6th International Congress on Scour and Erosion

Fully Resolved Simulation of erosion of circular particles in Couette flow

Romuald VERJUS [1], Sylvain GUILLOU [1]

1 Laboratoire Universitaire des Sciences Appliquées de Cherbourg (EA4253), Université de Caen, Esix,

Site Universitaire de Cherbourg, BP 78, 50130 Cherbourg-Octeville, France

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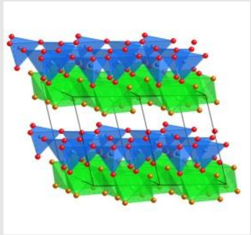
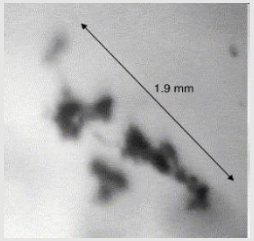
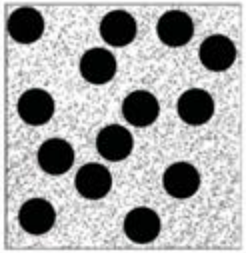
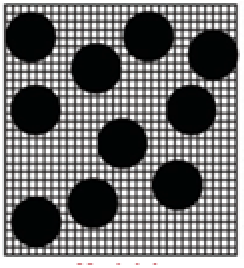
1. Sedimentary transport: scales and numerical strategies

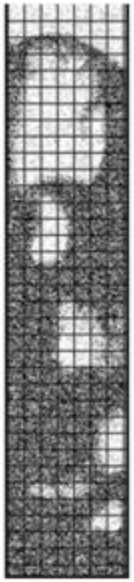
molecular | particle | suspension | estuary

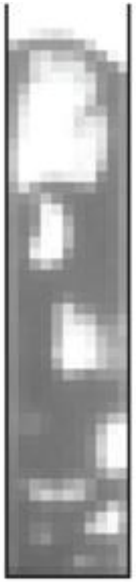


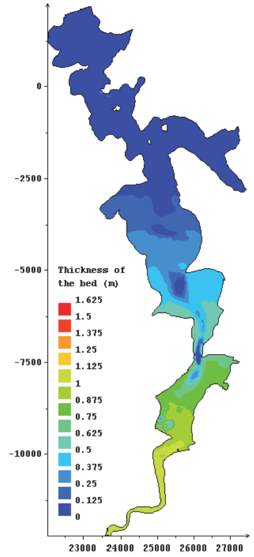












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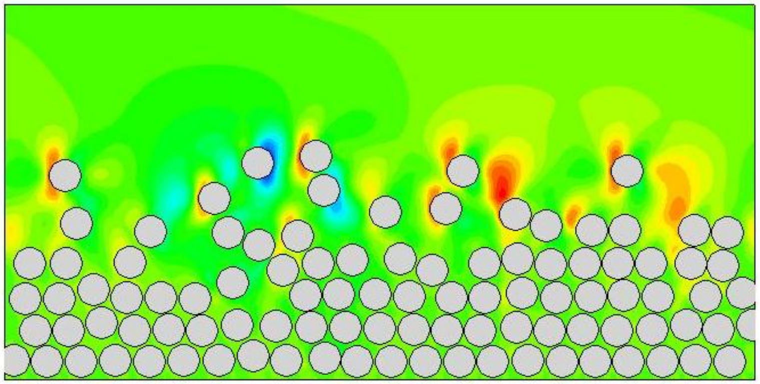
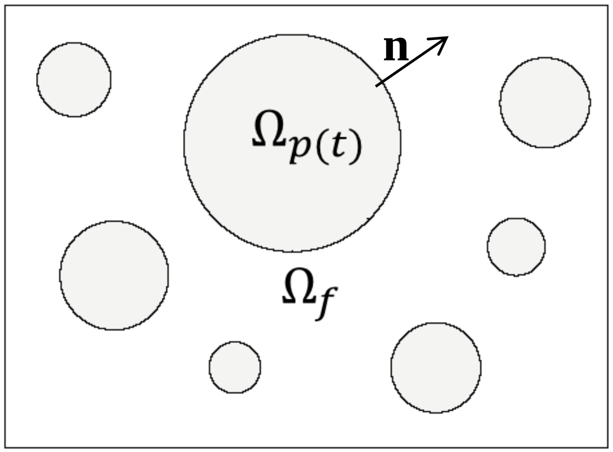
1. Sedimentary transport: Initiation of the motion of a grain in laminar flow



