the pressure term in run (1a). Moreover, the rheological viscosity is set to a constant value of 10^{-6} m²/s in simulation (1a) such that the resulting viscosities differ only slightly from the values of run (4). This results in similar velocity patterns to those described for simulation (1a) above. The numerical model approaches its limits with no or extremely small density differences between the isopycnal layers, as is the case in simulations (1a) and (4). The density layers would normally tend towards mixing. However, mixing is disabled in this test case. This is one reason for these particular velocity patterns.

The parameter variations of the seven model simulations lead to plausible results. The plausibility of the results is evaluated on the basis of:

- the influence of gravity and structural viscosity on the downslope flow by activating or deactivating these processes,
- the effects of gravitational forces by varying the density differences,
- the effects of the structural (rheological) viscosity by varying the density differences.

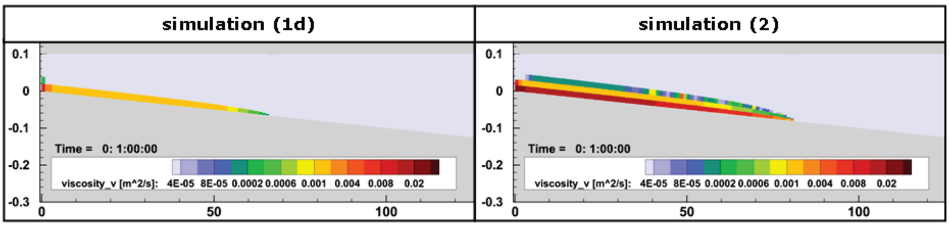

Figure 15: Rheological viscosity illustrated using a different color scale compared to Figs. 13 and 14 for simulation (1d) with lower suspension densities and simulation (2) with higher suspension densities. The viscosity varies within an isopycnal layer as a function of the shear impact and increases with increasing density according to the implemented rheological approach.

5.2 Application on the Ems Estuary – River Section Rhede to Herbrum

The Ems estuary is located along the border between Germany and the Netherlands. The domain can be seen in the lower right corner of Fig. 16. The estuary extends from the weir at Herbrum at the tidal limit to the outer coastline with a number of offshore islands. The freshwater discharge at Herbrum weir can vary between 20 and 400 m³/s while the most frequent discharge is about 40 m³/s.

The tidal range increases from 2.20 m at Borkum to 3.50 m at Papenburg. The tidal curve has become highly asymmetrical in the river Ems due to channel deepening. This asymmetry is characterized by a steep flood tide gradient and a short flood phase, a long slack water period from flood to ebb and a more gradual ebb tide gradient and longer ebb phase. This leads to a flood-dominated system with strong flood currents and much weaker ebb currents. Suspended sediments are thus transported upstream due to tidal pumping. The suspended particles, mostly muddy sediments, accumulate in the wide maximum turbidity zone and form dynamic fluid mud layers. However, fluid mud layers not only occur in the maximum turbidity zone, but may also be found further upstream of the maximum turbidity zone.

For further hydrological and morphological information, see WEILBEER (2005), KREBS and WEILBEER (2008), and WINTERWERP (2011). The fact that turbidity has increased significantly in the lower reaches of the Ems estuary over recent decades has led to a serious occurrence of suspended mud and fluid mud. SCHROTTKE (2006) showed that fluid mud layers with thicknesses of around 2 m or more can develop in the maximum turbidity zone, with concentrations of up to 300 kg/m³ or densities of up to 1190 kg/m³. Sediment echo sounding measurements (Fig. 5) show a highly-stratified water body. Two to three layers of different densities were detected, each layer being around one meter in thickness.

In the following, various characteristics of fluid mud dynamics in the Ems estuary are investigated by system studies using a sectional model. The model domain was chosen to focus on a specific area. Besides, the computational code is not parallelized and the required computational effort is reduced in this way. The model boundaries were chosen to coincide with the available boundary value data. The model domain extends from Rhede to the Herbrum weir in order to study the effects of tidal pumping and tidal movement of the lutocline. The location of the model domain is indicated in Fig. 16 along with the bathymetry and grid resolution of the model. The upstream reach of the model subdivides into a river arm leading to the weir at Herbrum and the harbor basin of a sluice. The sediment transport in this section of the model is dominated by mud suspensions. The fine sediments are mainly carried into this region by tidal pumping. Muddy sediments accumulate in the harbor basin.

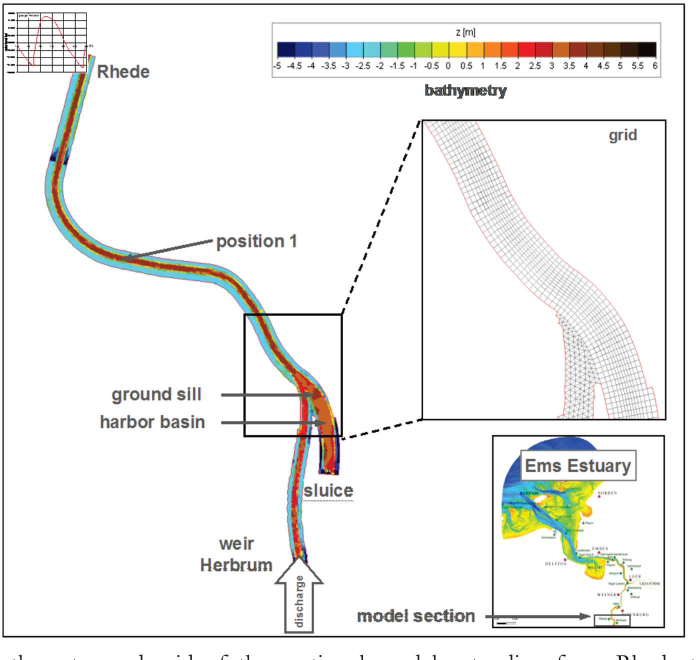

Figure 16: Bathymetry and grid of the sectional model extending from Rhede to Herbrum. A freshwater discharge is specified at the Herbrum weir while a tidal variation in water level is prescribed at the open boundary at Rhede.

Freshwater discharge is specified at the Herbrum weir from a source containing only clear water. Prescribed water levels are imposed at the open boundary at Rhede. Suspensions of different concentrations are introduced into the model domain at the open boundary. The isopycnal interfaces are triggered in the same way as the water surface. These move with the tidal signal at only half of the tidal range.

Three simulations were carried out with different boundary values (see Table 3). The first simulation (1) was forced by a measured time series for the freshwater discharge (50 to 60 m^3/s). Simulation (2) differs from simulation (1) on account of a reduced freshwater discharge set at a constant value of 25 m^3/s. The third simulation (3) was performed with a symmetrical M2-tide instead of measured tidal data. This permitted an evaluation of the effect of the asymmetrical tidal wave compared with a uniform M2 sine wave. The freshwater discharge in this case was the same as in simulation (1). The initial density levels (Table 4) are uniformly assigned throughout the entire model domain. The density values are determined according to the 16 predefined isopycnal layers and range from clear water at the surface to a dilute suspension, a concentrated suspension, fluid mud and finally to freshly consolidated mud at the bed. The movement of the mud suspension layers at the open boundary at Rhede is not based on measured data but rather on tidal motion.

The simulations cover several tidal cycles. The simulation time step is limited by the Courant time step criterion and the explicit solution for vertical mass transfer. From a preliminary study of varying the time step, it was found that a time step of 10 s yields sufficiently accurate results.

In the following, the simulation results are presented for a longitudinal profile following the channel centerline as well as for the selected reference position 1 (see Fig. 16).

Table 3: Simulation overview for the model Rhede to Herbrum.

simulation number	**boundary value: discharge**	**boundary value: water level**	**M**
1	measured data ~50-60 m^3/s	measured data of gauge Rhede	16
2	25 m^3/s	measured data of gauge Rhede	16
3	measured data ~50-60 m^3/s	M2 tide	16

Table 4: Initial density distribution over the depth with an initial water level of -1.15 m.

layer number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
density	**[kg/m³]**	1150	1130	1120	1110	1100	1090	1080	1070	1060	1050	1040	1030	1020	1010	1005	1000
mud concentration	**[kg/m³]**	240.9	208.8	192.7	176.7	160.6	144.6	128.5	112.4	96.4	80.3	64.2	48.2	32.1	16.1	8.0	0.0
isopycnal elevation	**[m]**	-4.00	-3.64	-3.47	-3.29	-3.11	-2.93	-2.75	-2.58	-2.40	-2.22	-2.04	-1.86	-1.68	-1.51	-1.33	-1.15

The vertical transport processes considered are entrainment and settling (hindered settling). The horizontal transport is driven by currents and gravitational forcing. Entrainment is determined according to the KRANENBURG and WINTERWERP (1997) approach with the coefficients $C_s = 0.50$ and $C_\sigma = 0.42$. The critical shear stress for entrainment is defined by the yield strength of the mud suspensions (first term of Equation (13)). The settling velocity with hindered settling is calculated by a formulation according to WINTERWERP and VAN KESTEREN (2004). The gelling concentration is set at 40 kg/m^3.

Consolidation takes place in the time range of weeks and months. This process has no effect during the simulation of a few tidal cycles. Consolidation can therefore be neglected in this case.

The rheological behavior of the mud suspensions is simulated by the parameterized Worrall-Tuliani model presented in Equations (14) and (19). The rheological viscosity approach is applied to every isopycnal layer because it is valid for clear water as well as for fluid mud and soft consolidated mud. A yield stress equal to zero is obtained for clear water and increases exponentially with increasing concentration (see Fig. 9). Accordingly, the rheological viscosity decreases to the molecular viscosity of clear water. An increase in the mud concentration as well as a decrease in the shear impact leads to an increase in the rheological viscosity. This is shown in Fig. 11. The horizontal turbulent viscosity is set to a constant value of 10^{-6} m²/s and the vertical turbulent viscosity to 10^{-2} m²/s. The calibration and sensitivity analysis of the model is presented in WEHR (2012).

The mud suspension transport is represented by the density layers. Their corresponding suspended particle concentrations are listed in Table 4. The isopycnal approach permits the modeling of high stratification during all tidal phases. The density distribution is shown in Fig. 17 on the left-hand side, and the velocity distribution on the right-hand side. These plots represent the longitudinal section along the centerline of the channel. Therefore, the results are only representative for this section. The simulation results reveal fluid mud formations comparable to the measured density gradients shown in Fig. 5. The mud suspension layers are influenced by the tide and move with the tidal current. During the flood tide, the mud suspension layers are transported in the direction of the harbor basin. The influence of the symmetry of the tidal signal and freshwater discharge on tidal pumping becomes apparent when the velocity and density distributions of the three simulations are compared. The symmetrical M2-tide in simulation (3) (first panel) produces an ebb current-dominated system because the freshwater discharge intensifies the ebb current, whereas tidal asymmetry leads to a longer ebb phase with much higher ebb currents compared to flood currents. Moreover, the upstream fluid mud intrusion is noticeably shorter, as evidenced by a comparison with simulations (1) and (2). Predominantly upstream transport due to tidal pumping is prevented by the tidal symmetry (simulation (3)). The mud suspensions advance rapidly and cover a wide area in the flood-dominated conditions of simulations (1) and (2). Simulation (1) (second panel) under measured asymmetric tidal conditions with a discharge of about 60 m³/s leads to higher values of the flood currents than ebb currents. Tidal pumping now takes effect and leads to predominant upstream transport. This effect is intensified in simulation (2) (third panel) because the discharge is reduced to 25 m³/s. In this case, the flood current becomes twice as high as the ebb current. The tidal pumping effect is more pronounced in simulation (2) because of the predominant flood currents.