- Need of a detailed description of physical variables of all particles
- Influence of the wall and the nearby particles on the initiation of the motion of a grain even at low Reynolds number

2. Fully resolved models for particulate flows: Constitutive equations



Fluid equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} = \frac{\nabla^2\mathbf{u}}{\mathrm{Re}} - \nabla p$$

$$\nabla.\mathbf{u} = 0$$

$$\boldsymbol{\sigma} = -p\mathbf{I_d} + 2\mu\mathbf{D(u)}$$

Rigid body motion

$$\mathbf{u} = \mathbf{U}_i + \boldsymbol{\omega}_i \times \mathbf{r}_i$$

Particulate equations

$$M_i \frac{d\mathbf{U}_i}{dt} = \mathbf{F_H} + \mathbf{F_c} + M_i\mathbf{g}$$

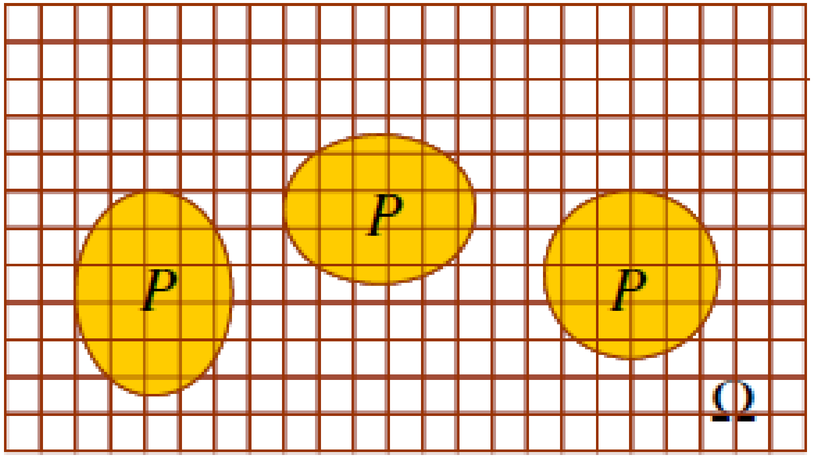
$$\frac{d(J_i\boldsymbol{\omega}_i)}{dt} = \mathbf{T_H} + \mathbf{T_C}$$

$$\mathbf{F_H} = \int_{\partial P_i} \boldsymbol{\sigma}\mathbf{n}\,dx$$

$$\mathbf{T_H} = \int_{\partial P_i} \boldsymbol{\sigma}\mathbf{n} \times (\mathbf{x} - \mathbf{x}_i)\,dx$$

– **fluid-particles coupling ? :** Arbitrary Lagrangian Eulerian Schemes (ALE) (Hirt, Hu et al.), Fictitious Domain Method (Glowinski, Hu, Patankar, Minev), Immersed Boundary Methods (Peskin, Ulhmann, Mittal)

2. Fully resolved models for particulate flows: Fictitious-Domain method



- Fixed mesh
- Simpler computational domain (particles+fluids)
- Rigid body motion is imposed inside particles with an additional constraint

FD formulation (Yu & Shao, 2007)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u} = \frac{\nabla^2 \mathbf{u}}{\mathrm{Re}} - \nabla p + \boldsymbol{\lambda}$$

$$\nabla.\mathbf{u} = 0$$

$$(\rho_r - 1)V_d\left(\frac{d\mathbf{U}}{dt} - \frac{1}{Fr}\frac{\mathbf{g}}{g}\right) = -\int_P \boldsymbol{\lambda} dx + \mathbf{F_c}$$

$$(\rho_r - 1)\frac{d(\mathbf{J_d}.\boldsymbol{\omega})}{dt} = -\int_P \mathbf{r} \times \boldsymbol{\lambda} dx + \mathbf{T}_c$$

$$\mathbf{u} = \mathbf{U} + \boldsymbol{\omega} \times \mathbf{r}$$

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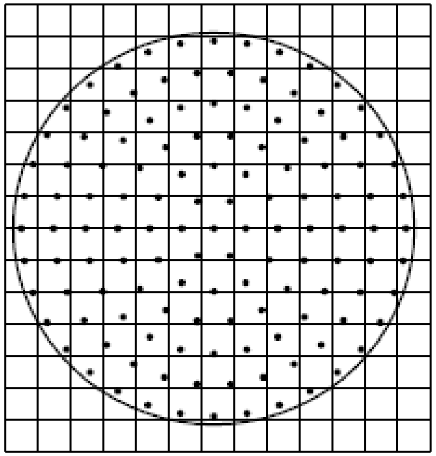
2. Fully resolved models for particulate flows: Algorithm

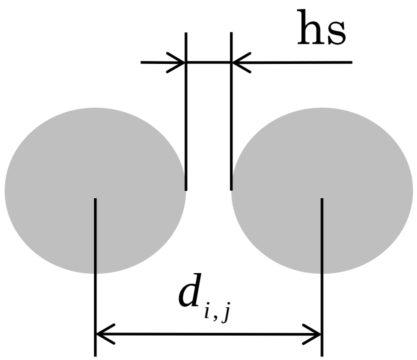


1. **Fluids equations** : finite difference method on a staggered grid and a projection technique (Guillou and Makhloufi, 2007) to obtain u and p
2. **Resolution of the particle's equation** : Calculus of the new particle's velocities and the new pseudo-body force
3. **Correction of the fluid phase**: Correction of the fluid velocities with the new pseudo body force

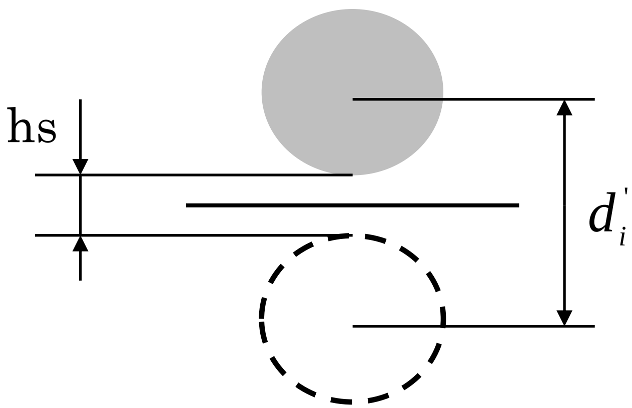
Validation : flow over a fixed cylinder, flow over a square particle, sedimentation of one and several particles at differents Reynolds numbers, sedimentation of square and rectangular particles at different Reynolds numbers… (Verjus et al. (2011), Verjus and Guillou (2012))

2. Fully resolved models for particulate flows: Contact modeling

Wan and Turek (2007)