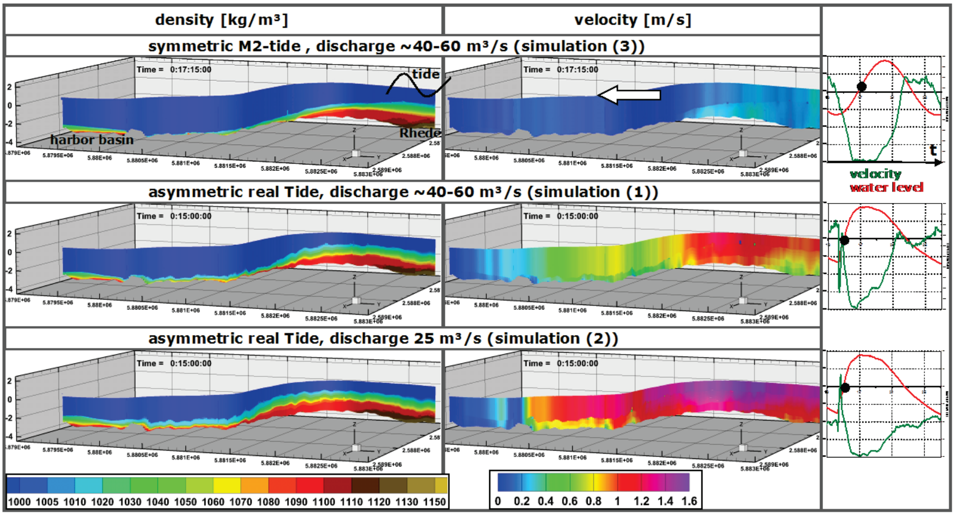

Figure 17: Density distribution (left-hand side) and isopycnal layer-averaged velocity (right-hand side) in the longitudinal section along the channel centerline for simulations (1), (2) and (3). The plots show simulation results for a flood situation with variation of the boundary values.

The velocity, density and rheological viscosity distributions illustrate the specific rheological behavior of the mud suspensions. These are presented exemplarily for simulation (1) in Figs. 18 and 19. The rheological behavior of fluid mud and mud suspensions is described in Sections 2.2 and 4.2. The parameterized Worrall-Tuliani approach is applied in the numerical model. This approach considers structural effects, which lead to an increase in the rheological viscosity with increasing suspended particle concentration and decreasing shear impact (shear-thinning behavior). A general impression of the viscosity as a function of density and shear rate is given in Fig. 11. In nature, the cohesive suspended particles accumulate to produce aggregates and these aggregates can form a structure due to the build-up of contacts. The internal structure is responsible for the flow resistance of the mud. This mechanism is parameterized in the numerical model by the constitutive law according to Worrall-Tuliani. High velocity gradients thus lead to the break-up of the particulate structure, which reduces the rheological viscosity of the mud suspension. This can be observed during the flood tide and especially at the beginning of increasing flood currents (see the first and second panels of Fig. 19). The resistance of the fluid mud increases (increase in the rheological viscosity) owing to hindered settling and decreasing shear rates on reaching slack water (see panels three and four of Figs. 18 and 19). The more highly-concentrated layers maintain their movement longer than the overlying clear water layer due to the difference in viscous behavior and density effects. A highly-stratified flow is temporarily formed during the ebb tide. Panels five and six illustrate the situation during ebb currents, where the velocity of the high-density layers is much lower than that of the low-concentration layers. Moreover, the rheological viscosity increases with increasing particle concentration.

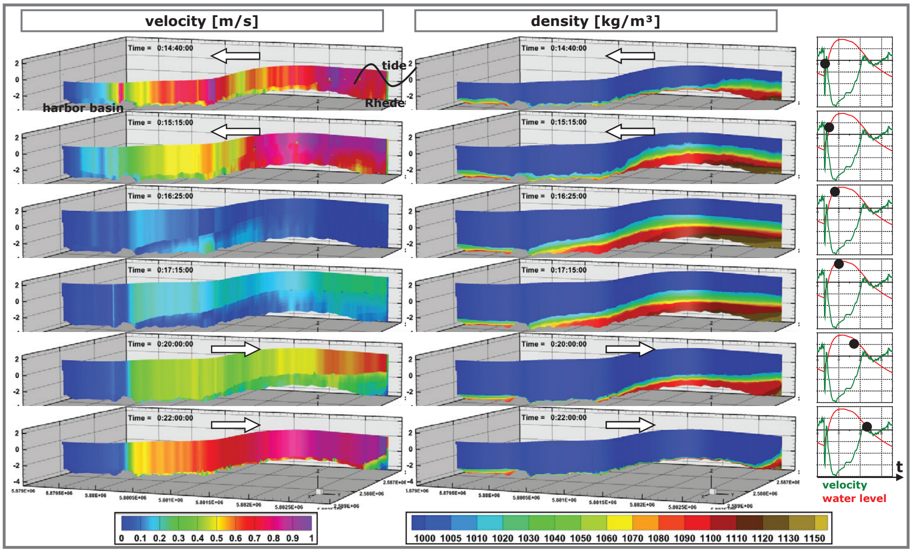

Figure 18: Isopycnal layer-averaged velocity (left-hand side) and density distribution (right-hand side) in the longitudinal section along the channel centerline for simulation (1). Panel one and two are during the flood tide, panels three and four are around slack water at high tide and panels five and six are during the ebb tide. Entrainment occurs from the near-bottom fluid mud layer, especially during the flood tide. Stratified flow occurs during flood and ebb tides. The inertia and resistance against the change of flow direction is more pronounced in the high-concentration mud suspension layers.