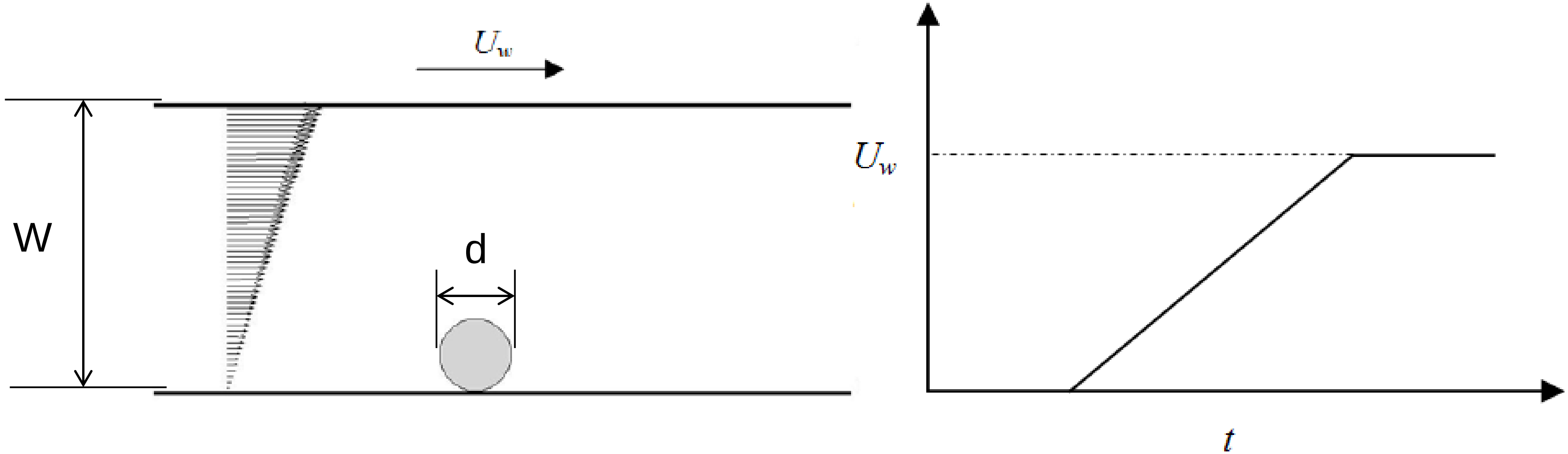


$$
\mathbf{F_C^{i,j}} = \begin{cases} \frac{1}{\varepsilon_p}(\mathbf{X_i} - \mathbf{X_j})(R_i - R_j - d_{i,j}) & \text{for} \quad d_{i,j} \le R_i + R_j \\ \frac{1}{\varepsilon_p}(\mathbf{X_i} - \mathbf{X_j})(R_i + R_j - d_{i,j} + hs)^2 & \text{for} \\ 0 & \text{for} \quad d_{i,j} > R_i + R_j + hs \end{cases}
$$



$$
\mathbf{F_W^{i}} = \begin{cases} \frac{1}{\varepsilon_w}(\mathbf{X_i} - \mathbf{X_i^{'}})(2R_i - d_i^{'}) & \text{for} \quad d_i^{'} \le 2R_i \\ \frac{1}{\varepsilon_w}(\mathbf{X_i} - \mathbf{X_i^{'}})(2R_i - d_i^{'} + hs)^2 & \text{for} \quad 2R_i \le d_i^{'} \le 2R_i + hs \\ 0 & \text{for} \quad d_{i,j} > 2R_i + hs \end{cases}