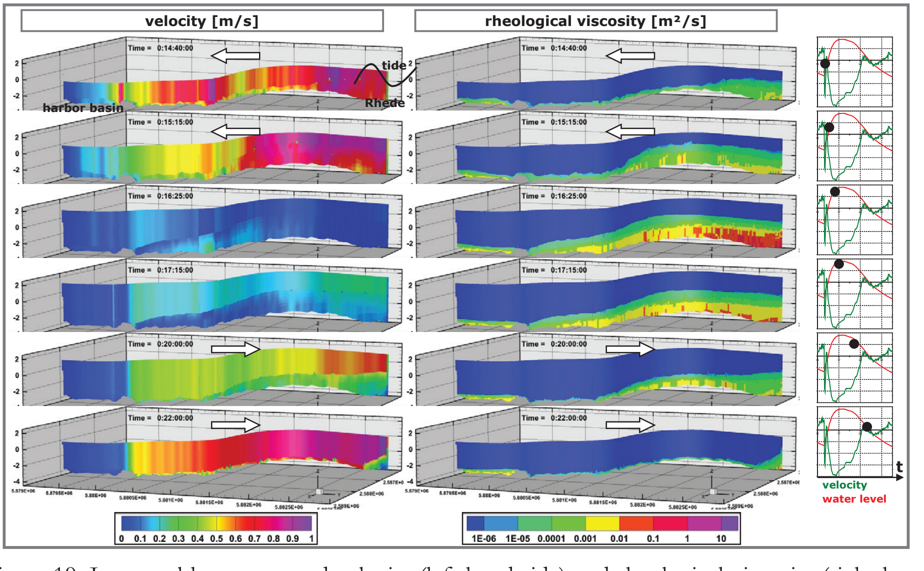

Figure 19: Isopycnal layer-averaged velocity (left-hand side) and rheological viscosity (right-hand side) in the longitudinal section along the channel centerline for simulation (1). Panel one and two are during the flood tide, panels three and four are around slack water at high tide and panels five and six are during the ebb tide. The viscosity decreases during periods of high shear forces (see panels one, two and five) whereas the internal structure can build up during slack water with a renewed increase in the viscosity (panels three and four).

Nevertheless, the system is influenced by the gravitational effects due to the highly- stratified flow. The internal Froude number often becomes less than unity, which indicates that gravitational flow predominates.

The fluid mud transport and development under tidal currents were evaluated qualitatively by comparing the simulation results with observations of the lutocline development according to WANG (2010). This is illustrated in Fig. 20. The observations were carried out over several tidal cycles at a specific location in the turbidity zone of the Ems estuary (Leerort) whereas the simulation results are taken from position 1. Apart from these different locations, the hydrodynamic conditions are not the same in each case. Therefore, only a phenomenological comparison is possible. However, the simulated and observed results both show the typical asymmetrical tide with high flood currents, long slack water at high tide as well as a long ebb phase. The different freshwater discharge conditions in the simulations demonstrate the effect of variable hydrodynamic conditions. The observed lutocline was obtained from ADCP measurements by analyzing the backscatter signal. A high backscatter gradient indicates a high-density gradient in the water column.

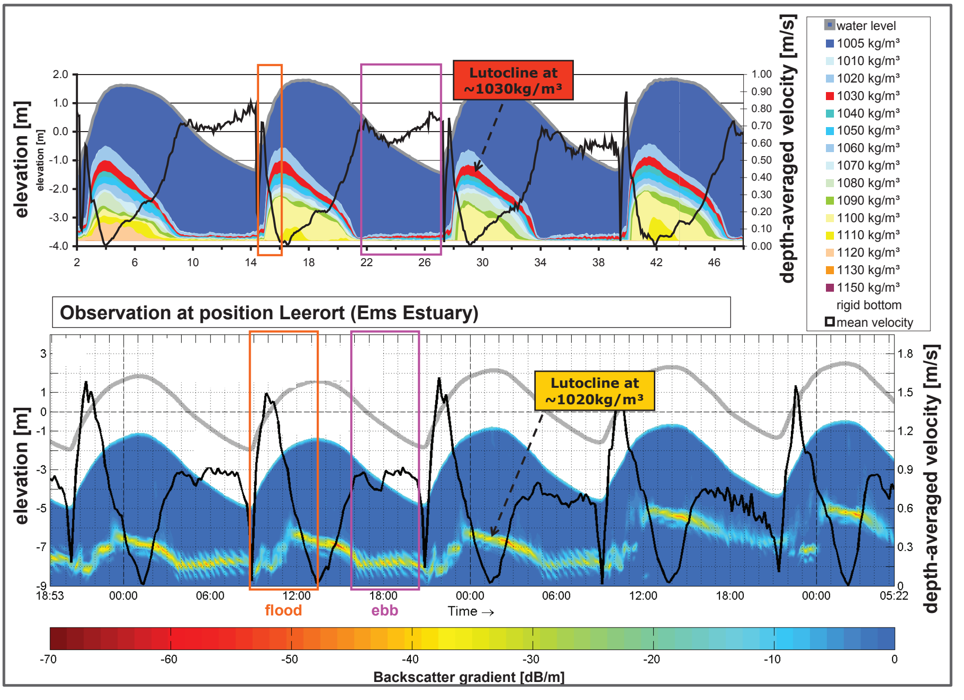

Figure 20: Tidal dynamics of the lutocline - comparison between simulation results (upper panel, simulation (1)) and observations based on 300 kHz ADCP measurements (lower panel). The density corresponding to the lutocline is 1030 kg/m³ in the simulations. The lutocline is indicated by a high backscatter gradient in the measurements. The water level is indicated in grey and the depth-averaged velocity in black. It should be noted that the simulated and measured data relate to different locations and different hydrological situations. However, the characteristic development of the lutocline is very similar. (illustration of observations by courtesy of WANG (2010)).

The suspended-matter concentration just below the lutocline is in the range of 30 kg/m³ (density ~1020 kg/m³) during slack water at high tide, as reported by WANG (2010). The concentration increases downward to the bottom. The simulated density stratification is followed by the subsurface elevation of the density layers. The lutocline is defined as the transition between Newtonian and non-Newtonian behavior, and is accompanied by a sharp density gradient. This corresponds to the layer with a density of 1030 kg/m³ in the simulations, as indicated in red in the graphics.