$$

3. Numerical results: one cylindrical particle in Couette flow



Numerical parameters:

$$\dot{\gamma} = \frac{U_w}{W} \qquad \mathrm{Re}\,\gamma = \frac{\dot{\gamma} d^2}{\upsilon} \qquad \rho_r = \frac{\rho_s}{\rho_f}$$

$$0.01 < \mathrm{Re}\,\gamma < 5 \qquad 1.00001 < \rho_r < 1.1 \qquad W = 8d$$

$$d/h = 16 \qquad 0.0005 < \Delta t < 0.005$$

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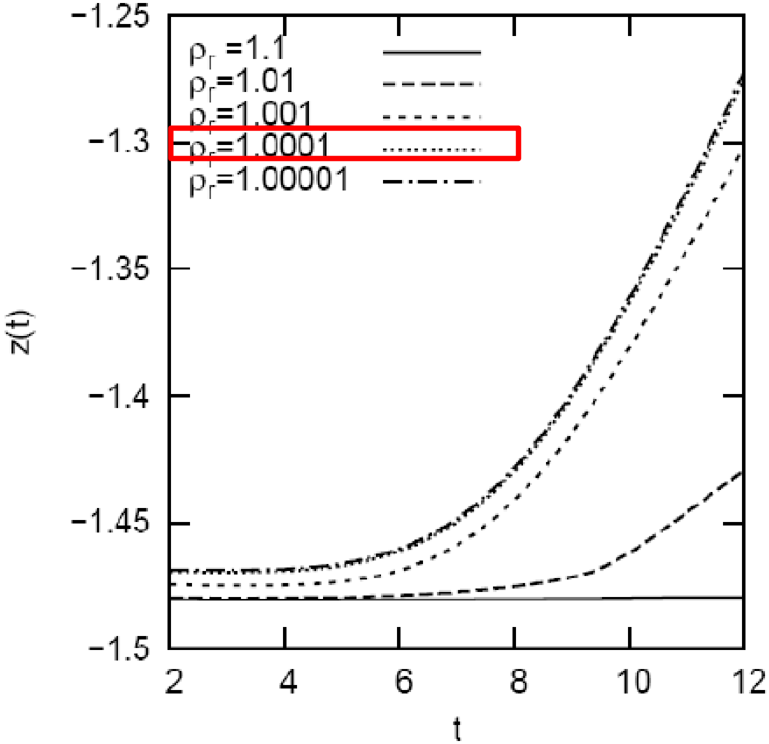
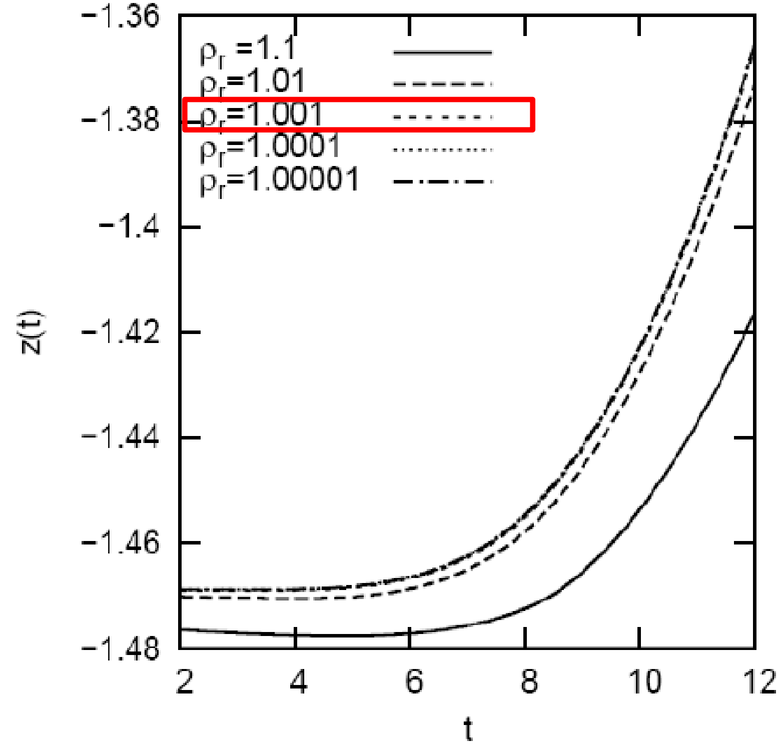
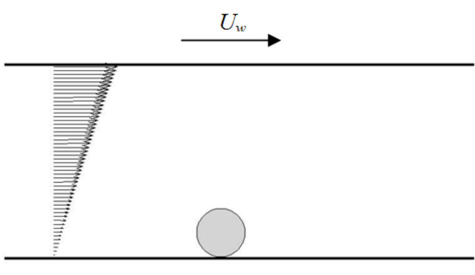
3. Numerical results: one cylindrical particle in Couette flow