The fluid mud suspension is entrained into the water column during the flood tide. The observations show low backscatter gradients over the entire water column. This mixing process is indicated in the model results by a rapid increase in the layer thicknesses (subsurfaces) of the mud suspensions. The increasing layer thicknesses result from higher concentration layers being mixed with lower concentration layers due to entrainment and horizontal transport. A highly-stable stratified system is then attained in both cases during slack water. The fluid mud is carried downstream with the ebb currents, which decreases the lutocline elevation. The intensifying ebb velocities progressively lower the lutocline level, as reflected in both the simulations and the observations. At the same time, the sharp transition between the fluid mud and the water body vanishes. The shapes of the simulated and observed lutoclines are very similar and reveal comparable reactions to the tidal flow, even though the mixing process should be intensified in the simulations.

5.2.1 Concluding Remarks

The numerical model is able to simulate tidally-influenced fluid mud dynamics in a more complex topographic domain. The rheological viscosity approach yields plausible results with respect to the velocity and density distribution. A comparison between the simulated and observed development of the lutocline leads to satisfying results. On account of the isopycnal approach, a highly-stratified multi-layered flow is achieved during the ebb tide and at slack water. The density layers interact due to interfacial shear stresses, momentum transfer and mass transfer, which permit the formation, development and movement of the stratified fluid mud flow. The density-driven flow is induced by the differences in the densities of the isopycnal layers, which especially influence the flow at slack water. The shear rate intensities increase during ebb and flood currents because the velocity differences are highest during these periods. However, the shear intensities may be much higher when considering a turbulence closure model, which would lead to higher entrainment rates. Further validation of the parameterized entrainment approach is necessary to clarify this matter.

The interfaces of the density layers are prescribed at the open boundary at Rhede. These move with half of the water-surface amplitude. This steering assumption should be replaced by measured data for the dynamic movement of the lutocline in more advanced and realistic studies. These measurements should investigate the movement of the lutocline as well as the density gradients of the lutocline. Moreover, the measurement of additional density horizons between the lutocline and the cohesive bed is required for data input and model validation.

5.3 Fluid Mud Formation in Troughs of Large Dunes in the Weser Estuary

The Weser estuary extends from the weir at Bremen-Hemelingen to Bremerhaven and then into the North Sea. The model domain of the Weser estuary is shown in Fig. 21. The maximum turbidity zone is located between Bremerhaven and Weser-km 50. The precise position of this zone varies, however, according to hydrological and meteorological conditions (detailed information on the hydrology and geomorphology of the Weser estuary may be found e.g. in BAW (2012)).

Large dunes occur in the upstream part of the maximum turbidity zone with wavelengths of around 50-100 m. Fine sediments settle in the troughs of these dunes during periods of low currents, and fluid mud layers form in these troughs. The fluid mud is resuspended during periods of high tidal currents. This process was observed by SCHROTTKE et al. (2006) and BECKER (2011). In this study, the formation of fluid mud in the troughs of the dunes is simulated using the fluid mud model, and the simulated results are compared with observations.

SCHROTTKE et al. (2006) and BECKER (2011) observed the occurrence of fluid mud layers in the troughs of dunes during three different periods and under different hydrological conditions. They detected density gradients in the water column using a hydroacoustic, parametric sub-bottom profiler. Selected results of these measurements are shown in Fig. 22.

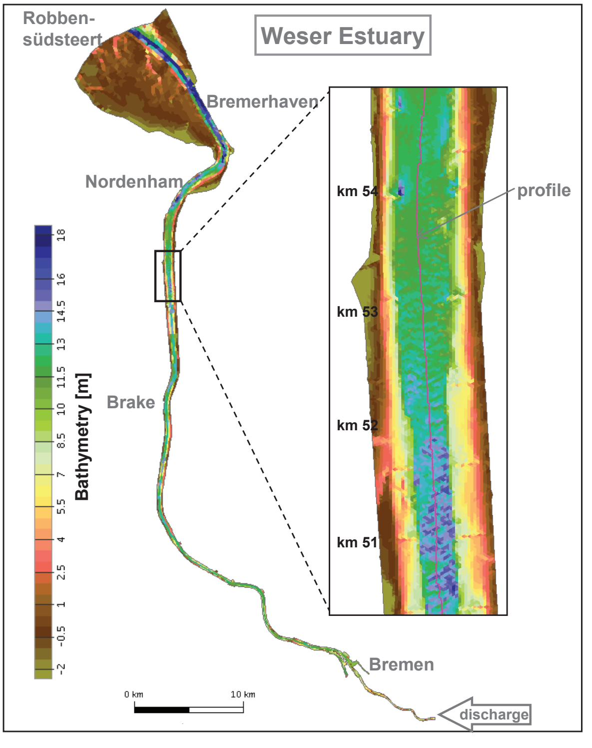

Figure 21: Bathymetry of the sectional model of the Weser estuary extending from Robbensüdsteert to Bremen-Hemelingen. A freshwater discharge is specified at the weir at Bremen-Hemelingen. A tidal water level variation is imposed at the open boundary at Robbensüdsteert.

In addition to the above-mentioned measurements, bulk densities were also determined. These were found to range from 1000.2 to 1000.3 kg/m³ above the lutocline, with maximum values of about 1017 kg/m³ below the lutocline.

The simulation period covers several tidal cycles in May 2002. Values of water level are imposed at the open boundary at Robbensüdsteert while a discharge is prescribed at Hemelingen weir. In a first step, the model is set up as a 2-D system by defining a single isopycnic layer with a density of 1000.1 kg/m³ (without mud layers). The numerical model is extended in a next step to a 3-D system by specifying five density layers with densities corresponding to measured density values ranging from 1000.1 to 1025 kg/m³. The near-bottom mud layers damp the bottom roughness, which thus is decreased. Time series of the water levels for the 2-D and 3-D simulations are given in Fig. 23 and compared with measured water levels. This comparison shows that better results for the water levels are obtained in the 2-D simulation. The velocity time series are given in Fig. 24 for the Nordenham position. A good fit is obtained between the measured and simulated velocities in terms of magnitude and period.