$\mathrm{Re}\,\gamma = 0.05$



$\mathrm{Re}\,\gamma = 0.5$

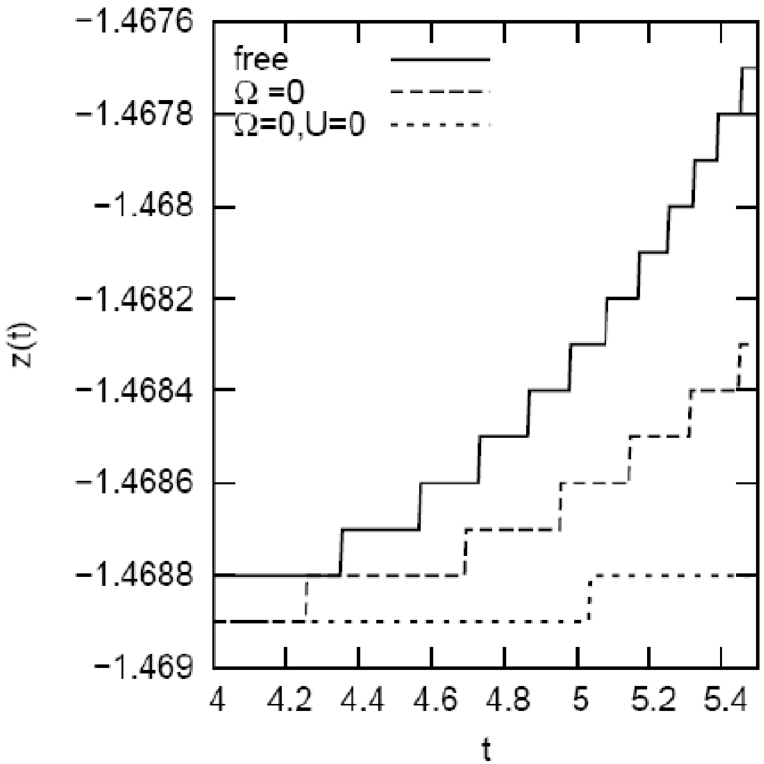




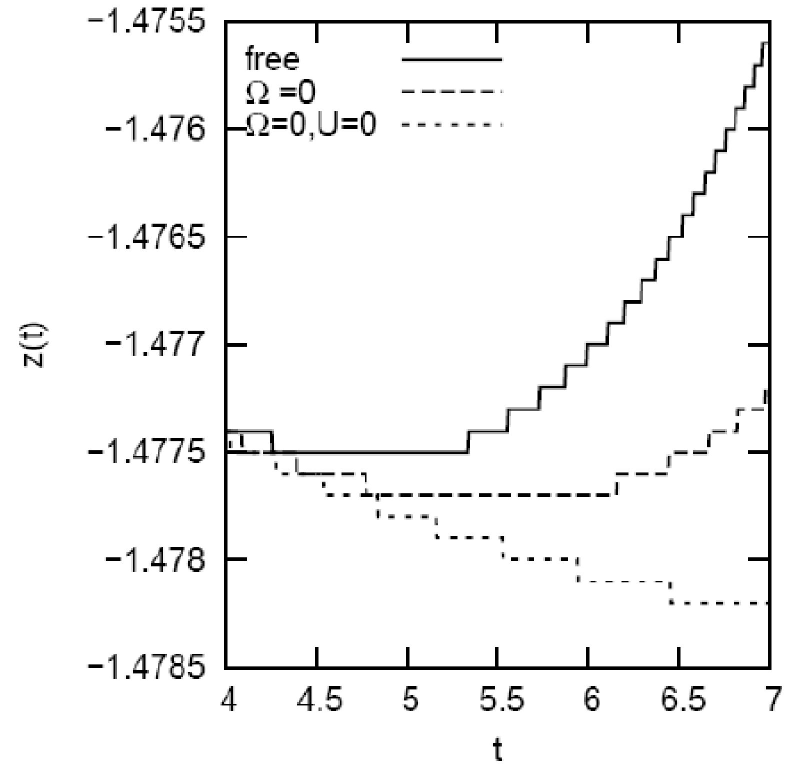
3. Numerical results: one cylindrical particle in Couette flow

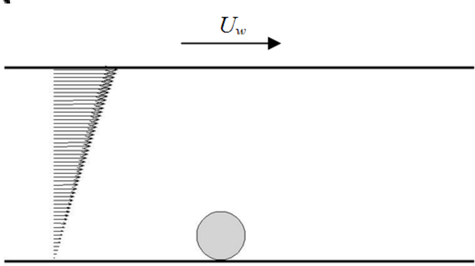
$Re\,\gamma = 0.5$

$\rho_r = 1.001$

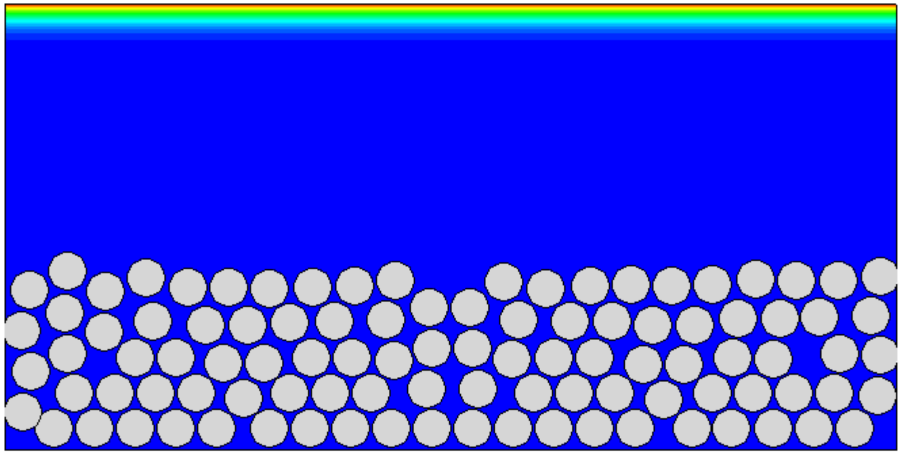


$\rho_r = 1.1$





3. Numerical results: erosion of 100 particles



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4. Conclusions and perspectives

Conclusions:

• Development of a direct numerical model for particulate flows

•First attempts of the detachment of a sliding particle in Couette flow at low shear Reynolds number (influence of the different processes)

• simulation of 100 particles in Couette flow

Perspectives:

• applied the code for more particles

• adding the friction force in the contact model

• make 3D realizations and compare them with experimental results

Thank you for your attention

3. Numerical results: benchmark of the migration of a neutrally buoyant cylinder in Couette flow