The vertical transport rates for mixing are determined according to the KRANENBURG and WINTERWERP (1997) approach with the coefficients $C_s = 0.80$ and $C_\sigma = 0.42$. The critical shear stress for entrainment is set to a constant value of 0.3 Pa. The settling velocity with hindered settling is calculated by a formulation according to WINTERWERP and VAN KESTEREN (2004) with a gelling concentration of 80 kg/m³.

The simulated development of the lutocline is presented in Fig. 25. The results are given for the longitudinal profile along the centerline of the river section, as indicated in Fig. 21. High velocity gradients between the water body and the mud layer induce mixing at the interface during flood and ebb tides (first and fourth panels of Fig. 25). This is also observed in the measured density gradients where the lutoclines become weaker with increasing ebb currents (third panel of Fig. 22).

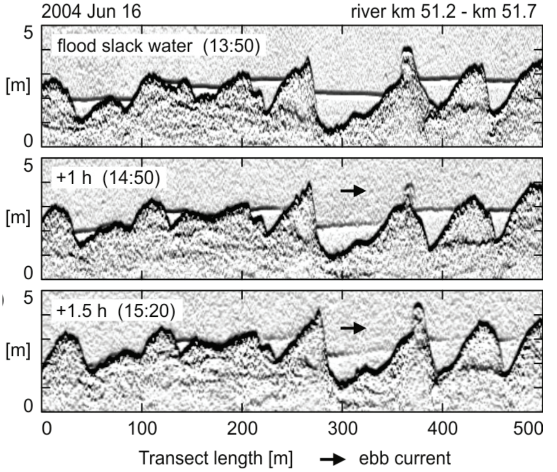

Figure 22: Observed lutoclines in the troughs of dunes detected by means of a hydroacoustic sub-bottom profiler (courtesy of M. Becker, see also SCHROTTKE et al. (2006) and BECKER (2011). The first panel shows fully built-up lutoclines in the troughs during slack water. The second (1 h after slack water) and third (1.6 h after slack water) panels indicate weaker lutoclines during the ebb phase, which incline with increasing ebb currents. The fluid mud begins to mix with the overlying water column due to increasing ebb currents.

Moreover, both measured data and simulation results show that the lutocline inclines according to the direction of the currents (see the second and third panels of Fig. 22 and Fig. 25).

Although the water body is resolved in a 2-D system, the application demonstrates that phenomena such as the development of fluid mud layers in the troughs of dunes can be simulated. Mixing and settling of the near-bottom fluid layer is obtained by a 3-D resolution. Further improvements may be achieved by applying a more sophisticated turbulence model and by modeling the water body in 3-D.

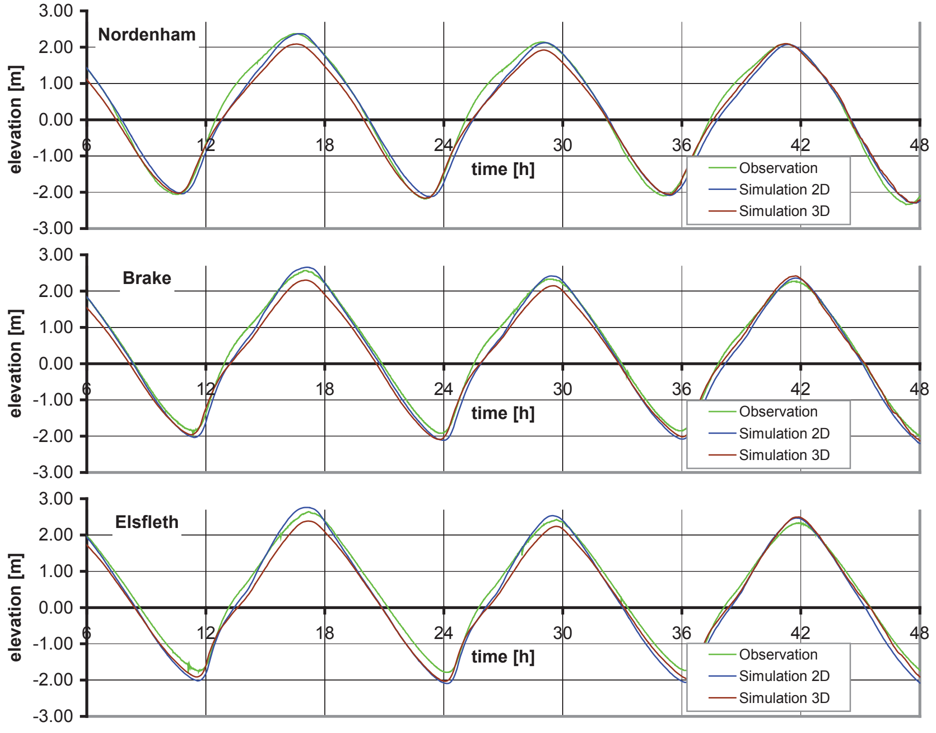

Figure 23: Time series of water level at Nordenham, Brake and Elsfleth. The simulated water levels result from a 2-D simulation considering a single isopycnic layer (blue) and a 3-D simulation with five density layers (red). The observed data were provided by tide gauges (green).

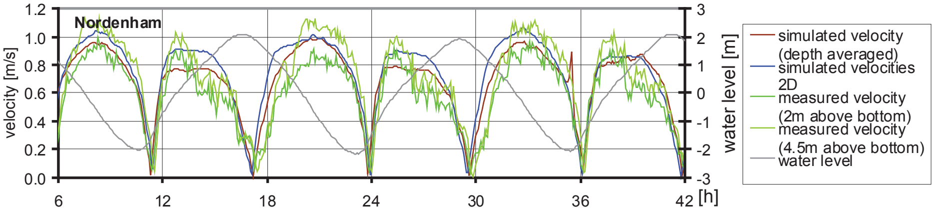

Figure 24: Time series of velocity at the Nordenham position. The simulated velocities shown here are depth-averaged values; the measured data were collected at a specific depth.

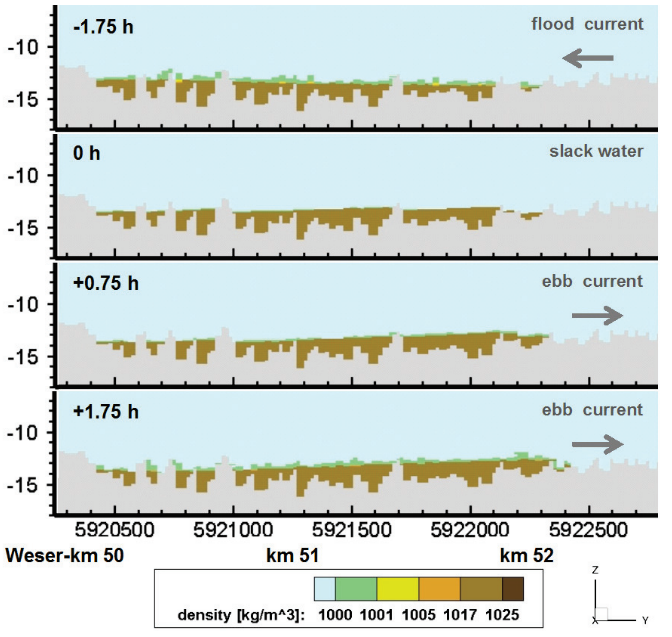

Figure 25: Simulated mud layers in dune fields. The plots depict the density distribution for different phases of the tide in the river section from Weser-km 50 to 52. The longitudinal section in the shipping channel is indicated in Fig. 21. High velocity gradients between the water body and the mud layer induce mixing at the interface during the flood and ebb tide (first and fourth panel). The lutocline inclines according to the direction of the currents.

6 Conclusions

The aim of the MudSim project was to further develop numerical simulations of the flow behavior, transport, formation and resuspension of fluid mud in order to improve our understanding of fluid mud dynamics. This has been realized by a hydrodynamic model in isopycnal coordinates. The extension of an existing numerical model contributes to achieving a better understanding of fluid mud dynamics in coastal areas, estuaries and harbors.

The following section summarizes the achievements of the project and outlines recommendations for future research.

6.1 Achievements

The occurrence of fluid mud is observed in single layer as well as in multi-layer systems. The fluid mud body can vary from a few decimeters to several meters in thickness. These fluid mud layers lead to stable stratified flow in a low-concentration suspension when mixing is insignificant. High density gradients then exist between the water body and the fluid mud body. For this reason, an isopycnal numerical approach proves to be promising. The developments for the simulation of fluid mud dynamics are therefore based on a **three-dimensional hydrodynamic isopycnal model** approach.

The characteristics of the numerical method are as follows:

- isopycnal discretization permits a three-dimensional resolution of the fluid mud body with a low degree of discretization and little computational effort
- the isopycnal approach resolves the density stratification and the velocity profile within the fluid mud body
- layer thicknesses vary with the transition to different states of suspension, thereby permitting a simulation of the formation, resuspension, settling, and advective and gravitational transport of fluid mud
- the numerical implementation is based on a numerical discretization in the vertical direction by ρ-layers, and in the horizontal direction by unstructured grids and in time
- interaction of the isopycnal layers is realized on the basis of momentum transfer, vertical mass transfer and interfacial shear stresses

Vertical transport processes, which lead to the formation and resuspension of fluid mud, are mainly governed by hindered settling and entrainment. This requires the thickness of the density layers to vary with time and in accordance with instantaneous mass transport rates. Vertical mass transfer between isopycnal layers was derived for a three-dimensional system (x,y,ρ) on the basis of volume and mass conservation in WEHR (2012). The functionality of the extended numerical model was evaluated by different system studies presented in WEHR (2012).

A method is described for the integration of **non-Newtonian flow behavior** in a numerical model based on the Reynolds-averaged Navier-Stokes equations. The model simulates the non-Newtonian flow of fluid mud by introducing a rheological viscosity to parameterize the rheology according to shear impact and particle concentration. The rheological model describes the structural break-up and recovery of aggregates in a mud suspension (Section 4.2). The rheological viscosity is no longer a constant such as the molecular viscosity but is now a time-dependent and process-descriptive parameter. It is possible to apply different rheological models in this way. Such rheological parameterizations can also be used for the transport of mud in pipelines or the determination of nautical depth.

The isopycnal numerical model described above was extended by using the rheological viscosity in the same way as the turbulent viscosity in the Reynolds-averaged Navier-Stokes equations. Although the rheological viscosity determines both the Newtonian and non-Newtonian flow behavior of suspensions, the character of the differential momentum equations remains unchanged. Internal friction and interfacial shear stresses are now related to the rheological behavior of the mud suspension and are taken into account in the numerical solution.

The rheology of fluid mud is described as a viscoplastic shear-thinning fluid by applying a parameterized Worrall-Tuliani model (Section 4.2). This model considers a yield stress and the break-up and recovery of the microscopic structure (aggregates of cohesive sediments). These parameters are calculated as a function of the shear impact and solid volume concentration. The entire water column is modeled by adopting this approach, as it not only covers the non-Newtonian behavior of high-concentration suspensions, but also the Newtonian behavior of low-concentration suspensions and clear water. The shear-thinning behavior has been studied phenomenologically and it was possible to reproduce shear-thinning behavior in a study of the Ems river section from Rhede to the weir at Herbrum, in which stratified flow in a tidally-influenced system was investigated (Section 5.2). The influence of rheological behavior on high-concentration flow was analyzed in a study of flow on an inclined plane (Section 5). This effect was compared to the influence of gravitational forcing due to density differences, which has proved to be the dominant process for this test case.

Fluid mud dynamics under the influence of tidal currents has been investigated for two model domains: the Ems estuary (Section 5.2) and the Weser estuary (Section 5.3):

- Fluid mud formation, advective and gravitational transport, and resuspension are periodic processes in tidal systems.
- Highly-stratified flow develops during slack water at high tide and during the ebb tide in the shipping channel.
- The rheological viscosities determined as a function of the shear rate and density yield plausible results and influence the velocities of the stratified flow.
- A qualitative comparison of simulated fluid mud formation and the observed development of the lutocline in the river sections of the Ems and Weser estuaries shows similar results.

These applications demonstrate that the developed numerical model approach permits the simulation of three-dimensional fluid mud dynamics. The developed numerical model is capable of simulating fluid mud dynamics in systems such as harbor basins and river sections where high-concentration flow and fluid mud formations dominate the system. Such simulations can contribute to classical 3-D hydrodynamic and morphodynamic simulations of estuarine systems for evaluating sediment transport analysis and maintenance strategies.

6.2 Recommendations

The presented numerical model applies an appropriate resolution of the fluid mud body using isopycnal layers. Each isopycnal layer represents a single phase fluid/suspension with a specific particle concentration and specific rheological properties. The isopycnal layer may become very thin or even attain zero thickness depending on the transport rate and the development of cohesive mud suspensions.

The three-dimensional isopycnal model is applied to the entire water column from the consolidated bed to the free surface in the presented model applications. Simulations of the dynamics of highly-concentrated mud suspensions show reasonable results. However, the numerical approach limits the flow to stable stratification. This assumption does not always apply in highly-turbulent flows with suspended sediments. In particular, the presence of suspended sediment transport and baroclinic processes may result in an **unstable stratification** in estuaries. Further investigations on the simulation of the low-concentrated water body are necessary to permit the comprehensive modeling of estuarine systems. A possible solution to achieve a more sophisticated model of the water body is described in the following.

A solution can be achieved by coupling the isopycnal model with an existing and established three-dimensional hydrodynamic model such as UnTRIM (CASULLI and WALTERS 2000; CASULLI and LANG 2004), Telemac (HERVOUET and BATES 2000; ELECTRICITE DE FRANCE 2000) or Delft3D (LESSER et al. 2004; GERRITSEN et al. 2007).

The isopycnal numerical model then functions as a module representing the fluid mud body. The suspended sediment and salt transport simulation is performed by the hydrodynamic model. The isopycnal fluid mud module would then become active in the case of fluid mud formation once the threshold from Newtonian to non-Newtonian flow or a specific mud concentration is exceeded. This module would only be activated in model domains with cohesive sediment accumulations, thereby reducing computational effort for large model domains with different transport regimes such as those in estuaries. This concept will require further developments, research and software engineering for the comprehensive modeling of estuarine systems.

Coupling of the isopycnal fluid mud model with an advanced hydrodynamic model appears to be a promising and practicable solution for modeling fluid mud dynamics in practical applications, e.g.in estuarine environments. This still permits the use of optimal vertical discretization for each particular problem - isopycnals for fluid mud dynamics and z- or σ-layers for low-concentration hydrodynamics, applied independently. In this case, communication between the models will require further investigations on software engineering as well as the description of physical processes. One aspect of the latter is outlined in the following.

This work focuses on the interfacial and internal friction resulting from rheological behavior. However, the internal shear stresses are also influenced by turbulence.

In nature, fluid mud flows become laminar as the turbulence is destroyed due to density stratification. On the other hand, the rheological behavior changes from non-Newtonian to Newtonian as the mud concentration decreases in the water body, with the possible creation of turbulence. Turbulence interacts with the suspended particles due to turbulence damping and buoyancy effects, which in turn influence the settling velocity. Thus, in the high-concentration, stratified areas, the flow behavior is characterized by the rheological viscosity whereas the turbulent viscosity is dominant in low-concentration, mixed areas. Both **rheology and turbulence** are modeled with a similar conceptual model as described above. They are taken into account through a viscosity and result in a deceleration of the average velocity with increasing viscosities (internal friction). However, their physical effect is contradictory. Whereas the rheological viscosity leads to laminar and stratified flow as its magnitude increases, increasing turbulent viscosity, on the other hand, intensifies turbulent mixing and may cause unstable stratifications. Accordingly, research on the interaction between rheological and turbulent viscosity will be important for progressive fluid mud and suspended sediment transport modeling. The focus should be on the transitional area between fluid mud and dilute suspension as well as on the formation process and the resuspension of fluid mud as both quantities may reach considerable magnitudes during resuspension or entrainment. A general approach using the viscosity should combine rheology and turbulence modeling and take account of the solids concentration, shear conditions and structural mechanisms (e.g. flocculation) for the overall water body.

The improvement of the turbulence model will also affect the entrainment of fluid mud which is basically induced by turbulent interfacial shear stresses. Another aspect worth investigating is the influence of fluid mud formation in large areas and of sizable thickness on the internal friction in estuarine systems. Turbulence will be damped during periods of high stratification and internal friction is built up by the rheological viscosity. The shear-thinning behavior of fluid mud may then lead to relatively small rheological viscosities once the fluid mud moves with the tidal currents. Compared to the magnitudes of the rheological viscosities, the turbulent viscosities can reach much higher magnitudes in a turbulence-dominated system. This aspect and the reduced bottom friction of the water body flowing above the fluid mud body may lead to a larger tidal range in estuaries (see description of tidal dynamics in MALCHEREK (2010).

Further process-based improvements and validation of the fluid mud model will require additional comparisons with laboratory studies and **field measurements**. Observation of the development of fluid mud involves measurements not only of the lutocline movement but also of the density stratification below the lutocline and the velocity distribution inside the fluid mud body. These types of measurement are subject of ongoing research into highly dynamic systems as it is difficult to perform measurements in high-concentration suspensions. These measurements should allow to relate specific observed phenomena to physical processes. In tidal systems, the physical processes are strongly related to the tidal cycle. Therefore, it is necessary to obtain continuous information in tidal systems (e.g. at least one tidal cycle).

The characteristic flow behavior of fluid mud can be reproduced by considering a mud suspension comprising only cohesive sediments and water. However, there are several aspects from which we can learn and gain a better understanding of the flow behavior under different conditions. Some of these aspects are **biology**, **dissolved oxygen** and **mud-sand mixtures** (considering grains larger than 63 μm).

7 Acknowledgement

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